The mysterious appearance of objects

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Abstract. Moving from some reflections on the empirical practice of measurement and on the nature of visual perception, we present a constructivist approach to objects. At the basis of such approach there is the idea that all we may know about what is out there is always mediated by some sort of apparatus, being it a measurement instrument or our perceptual system. Given this perspective, some questions are in order: how are objects identified and re-identified through time from the outcomes of apparatuses? How can we distinguish different (kinds of) objects? Our first goal will be to make explicit the mechanism used to build objects from the apparatuses' outcomes, emphasizing what are the ontological and representational problems this construction faces. A second contribution will be a preliminary discussion of some possible ways to distinguish social objects, the constructed objects *par excellence*, from physical ones. A third contribution will be an attempt to make a bridge between two scientific communities that rarely seek contact or mutual recognition: that of formal ontologies and that of formal concept analysis.

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The suspicion that the world, or, if you prefer, reality, or, simply, what is 'out there', is not how it appears to be, is not just philosophical or fictional; it is something deeper, a doubt that we had at least once in our life. *The Matrix* movie can be seen as the popular version of Arthur Schopenauer's *Veil of Maya*: our eyes are clouded by illusion (*maya*) that does not allow us to see objects in themselves. The mere possibility that objects could mysteriously appear to us only through the veil, is enough to bring us into an uncomfortable state of incertitude about what exists out there.

Tearing this veil away, showing reality as it is, is a goal shared by many scientists. In this enterprise, sciences—physical sciences, *in primis*, but also psychological and social sciences—ascribe a central role to *measurement*. The monumental work of Krantz, Luce, Suppes & Tversky [9] seems to dispel any doubt on the level of understanding and agreement sciences have reached about measurement. However, according to Kyburg Jr [11] the majority of the literature on measurement is concerned with specific technical problems that regard the construction of numerical structures representing given attributes (governed by some axioms). Less attention has been devoted to, and less agreement achieved on, foundational aspects of measurement, e.g. induction (of the governing axioms) and scientific inference, or realist vs. operationalist (conventionalist) view on measures. The latter debate is of interest for us, however we will not enter into it here. Rather, we will show that a constructivist position on objects, the acceptance of the veil, could dissolve some mysteries, enlighten some classical problems by forcing the emergence of hypotheses that often remain implicit in the *empirical* practice of measurement.

From an empirical point of view, measurement can be seen as "a physical interaction, set up by agents, in a way that allows them to gather information. The outcome of a measurement provides a representation of the entity (object, event, process) measured, selectively, by displaying values of some physical parameters that—according to the theory governing this context—characterize that object" ([25], p.179). On the basis of the measures, objects[1] are placed in a (structured) space of possible states that allows their classification and comparison. In this perspective, the out there is *mediated* by apparatuses: the observer has direct access only to the measurement outcome[2], not to the input of the sensor, "physics has nothing to say about a possible real world lying behind experience (...) the task of physics as of all science is found in the coherent *description* of experience" ([12], p.2). A whole new world, a world of 'public hallucinations' emerges: "the microscope *need not* be thought of as a window, but is *most certainly* an engine creating new optical phenomena. It is accurate to say of what we see in the microscope that we are "seeing an image" (...) that could be *either* a copy of a real thing not visible to the naked eye or a mere public hallucination" ([25], p.109).[3] Analogously, studies in the cognitive sciences show that the link between our sensations and the out there is far from obvious: "the visual system does not have direct access to facts about the environment; it has access only to facts about the image projected onto the retina. That is, an organism cannot be presumed to know how the environment is structured except through sensory information. (...) The confusion that underlies the experience error is typically to suppose that the starting point for vision is the distal stimulus rather than the proximal stimulus (...) the structure of the environment is more accurately regarded as the *result* of visual perception rather than its starting point" ([15], p.191). But, why does the perceptual system organize perceptual information in a certain way rather than another? Why do scientists rely on specific apparatuses to discover the out there? Palmer [15] argues that, in the case of perception, one could appeal to evolutionary reasons: certain organization patterns establish a sort of *coordination* with the environment that makes possible the organisms' survival. Scientific apparatuses and measurement procedures develop hand in hand with a theoretical framework and they aim at "the reliability of the predictions concerning these [procedures] and their correlation with other measurement procedures derived from the mature theory in which they are theoretically embedded." ([25], p.124).

Following these ideas, in order to 'know' the out there, the observer must begin from the apparatuses' outcomes, the public hallucinations, the sensations, the proximal stimuli. This is also our starting point. In [13], extending the *empirical measurement theory* [6], we showed that ascribing a central role to instruments and their description, the empirical attribution of properties to objects at a given time—the classification of objects at a time—can attain an *inter-subjective* dimension—via calibration of instruments and conventional standards of measurement—without presupposing an objective reality. Here we want to explore a more radical position. A position that, differently from the formal theories on measurement we are aware of, does not presuppose the *a priori* identification of objects under-measurement.[4] We want to move the observer '*inside*' the instrument. From the inside, the observer does not see what the instrument is connected to, "gauges do not supply (...) information about these external connections (...) [about]

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[1] In the following we will concentrate on objects even though the framework can be extended to take into account events, processes, etc.

[2] We assume that the internal states of the apparatuses are empirically discernible without errors.

[3] van Fraassen is making a stronger point here: in some cases, the instruments of measure *produce* the phenomena that sciences use to make sense of what is out there.

[4] Actually, also [10,16]—two approaches concerned with the ontological foundations of observation and measurement in a constructivist context—do not explicitly consider where objects come from.

which tire (if any) the gauge represents" ([3], p.23). Furthermore, an apparatus does not necessarily interact with one 'single' object; for instance, a camera collects information about whatever is in front of the lens.[5] Such a radical position raises some questions. How are objects (under measurement) identified and re-identified through time from the outcomes of apparatuses? How are different (kinds of) objects distinguished? These questions are not new in physical sciences: "by what principles shall we select certain data from the chaos, and call them all appearances of the same thing?" ([20], p.86). Similar questions are considered in cognitive science. How is vision able to structure the receptors' outputs, e.g. luminance and color of retinal positions, in meaningful surfaces, objects, or groups and track them across time [17,15]?

As observed by Pylyshyn [17], these questions bring the observer into a *vicious circle*. To decide in virtue of what 'that' is an object of a certain kind, one needs to refer to some properties, e.g. the mass or the spatio-temporal trajectories. To define what it means to have these properties one needs to refer to other properties, etc. Similarly, to understand which is the tire the gauge is connected to, one needs additional information whose collection must rely on a second apparatus. But to understand that the second apparatus is connected to both the gauge and the tire, one needs a third apparatus, etc. To avoid the vicious circle, the recursion needs to stop somewhere. Pylyshyn hypothesizes that, to pick up and track objects, our perceptive system relies on a *non conceptual* mechanism. This mechanism is not a *classification* system, one does not know *what* (what kind of objects) is selected, even though one knows *which* object it is, but this is enough to have the possibility to attribute properties to 'that'. Matthen [14] considers an *auto-calibration process* founded on our ability to predict (in an inductive way) the behavior of the environment. The authors support their answers with experimental results which we don't have the competences and the space to evaluate. However, these answers are not very satisfactory from the point of view of measurement. First of all, measurement apparatuses don't have the 'innate' ability of carving-up the world into objects (and properties). Outputs need to be explicitly selected and manipulated to identify objects. Second, reliable predictions seem not enough to select a way of (re-)identifying objects that is unique. Actually, this seems also true for our perceptual system. As some Gestaltists have tried to demonstrate, there are various alternative ways in which the visual system may arrange and organize into a coherent image a single perceptual experience.

Which possibility remains for the physicists, or, more generally, for those who start from data? Here we will explore the idea that objects (and properties) are conventionally built. Again, this is not a new idea. Russell [20] claims that physics considers the series of data which obey the law of physics and the things that render all sense-data calculable from a sufficient collection of data. Goodman [8] suggests that the re-identification of physical objects relies on the re-identification of qualia that, ultimately, is a matter of *decrees*, the result of some (conventional) choices. Quine [18] talks of *posits*, objects just built for convenience: "As an empiricist I continue to think of the conceptual scheme of science as a tool, ultimately, for predicting future experience in the light of past experience. Physical objects are conceptually imported into the situation as convenient intermediaries—not by definition in terms of experience, but simply as irreducible posits comparable, epistemologically, to the gods of Homer" ([18], p.44). This move raises an additional question. If the physical world is conventional, what is the difference

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[5] In this case, the *segmentation* of the output into sub-outputs relative to single objects must be considered.

with respect to the social world? The social world too is governed by laws that allow for prediction and objects have a conventional nature and a coordination role.

Our first goal will be to make explicit the general mechanism used to build objects from the outcomes of apparatuses, emphasizing what are the ontological and representational problems this construction faces. A second contribution will be a preliminary discussion of some possible ways to distinguish social objects from physical ones (see Section 4). A third contribution will be an attempt to create some links between two scientific communities that rarely seek contact or mutual recognition: that of formal ontologies and that of *formal concept analysis*. In the following we will try to apply the theoretical tools of both traditions in order to approach the questions listed above.

1. Formal concept analysis and outcomes of apparatuses

Our main objective is to investigate alternative ways of building objects from *observations*—from the descriptions of the outcomes of the apparatuses one disposes of[6]—making explicit the *conventions* on which this construction is based. Some of these conventions are necessary to allow the inter-subjective sharing and communication of observations, their diachronic comparison, etc. With respect to measurement instruments, they consist in establishing reference standards, units of measurement, calibration and operational procedures, etc. These aspects are discussed in detail in [13]. Here we focus only on the conventions necessary to build objects from observations that are then *assumed* as inter-subjective, sharable, and shared. This means that if observations are represented as rows in a table where the columns are (multi-valued) attributes of the outcomes of some apparatus—a quite natural choice indeed—then we assume that the 'meaning' of these attributes and their values is already established, allowing observations taken at different times or by different apparatuses to be compared, while maintaining the perspective on the out there from the standpoint of given types of apparatuses.

The idea of representing observations as rows characterized by attributes evokes the *formal concept analysis* (FCA), a field in applied mathematics, started by Wille, with the goal of precisely defining the notions of *concept* and *conceptual hierarchy* to apply mathematical methods to conceptual data analysis and knowledge processing [7]. Because of our focus on the re-identification through time, we will also consider the *temporal concept analysis*, an extension of FCA introduced by Wolff in [26,27].[7]

A *formal context* $\langle O, A, \mathit{IN}\rangle$ consists of a set of (formal) *objects* O, a set of (formal) *attributes* A, and an *incidence* relation IN between O and A: $\langle o, a\rangle \in \mathit{IN}$ can be read as "the object o has the attribute a". A *formal concept* of the context $\langle O, A, \mathit{IN}\rangle$ is a pair $\langle E, I\rangle$ with $E \subseteq O, I \subseteq A, E' = I, I' = E$, where $E' = \{a \in A \mid \langle o, a\rangle \in \mathit{IN}$ for all $o \in E\}$ and $I' = \{o \in O \mid \langle o, a\rangle \in \mathit{IN}$ for all $a \in I\}$. E is called the *extent* and I the *intent* of the concept $\langle E, I\rangle$. A formal concept $\langle E, I\rangle$ abstracts from single objects by collecting in E all the objects possessing all the attributes in I. Table 1.a illustrates a formal context with $O = \{o_1, o_2, o_3, o_4\}$, $A = \{t_1, t_2, t_3, t_4, 1, 2, B\}$, where '×' indicates the IN relation, e.g. $\langle o_1, t_1\rangle \in \mathit{IN}$. In this example, $\langle\{o_1, o_3\}, \{1, B\}\rangle$, $\langle\{o_2, o_4\}, \{2, B\}\rangle$, and $\langle\{o_1\}, \{t_1, 1, B\}\rangle$ are formal concepts, but not $\langle\{o_1, o_2, o_3\}, \{B\}\rangle$ or $\langle\{o_1\}, \{t_1\}\rangle$.

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[6] Clear examples of apparatuses are measurement or perceptual systems. However, what follows can be easily adapted, for instance, to systematic procedures to collect sociological or economical data.

[7] Where not explicitly noted, we will refer to [7] for the formal details.

Table 1. Some examples of formal contexts

	t_1	t_2	t_3	t_4	1	2	B
o_1	×				×		×
o_2		×				×	×
o_3			×		×		×
o_4				×		×	×

(**a**)

	tme	*pos*	*col*	1	2
o_1	t_1	1	B	■	
o_2	t_2	2	B		■
o_3	t_3	1	B	■	
o_4	t_4	2	B		■

(**b**)

	tme	*pos*	*col*	1	2
o_1	$t_1 \mid t_2$	1	B	■	■
o_2	$t_1 \mid t_2$	2	B		
o_3	$t_3 \mid t_4$	1	B	■	■
o_4	$t_3 \mid t_4$	2	B		

(**c**)

A *many-valued context* $\langle O, A, V, IN\rangle$ consists of a set of objects O, a set of attributes A, a set of *values* V, and an *incidence* relation IN between O, A, and V such that $\langle o, a, v\rangle \in IN$ and $\langle o, a, v'\rangle \in IN$ then $v = v'$. We write $a(o) = v$ instead of $\langle o, a, v\rangle \in IN$ for "the attribute a has the value v for the object o". A many-valued context can be translated in a (one-valued) formal context through *scaling*, i.e. by mapping *ranges* of attribute values into a one-valued attribute. Finite many-valued contexts can be translated into one-valued contexts without loss of information by assuming a *plain* scale that separately maps each attribute value to a single attribute. The many-valued context in Table 1.b, $A = \{tme, pos, col\}$ and $V = \{t_1, t_2, t_3, t_4, 1, 2, B\}$, can be plainly 'scaled' into the one in Table 1.a (the last column can be omitted for the moment). We will consider only finite many-valued contexts.

A *conceptual time system* (CTS) [26] on objects O consists of two (scaled) many-valued contexts, a *temporal* one (the *time part*) and a *non-temporal* one (the *event part*) both with the same objects O. The time part provides a complete temporal description of objects in terms of time attributes here reduced to a unique one, *tme*, whose values refer to an absolute clock.[8] The event part contains all the other attributes. The *states* of the outputs of the system can then be seen as the *concepts* of the non-temporal context. For the sake of conciseness, temporal and non-temporal attributes are depicted in the same table, where the last column graphically illustrates the states as, for instance, in Table 1.b.

In [26], the objects of CTSs are interchangeably called 'time objects', 'time granules', or 'observations', i.e. 'elementary pieces of time', 'time instants' during which some measurement has taken place. This is partially motivated by the fact that objects are *individuated* by the temporal context, i.e. we have a single output at a given time, the *complete* output of the system. To avoid misunderstandings, we will use the term *object* to indicate the entities built from observations, e.g. common sense, physical, or perceptual objects. In the perspective of describing the outputs of apparatuses, the objects of CTSs will be called *outputs* or *observations*—even though observations are, more precisely, the descriptions of the outputs–while the values of the *tme* attribute will be called *times*. These choices reflect some underlying ontological assumptions.

First, the outputs snap the out there by an apparatus at a time[9], but they are neither times nor ontological properties of objects. Different apparatuses can collect information simultaneously and the temporal context allows outputs coming from different apparatuses to be synchronized or ordered (if times are ordered, see below). Furthermore, outputs say how, at a time, a part of the out there *looks* like through the apparatus, and in this sense they are similar to *qualia* [8].

Second, non-plain temporal scales can group times together. The context in Table 1.c is obtained from the one in Table 1.b by mapping both t_1 and t_2 to $t_1|t_2$ and both t_3 and t_4

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[8] We cannot enter here into the huge problem of time construction.

[9] Therefore, outputs (observations) are dependent on a time and an apparatus.

to $t_3|t_4$ (plainly scaling the non-temporal part). We obtain different outputs with the same non-instantaneous *tme*-values. What is the state of the system at time $t_1|t_2$? In the example in Table 1.c states can be consistently reconstructed as depicted in the last column, by assuming that o_1 and o_2 are just parts of the complete output. But what happens if $pos(o_1) = pos(o_2)$ but $col(o_1) \neq col(o_2)$? This can be problematic in the *aggregation* of outputs coming from different apparatuses, modeled by contexts that are *locally* consistent but *globally* inconsistent. The solution to this problem, not addressed here, requires drastic losses of data or a lot of additional information about the apparatuses.

Third, in the attempt to build objects from observations one could have the necessity to refer to a part of the output. The output of a camera could concern a multitude of objects. One needs to *segment* it to individuate the specific 'pixels' that refer to a single object. We will come back to this problem in Section 3.

Fourth, the values of attributes can be structured or have dependencies. This is information *about* the apparatus that is not represented in the formal context. If we say that the output o_1 in Table 1.b represents the detection of black at position 1, we are assuming a dependence between the values of *pos* and *col*. Goodman [8] would say that, in o_1, 1 and B (and t_1) are *together* and *pos* identifies the portions of the 'visual-field' of the apparatus. The physical structure of the sensor or its orientation can be explicitly taken into account by some relations among *pos*-values. The conventional structure of *measures*, e.g. the order between times and weights or the color splinter, can be represented in a similar way. We will see how to represent this information and its role in specifying (i) the unity criteria —to segment complex outputs in basic units linked to single objects—and (ii) the re-identification criteria—to individuate one single object at different times.

2. The construction of objects

Consider the observations in Table 1.b. Which objects can be built from them? On the basis of which criteria? In [27], Wolff seems to think that it is a matter of choice. Goodman would probably say that it is a matter of decrees. Objects help us in *explaining* our basic observations—what we know about the out there—and, in general, there are alternative coherent explanations. Technically, Wolff proceeds in the following way. (i) He labels each observation with an 'object name', obtaining the couples $\langle object, observation\rangle$ called *actual objects*, sort of *temporal slices* of objects [24]. (ii) He builds the life tracks of objects by temporally ordering the actual objects using the *genidentity* relation R: $\langle obj_1, obs_1\rangle \mathrm{R} \langle obj_2, obs_2\rangle$ iff $obj_1 = obj_2$ and $obs_1 < obs_2$, where '<' is a temporal order between observations.[10] In Wolff's words: "[given a (scaled) many-valued context **S**] one has to construct a 'conceptually meaningful' genidentity R leading to 'meaningful' directed paths in the transition digraph of the basic [system] (**S**, R). These direct paths are then taken as the new objects, and the 'nodes' on these paths are interpreted as its 'actual objects"' ([27], p.7). Coming back to our example, from the observations in Table 1.b different objects can be built: a single object switching from position 1 to position 2;[11] two static objects;[12] four instantaneous objects; etc. This example is isomorphic to the classical one of the morning/evening star [5] as described in [26] where 1 is the east, 2 is

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[10] The relation $<$ among observations can be induced by an order among the values of the temporal attributes.

[11] We have the four actual objects $\langle obj_1, o_1\rangle$, $\langle obj_1, o_2\rangle$, $\langle obj_1, o_3\rangle$, $\langle obj_1, o_4\rangle$.

[12] We have the four actual objects $\langle obj_1, o_1\rangle$, $\langle obj_2, o_2\rangle$, $\langle obj_1, o_3\rangle$, $\langle obj_2, o_4\rangle$.

the west, B is luminous, t_1 and t_3 are in the morning while t_2 and t_4 are in the evening. If the observer knows only the data in Table 1.b, all the previous constructions are perfectly coherent. To understand that the morning star is the same as the evening star, keeping only the context with one object, additional information is needed.

This approach is general and simple but, from the perspective of object construction, it has some drawbacks. First, the criteria adopted to build objects—*why*, for instance, one should choose one object rather than two—are not explicit. This prevents one to distinguish *kinds* of objects on the basis of their *identity criteria*, on the basis of the way observations are grouped. Notice that the grouping via formal concepts is not enough. For example, complex *patterns* of observations that, to be specified, have to take into account the *way* an attribute changes with respect to another (e.g., the way *col* or *pos* 'change through' *tme*) cannot be, in general, captured with concepts. Second, the approach in [27] does not allow to manage observations of multiple objects, like camera outputs, that must be segmented to be associated to single objects. We consider this problem in Section 3 while, to address the first one, we introduce the grouping criteria.

A *grouping criterion* γ is a set of sets of sets of attributes of a given formal context. The elements of γ are called *basic constraints*. Grouping criteria *specify* patterns of observations in a *disjunctive normal form*. In particular, they are a disjunction of basic constraints that, in turn, are conjunctions of attribute ascriptions.[13] E.g., in the context in Table 1.a, $\gamma_1 = \{\{\{1, \mathrm{B}\}, \{1, \mathrm{B}\}\}, \{\{2, \mathrm{B}\}, \{2, \mathrm{B}\}\}\}$ specifies the patterns of (exactly) two observations both with attributes 1 *and* B (the first basic constraint $\{\{1, \mathrm{B}\}, \{1, \mathrm{B}\}\}$) *or* both with attributes 2 *and* B (the second basic constraint $\{\{2, \mathrm{B}\}, \{2, \mathrm{B}\}\}$).

Given a grouping criterion γ, the set of γ-patterns (π_γ) is the set of all sets of observations (in the given context) that satisfy at least one basic constraint in γ, i.e.:

$$\begin{aligned}\pi_\gamma = \{\{o_1, \ldots, o_n\} \mid\ & \text{all } o_i \text{ are different and there exists a constraint } \{c_1, \ldots, c_n\} \in \gamma \text{ s.t.}\\ & \text{for all } o_i \text{ there exists } c_j \text{ such that, for all } a \in c_j,\ \langle o_i, a\rangle \in \mathit{IN}, \text{ and}\\ & \text{for all } c_j \text{ there exists } o_i \text{ such that, for all } a \in c_j,\ \langle o_i, a\rangle \in \mathit{IN}\}\end{aligned}$$

For instance, in the context in Table 1.a, $\pi_{\gamma_1} = \{\{o_1, o_3\}, \{o_2, o_4\}\}$. The criterion $\gamma_2 = \{\{\{t_1,1\},\{t_2,2\}\},\{\{t_1,2\},\{t_2,1\}\},\{\{t_2,1\},\{t_3,2\}\},\{\{t_2,2\},\{t_3,1\}\},\{\{t_3,1\},\{t_4,2\}\},\{\{t_3,2\},\{t_4,1\}\}\}$ specifies patterns of two observations with different positions and contiguous times. Applied to the context in Table 1.a, γ_2 generates $\pi_{\gamma_2} = \{\{o_1, o_2\}, \{o_2, o_3\}, \{o_3, o_4\}\}$.

The last example shows that γ-patterns can overlap. Consequently, the same observation can be a member of, can *support*, a multitude of different patterns. Grouping criteria can also generate patterns that are included one in the other. For instance, $\gamma_3 = \{\{\{1, \mathrm{B}\}\}, \{\{1, \mathrm{B}\}, \{1, \mathrm{B}\}\}, \{\{1, \mathrm{B}\}, \{1, \mathrm{B}\}, \{1, \mathrm{B}\}\}\}$, in the context in Table 1.a, generates $\pi_{\gamma_3} = \{\{o_1\}, \{o_3\}, \{o_1, o_3\}\}$ that, in addition, does not cover the whole set of observations. Therefore, observations are not necessarily partitioned by patterns.

Grouping criteria can be seen as 'extensional' (in terms of the attributes the observations need to have) specifications of constraints that concern observations' attributes: γ_1 corresponds to $col(o_1) = col(o_2) = \mathrm{B}$ and $pos(o_1) = pos(o_2)$; γ_2 corresponds to $pos(o_1) \neq pos(o_2)$ and $tme(o_1) \lessdot tme(o_2)$ assuming times are ordered by $<$ and $x \lessdot y$ iff $x < y \wedge \neg\exists z(x < z \wedge z < y)$.[14] In our framework, *chains*—patterns that have a maximal, not specifiable a priori, length—can be built relying on a (recursive) *closure* operator on

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[13] We do not consider negative conjuncts. However, the framework can be extended to take them into account.

[14] In principle, given a formal context, it is possible to generate the grouping criteria starting from the constraints expressed in this intensional form.

Table 2. Different changes in color and position

	tme	*pos*	*col*	1	2	3
o_1	t_1	1	B	■		
o_2	t_2	2	B		■	
o_3	t_3	3	G			▨
o_4	t_4	3	G			▨

(**a**)

	tme	*pos*	*col*	1	2	3
o_1	t_1	1	B	■		
o_2	t_2	1	B	■		
o_3	t_3	3	G			▨
o_4	t_4	3	G			▨

(**b**)

π_γ, a sort of generalization of the transitive closure: if $p_1, p_2 \in \pi_\gamma$ and $p_1 \cap p_2 \neq \emptyset$, then delete p_1 and p_2 from π_γ and introduce in it $p_1 \cup p_2$. We will note with $\bar{\pi}_\gamma$ the closure of π_γ. For instance, $\bar{\pi}_{\gamma_2} = \{\{o_1, o_2, o_3, o_4\}\}$ contains only one chain of black observations with different positions at contiguous times, and $\bar{\pi}_{\gamma_3} = \{\{o_1, o_3\}\}$ contains only one chain of black observations with position 1.

Different grouping criteria can generate the same patterns. E.g., in the context in Table 2.b, $\gamma_4 = \{\{\{1\},\{1\}\}, \{\{2\},\{2\}\}, \{\{3\},\{3\}\}\}$ (corresponding to $pos(o_1) = pos(o_2)$) and $\gamma_5 = \{\{\{\mathrm{B}\},\{\mathrm{B}\}\}, \{\{\mathrm{G}\},\{\mathrm{G}\}\}\}$ (corresponding to $col(o_1) = col(o_2)$) both generate $\pi_{\gamma_4} = \pi_{\gamma_5} = \{\{o_1, o_2\}, \{o_3, o_4\}\}$. We have again observations that support different patterns, but now these patterns are generated by different criteria.

We use grouping criteria to group observations into objects. In particular, we distinguish *synchronic* criteria, called *unity* criteria and noted γ^{u} (see Section 3), that collect observations having the same temporal attribute, and *diachronic* criteria, called *re-identification* criteria and noted γ^{r}, that collect observations having different temporal attributes, e.g., γ_2. For instance, by modifying γ_4 and γ_5 accepting only patterns with contiguous times[15] and closing them, we re-identify objects on the basis of the position (γ_4^{r}) or the color (γ_5^{r}). Applying these new criteria to the context in Table 2.b we obtain $\pi_{\gamma_4} = \pi_{\gamma_5} = \bar{\pi}_{\gamma_4^{\mathrm{r}}} = \bar{\pi}_{\gamma_5^{\mathrm{r}}}$. Notice that this is only a factual situation: by applying γ_4^{r} and γ_5^{r} to the context in Table 2.a, we obtain $\bar{\pi}_{\gamma_4^{\mathrm{r}}} = \{\{o_3, o_4\}\} \neq \bar{\pi}_{\gamma_5^{\mathrm{r}}} = \pi_{\gamma_5}$.

Similarly as for formal concepts, an object kind k seems to have an *intensional* component—the re-identification criterion γ_k^{r}—and an *extensional* component—the patterns (chains) in $\pi_{\gamma_k^{\mathrm{r}}}$ ($\bar{\pi}_{\gamma_k^{\mathrm{r}}}$). In this view, objects are not reducible to patterns or chains, but, following a sort of constructivist (in a logical sense) approach, the building mechanism, the re-identification criterion, has to be considered as a 'part' of objects.[16] A kind k of objects (in a formal context) is then a couple $\langle \pi_{\gamma_k^{\mathrm{r}}}, \gamma_k^{\mathrm{r}} \rangle$, $\pi_{\gamma_k^{\mathrm{r}}}$ is the *extent* and γ_k^{r} the *intent* of k. The same holds if we consider the closure.[17] In this way $k_4 \neq k_5$ even though $\bar{\pi}_{\gamma_4^{\mathrm{r}}} = \bar{\pi}_{\gamma_5^{\mathrm{r}}}$ because $\gamma_4^{\mathrm{r}} \neq \gamma_5^{\mathrm{r}}$, i.e., the patterns are generated by different grouping criteria that, typically, are founded in the theory behind the instruments or in the way observers 'interpret' the observations. Objects of kind k, k-objects, have then to take into account the intensional part. We will represent them as couples $\langle x, k \rangle$ where k is a kind with re-identification criterion γ_k^{r} and $x \in \pi_{\gamma_k^{\mathrm{r}}}$ (or $\bar{\pi}_{\gamma_k^{\mathrm{r}}}$). Considering the previous γ_4^{r} and γ_5^{r}, even in the context in Table 2.a, we can now distinguish $\langle \{o_1, o_2\}, k_4 \rangle$ from $\langle \{o_1, o_2\}, k_5 \rangle$. In this way, it is possible to address the classical ontological problem of *material constitu-*

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[15] By substituting the first basic constraint in γ_4 (similarly for the other cases) with the three basic constraints $\{\{t_1, 1\}, \{t_2, 1\}\}, \{\{t_2, 1\}, \{t_3, 1\}\}, \{\{t_3, 1\}, \{t_4, 1\}\}$.

[16] In [21], Scheider and colleagues follow a similar approach in a perceptive context. However, it is not clear to us how the intensional dimension of objects is captured, and only the private dimension is considered.

[17] Note that the closure operator contributes to the identity of the kind.

Table 3. Segmentation of observations

	tme	*pos*	*col*	1	2	3
o_1	t_1	1	B	■		
o_2	t_2	2	G		▨	
o_3	t_3	3				□
o_4	t_4	1		□		
o_5	t_5	2	B		■	
o_6	t_6	3	G			▨

(**a**)

	tme	col_{p_1}	col_{p_2}	col_{p_3}	1	2	3
$\bar{o}_1$	t_3	B	G		■	▨	□
$\bar{o}_2$	t_6		B	G	□	■	▨

(**b**)

	tme	col_{p_1}	col_{p_2}	col_{p_3}	1	2	3
$\bar{o}_{11}$	t_3	B			■	□	□
$\bar{o}_{12}$	t_3		G		□	▨	□
$\bar{o}_{21}$	t_6		B		□	■	□
$\bar{o}_{22}$	t_6			G	□	□	▨

(**c**)

tion (see [19] for a review), the idea that (at a given time) objects of different kind can *coincide*, e.g., a statue and the amount of clay that constitutes it.[18]

3. Segmentation and unity criteria

Let us now suppose to have at our disposal two structurally different apparatuses, calibrated with respect to time (*tme*) and color (*col*) attributes. The first apparatus has an orientable (in three different directions) one-pixel sensor that is very quick in detecting black or gray shades of color. The second apparatus has a slower (non-orientable) three-pixels sensor. The two sensors are simultaneously turned on. The first sensor sequentially detects the colors at its orientations (represented by *pos*-values), producing six outputs from t_1 to t_6, see Table 3.a. The second sensor produces two outputs, at t_3 and t_6, that however regard the colors detected by three different pixels, see Table 3.b. Re-identification criteria can generate up to 15 objects in the first case but only 3 in the second case. In particular, from the data in Table 3.b, one can construct only bi-colored objects because data cannot be *segmented*,[19] objects cannot be built starting from relevant *parts* of observations (called *partial* observations).[20]

Partial observations are built first by decomposing observations in *atomic parts* and then by grouping them according to *unity criteria*.An observation o_j is an atomic part of o_i if and only if (1) $tme(o_j) = tme(o_i)$ and (2) there exists a non temporal attribute att ($att \neq tme$) with $att(o_i) \neq \varnothing$ ($\varnothing$ is the undefined value) such that $att(o_j) = att(o_i)$ and for all non-temporal attributes $att' \neq att$ we have $att'(o_j) = \varnothing$.[21] The atomic parts of the observations in Table 3.b are reported in Table 3.c. We can then use *unity* criteria γ^{u} to group *synchronous* atomic observations—observations with the same temporal attribute—into partial ones. In general, unity criteria take into account some information about the apparatus and/or the observer—his/her intended interpretation of data.[22]

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[18] At time t, a k_1-object x *coincides* with a k_2-object y, if there exists an observation belonging to both the observations grouped in x and the ones grouped in y with temporal attribute t.

[19] Vice versa, the observations in Table 3.a can be grouped by scaling sequences of three successive times in a single one, e.g. t_1, t_2, and t_3 are scaled in $t_1|t_2|t_3$ (see Table 1.c).

[20] Segmentation algorithms are central in computer vision, see [2], that contains also a useful overview.

[21] This definition works only when all attributes are independent. If, for instance, the attribute *col* in Table 3.a is *together* (in Goodman's sense) with *pos*—i.e. colors are 'positioned'—then atomic observations o_i with $col(o_i) \neq \varnothing$ and $pos(o_i) = \varnothing$ satisfy the definition but not the dependence (while the ones with $col(o_i) = \varnothing$ and $pos(o_i) \neq \varnothing$ still make sense). A more complex definition is needed to take into account dependencies.

[22] It is also possible to layer unity criteria. For instance, uniformly colored areas of a picture can be built by grouping self-connected sets of pixels with the same color, and uniformly colored areas can, on their turn, be grouped in some complex patterns on the basis of some additional criteria. We do not address this topic here.

Let us consider the criterion (X_i stands for “the value of the attribute col_{p_i} is X”) $\gamma_1^{\mathrm{u}} = \{\{\{t_3, \mathrm{B}_1\}, \{t_3, \mathrm{G}_2\}\}, \{\{t_3, \mathrm{G}_1\}, \{t_3, \mathrm{B}_2\}\}, \{\{t_3, \mathrm{B}_2\}, \{t_3, \mathrm{G}_3\}\}, \{\{t_3, \mathrm{G}_2\}, \{t_3, \mathrm{B}_3\}\}, \ldots\}$ that corresponds to the constraint: there exists n such that $col_{p_n}(o_1) \neq \emptyset$, $col_{p_{n+1}}(o_2) \neq \emptyset$, $col_{p_n}(o_1) \neq col_{p_{n+1}}(o_2)$, and $tme(o_1) = tme(o_2)$, i.e., we are here grouping atomic observations with adjacent non uniformly colored pixels (supposing pixels are discrete and ordered). In the context in Table 3.c, γ_1^{u} generates $\pi_{\gamma_1^{\mathrm{u}}} = \{\{\bar{o}_{11}, \bar{o}_{12}\}, \{\bar{o}_{21}, \bar{o}_{22}\}\}$. The unity criterion that corresponds to the constraint $col_{p_n}(o_1) \neq \emptyset$, $col_{p_{n+1}}(o_2) \neq \emptyset$, $col_{p_n}(o_1) = col_{p_{n+1}}(o_2)$, and $tme(o_1) = tme(o_2)$, does not generate any pattern, and $\bar{o}_1$ and $\bar{o}_2$ are not segmented, because no adjacent pixels with uniform color exist.

As in the case of re-identification criteria, different unity criteria may generate the same patterns. *Partial* observations[23] have then to explicitly refer to the unity criteria that generated them. In the previous case, we end up with two partial observations: $\langle\{\bar{o}_{11}, \bar{o}_{12}\}, \gamma_1^{\mathrm{u}}\rangle$ and $\langle\{\bar{o}_{21}, \bar{o}_{22}\}, \gamma_1^{\mathrm{u}}\rangle$. Note that these partial observations are different from the initial observations $\bar{o}_1$ and $\bar{o}_2$ because their unity criterion is not based on time, as it is implicitly the case of the original ones instead.[24]

At this point, re-identification criteria can be used to group partial observations instead of complete ones. The *intent* of a kind k has then two components: a unity criterion γ_k^{u} *and* a re-identification criterion γ_k^{r}. The criterion γ_k^{u} groups synchronic atomic observations, while γ_k^{r} groups diachronic partial observations *all generated by* γ_k^{u}, i.e. only partial observations uniformly built can be grouped to form an object. The previous definition (see Section 2) of a kind k needs then to be extended: $k = \langle \mathcal{E}_k, \gamma_k^{\mathrm{u}}, \gamma_k^{\mathrm{r}} \rangle$ where $\mathcal{E}_k$ is the *extent* of k, the set of patterns generated by applying γ_k^{r} to the set of partial observations generated by applying γ_k^{u} to the set of the atomic observations of a given context. We can then define *object systems*. An object system $\langle \theta, K \rangle$ consists of a CTS θ and a finite set K of object *kinds* on θ.

3.1. No observation, no object?

In our account no object can obtain independently from our constructions and observations. However, science often refers to things that were long before human species appeared, or to things that will be long after humans will be gone. How can such things be the result of construction and observation? Boghossian exemplifies a common line of attack from the realist’s camp: “[T]here were mountains on earth well before there were humans. How, then, could we be said to have constructed the fact that there are mountains on earth?” ([1], p. 26).

First, notice that the kind of constructivism we are subscribing to does not necessarily imply anti-realism, because we aim at being tuned with the out there. The observations (partially) depend on instruments *that point to the out there*, and objects and laws depend on observations.

Second, it is not necessary to assume that k-objects are put into existence when the identity (unity and re-identification) criteria of k are introduced, i.e. when the ‘concept’

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[23] Partial observations include also whole observations as limit cases.

[24] Original observations can be seen as patterns generated by grouping all synchronous atomic observations, i.e. the unity criterion takes into account only the temporal attribute.

k is created or explicitly considered.[25] If one disposes of observations collected before the introduction of k, still one could find k-objects in them.[26]

'Incomplete observation' is more problematic. Suppose that, from t_0 to t_n, we collected a set of observations with the same apparatus and that, according to our criteria, objects of kind k are identified only between t_i and t_n (with $t_0 < t_i$). One could infer that k-objects *come into existence* at t_i, as in the case of $k =$ 'Airbus A380' and $t_i =$ 'April 27, 2005'. But the position that assumes that k-objects are *discovered* at t_i—they already existed, but the instrument was pointing in the wrong direction—cannot be ruled out. For the same reason, if k is introduced at t_0, it is not possible to claim that no k-objects exist between t_0 and t_i, we just did not observe them. For instance, Neptune has been found by mathematical prediction and successively observed on September 23, 1846 by Johann Galle, but nobody claims that in August 1846 Neptune was not present. More drastically, did Neptune exist before the first telescope was invented? This is a general problem the constructivist (instrumentalist) faces: "no observation, no object".

A *partial* way out considers *prediction and retrodiction laws*. One predicts (or retrodicts) what would be the outputs of instruments at different times or 'aimings'—the *expected* observations—given the *actual* observations collected by them. The expected observations are the result of the application of laws to the actual ones. While identity criteria produce objects—convenient intermediaries, in Quine's words—that help our understanding and interpretation of observations, laws[27] produce *new* observations, they add 'expectations' about the out there by *projecting* observations into observations. This mechanism allows to 'virtually' use the instrument also before its creation (or in the future) and to 'virtually' point it in a given direction. We are here capturing a sort of *modal* reasoning regulated by laws. A kind *k comes into existence* at t iff, at t, there exist some k-objects but no k-object can be built from any retrojection. However, the revision of laws or the acquisition of new data could impact on the retrojections and consequently change the time of appearance of k-objects. Therefore, the existence of k-objects at time t does not require any observation at t, but, in any case, it depends on the laws and on the actual data one possesses at a given time. This can be seen as a form of *fallibilism*, still at center of a lively debate.

4. Social objects in a constructed realm

The common conception is that social reality is something constructed *par excellence*. More than this, often the slogan "if you construct it, it is social, if it is given, then it is not" is assumed. Realists, e.g. [23], endorse such a slogan with some kind of ease, but in our approach also *non* social objects are built. Moreover, for standard accounts, to have a social object means to have a plurality of agents accepting something as being the case. But, as we have seen, in our account the identifiability of 'regular' objects relies on some sort of conventionalism too. Finally, social objects are usually intended as means to achieve some sort of coordination. But, again, this seems to hold also for some *non* social

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[25] Our framework can be easily extended to account for this.

[26] One can also assume that the identity criteria associated to k evolve in time, i.e. they can be only applied to observations collected during a specific interval of time. We will not deal with this important topic.

[27] Also laws are constructed, they are part of a theory which is built to make sense of the observations. To cope with prediction/retrodiction laws and the way they are built, our framework needs to be heavily extended.

Table 4. A chess game?

	1	2	3	4	5	6	7	8
1	☒	☒	☒	☒	☒	☒	☒	☒
2	☒	☒	☒	☒	☒	☒	☒	☒
3	☐	☐	☐	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	☐	☐	☐
7	☒	☒	☒	☒	☒	☒	☒	☒
8	☒	☒	☒	☒	☒	☒	☒	☒

(**a**)

	1	2	3	4	5	6	7	8
1	☒	☒	☒	☒	☒	☒	☒	☒
2	☒	☒	☒	☒	☒	☒	☒	☒
3	☐	☐	☐	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	☐	☒	☐
6	☐	☐	☐	☐	☐	☐	☐	☐
7	☒	☒	☒	☒	☒	☒	☐	☒
8	☒	☒	☒	☒	☒	☒	☒	☒

(**b**)

objects, since they can be the products of (public) decrees or perceptive mechanisms used within a community of agents—who share similar ‘sensors’, as the members of our species—and they are used to coordinate with the out there and among each other. How, then, to tell the difference between what is social and what is not?

A first answer could be based on identity criteria: for social objects they seem to be more elusive than those of other ordinary objects, as the physical ones. The difference here is more a matter of degrees than of kind: social objects seem to be more ‘abstract’ or ‘physically pliable’. A basketball team, for example, can change in a meaningful and continuous way through time: players, coaches, sometimes even city. But their behavior in time is not the only peculiarity of social objects. A collection of paintings, such as the British Royal Collection, can be scattered, located in different places, such as Windsor Castle, Buckingham Palace and Hampton Court Palace, at the same time.

Let’s try to make sense of such more relaxed degrees of freedom in light of the just presented framework. Consider a 8 × 8 grid sensor that detects which positions of the grid are occupied. The states at t_0 and t_1 are depicted in Tables 4.a-b respectively. Could we say that there are objects positioned on the grid and that one of them moved between t_0 and t_1? Suppose now the sensor is declared to point to a chessboard. Chess players could now say that there are 4 rooks, 4 knights, etc. and that a white pawn moved. This amounts to say that the re-identification of chess pieces, in addition to rules for moving them, is based on their starting positions on the chessboard.

Suppose now that during a chess game the figurine used as white queen is lost and the players decide to substitute it with a pebble, relying on the pliability of chess pieces—they can survive a change in shape, material, color etc. without loosing their own identity. If someone is observing the game by only looking at the mentioned 8 × 8 grid sensor, still she is able to track the white queen as well as the other pieces. This is not only because the sensor’s output is interpreted using chess rules, but also because the sensor cannot detect the difference between the figurine and the pebble, being it just able to discriminate on the basis of positions and not of shapes or colors.

By disposing of a better sensor that detects also shapes or other physical attributes, to re-identify the white queen—but not the figurine or the pebble—the observer has to abstract from them. In addition, at the beginning the white queen coincides (in the sense introduced in Section 2) with the figurine, while at the end, with the pebble.[28] The pebble (and the figurine) is pliable in a different sense, it can play different roles or it can constitute different entities at different times. For example, the pebble used for the queen can be also used, in a different phase of the game, for a rook. Therefore, queens seem physically more pliable than figurines or pebbles, they are weakly linked to their

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[28] Note that the figurine and the pebble *are not* queens, i.e. the kind ‘queen’ does not subsume the kinds ‘figurine’ and ‘pebble’. Following [4], the queen could be seen as a sum of two different *qua-objects*, one inhering in the figurine and the other one inhering in the pebble.

physical substratum. Vice versa, figurines and pebbles can be used in different contexts but are physically stable.

Another characteristic of social objects is their *intentional* dimension. Not only social objects get necessarily into existence at a given time (see Section 3.1)—for instance when a declarative, institutional act is performed—but they are created with some purpose in mind. The *intentional* dimension can then be represented as a selection of the expected observations, called *intended* observations. These are the observations intended to be produced, from the actual ones, by 'using' the social object. Since we are constrained by physical laws, our purposes are realizable only if they are compatible with them, i.e. the intended observations need to be included in the expected observations of physics.

In this view, social objects can be seen as ways of 'exploiting' (our knowledge of) the out there to achieve a goal. For example, chess can be seen as a complex object that, through some rules, exploits figurines and boards to allow two persons to play a game. Once we have the purpose of chess, after the game, a possible mismatch between the intended observations and the *current* ones, can be interpreted in different ways: (1) chess is wrong by design as the intended observations are not physically realizable, e.g. rules are contradictory or impossible to follow for humans; (2) the design is underspecified, it guarantees to achieve the goal only in some cases, e.g. rules do not constrain the moves enough; (3) there is a 'malfunctioning', the design is correct but the particular physical realization does not satisfy the specifications, e.g. figurines for queens are missing; (4) there is a 'misuse', the design is correct but the particular 'users' behavior does not satisfy the specifications, e.g. the players do not follow the rules. The mismatch requires a change of the design in the first case[29], while in the second one there is at least a partial fulfillment of the goal that can motivate a change, but not necessarily.

Assume that an alien, say E.T., has access to the very same observations as Yuri, a chess champion. Differently from Yuri, E.T has no knowledge of the intended goal and of the design of chess. By means of induction, E.T. can find patterns of regularity in data and come up with some prediction's laws. When he finds some mismatches between the newly acquired observations and the projections generated by his laws, he can only conclude that his laws are wrong and need to be revised. This situation seems close to the one described by Searle in [22], where a detective writes down the items a man selects according to his shopping list. At the end of the day, both shopper and detective have identical lists. However, their function is different, they have different *directions of fit*. As in the case of E.T.'s prediction laws, the detective's list, but not the shopper's, has to *fit* the world by matching the observations of the shopper's actions.

5. Conclusions

Constructivism, as we intend it, is an attempt to understand what is out there without relying on the naive illusion that the veil will be one day teared away. We extended some tools imported from FCA to make explicit how objects can be (re-)identified from observations, making a clear distinction between their extensional and intensional dimensions. In addition, we analyzed how our framework can be extended—with projections, laws, and intentionality—to distinguish, among constructed objects, social objects.

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[29] These mismatches can also provide evidences that the assumed physical laws are wrong.

The possible applications of this approach may span a wide variety of directions: from the identification of the entities the web data refer to, to enhancement of algorithms for the tracking of objects in computer vision systems, to the use of formal methods for object recognition in cognitive sciences.

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References

[1] Paul Boghossian. *Fear of Knowledge: Against Relativism and Constructivism.* OUP, 2006.

[2] C. Carson, S. Belongie, H. Greenspan, and J. Malik. Blobworld: Image segmentation using expectation-maximization and its application to image querying. *IEEE Transaction on Pattern Analysis and Machine Intelligence*, 24(8):1026–1038, 2002.

[3] F. Dretske. *Naturalizing the Mind.* MIT Press, 1995.

[4] Kit Fine. Acts, events and things. In *Sixth International Wittgenstein Symposium, Kirchberg-Wechsel (Austria)*, pages 97–105, 1982.

[5] G. Frege. *On Sense and Reference*. Blackwell, third edition, 1980.

[6] A. Frigerio, A. Giordani, and L. Mari. Outline of a general model of measurement. *Synthese*, Published online: 28 February 2009.

[7] B. Ganter and R. Wille. *Formal Conceptual Analysis. Mathematical Foundations*. Springer, 1999.

[8] N. Goodman. *The Structure of Appearance*. Harvard University Press, Cambridge MA., 1951.

[9] D. M. Krantz, R. D. Luce, P. Suppes, and A. Tversky. *Foundations of Measurement*, volume I - Additive and Polynomial Representations. Academic Press, 1971.

[10] W. Kuhn. A functional ontology of observation and measurement. In K. Janowicz, M. Raubal, and S. Levashkin, editors, *GeoSpatial Semantics (GeoS 2009)*, pages 26–43. Springer Verlag, 2009.

[11] H. E. Kyburg Jr. *Theory and Measurement*. CUP, 1984.

[12] R. B. Lindsay and H. Margenau. *Foundations of Physics*. John Wiley & Sons, 1936.

[13] C. Masolo. Founding properties on measurement. In A. Galton and R. Mizoguchi, editors, *International Conference on Formal Ontology and Information Systems (FOIS 2010)*, pages 89–102. IOS Press, 2010.

[14] M. Matthen. *Seeing, Doing, and Knowing. A Philosophical Theory of Sense Percetion.* OUP, 2005.

[15] S. E. Palmer. Organizing objects and scenes. In D. J. Levitin, editor, *Foundations of Cognitive Psychology, Core Readings*, chapter 8, pages 189–212. MIT Press, 2002.

[16] F. Probst. Observations, measurements and semantic reference spaces. *Applied Ontology*, 3(1-2):63–90, 2008.

[17] Z. W. Pylyshyn. *Things and Places. How the Mind Connects with the World.* MIT Press, 2007.

[18] W. V. O. Quine. Two dogmas of empiricism. In *From a Logical Point of View*, pages 20–46. Harper and Row, second edition, 1961.

[19] M. Rea, editor. *Material Constitution.* Rowman and Littlefield Publishers, 1996.

[20] B. A. W. Russell. *Our Knowledge of the External World.* Allen and Unwin, 1914.

[21] S. Scheider, F. Probst, and K. Janowicz. Constructing bodies and their qualities from observations. In A. Galton and R. Mizoguchi, editors, *International Conference on Formal Ontology and Information Systems (FOIS 2010)*, pages 131–146. IOS Press, 2010.

[22] J. R. Searle. *Expression and Meaning. Studies in the Theory of Speech Acts.* CUP, 1979.

[23] J. R. Searle. *Making the Social World: The Structure of Human Civilization.* OUP, 2010.

[24] T. Sider. *Four-Dimensionalism. An Ontology of Persistence and Time.* Clarendon Press, 2001.

[25] B. C. van Fraassen. *Scientific Representation: Paradoxes of Perspective*. Clarendon Press, 2008.

[26] K. E. Wolff. Temporal concept analysis. In E. Mephu Nguifo et al., editor, *Workshop on Concept Lattices-Based Theory, Methods and Tools for Knowledge Discovery in Databases*, pages 91–107, 2001.

[27] K. E. Wolff. Towards a conceptual theory of indistinguishable objects. In P. W. Eklund, editor, *International Conference on Formal Concept Analysis (ICFCA 2004)*, pages 180–188. Springer, 2004.