

# Action and Agency

Nicolas Troquard<sup>1,2,3</sup>, Robert Trypuz<sup>1,3</sup>, Laure Vieu<sup>1,2</sup>

<sup>1</sup>Laboratorio di Ontologia Applicata, ISTC, CNR, Trento

<sup>2</sup>Institut de Recherche en Informatique de Toulouse,  
Université Paul Sabatier & CNRS

<sup>3</sup>Università di Trento

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# Introduction

- ▶ Action and agency are crucial notions for a variety of application domains
- ▶ Although many different research areas have proposed theoretical accounts, no well-developed formal ontology of action *and* agency is currently available
- ▶ Agency has been studied essentially in modal logics, however, the FOL framework is more adequate for investigating ontological issues
- ▶ We propose here to build bridges between the modal logic and the first-order axiomatic theory traditions

# Outline

- ▶ Motivations and approach
- ▶ STIT, a modal logic of Agency
- ▶ OntoSTIT, a first-order equivalent
- ▶ Limitations of STIT and OntoSTIT
- ▶ Adding actions into the picture: OntoSTIT+
- ▶ Discussion

# Agency and Action

## ► Agency logics

- Anselm: acting is best described by what an agent brings about or sees *to it that* (STIT).
- STIT logic (in short: STIT) is one of the most suitable logical systems dealing with *agency*, both in terms of
  - expressivity (more expressive than ATL and CL)
  - calculability (decidable fragment)

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  - calculability (decidable fragment)

## ► No action in agency logics

- Focus on a “responsibility” relation between agents and states of affairs.
- Actions are only implicit.  
No distinction between different ways of bringing about the same result.



# Agency and Action

## ► Theories of action

- Reifying events (and actions) allow expressing properties such as participants and modes, temporal and causal relations (Davidson)
- FOL is the preferred framework for event theories
  - high expressivity
  - transparency of ontological commitment

# Agency and Action

## ► Theories of action

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- FOL is the preferred framework for event theories
  - high expressivity
  - transparency of ontological commitment

## ► No agency in theories of action

- A participant identified as “agent”; causation between eventualities, but
- No direct link between agent and caused state of affairs

# Modal Logic vs. Ontology?

Belnap et al., 2001:

- ▶ “The modal logic of agency is not popular. Perhaps largely due to the influence of Davidson, but based also on very different work of such as Goldman’s and Thomson’s, the dominant logical template takes an agent as a wart on the skin of an action, and takes an action as a kind of event. This ‘actions as events’ picture is all ontology, not modality, and indeed, in the case of Davidson, is driven by the sort of commitment to first order logic that counts modalities as Bad.”
- ▶ Each modal logic of agency, as STIT, “has the advantage that it permits us to *postpone attempting to fashion an ontological theory*, while still advancing our grasp of some important features of action...”.

# Modal Logic vs. Ontology?

- ▶ No reason for considering the modal logic and the FO theory approaches as competing
- ▶ No reason for not studying action and agency together
- ▶ As any logical framework, STIT:
  - ▶ Does carry ontological assumptions - mostly hidden in properties of its models
  - ▶ Can therefore be seen as an ontology of agency
- ▶ If we want to focus on ontology issues, before dealing with reasoning, it is nevertheless easier and clearer to do it in a FOL framework

# The approach

- ▶ Propose a FOL equivalent to STIT, the theory *OntoSTIT*
- ▶ Extend the domain of *OntoSTIT* and the signature of the language to include actions, yielding the theory *OntoSTIT+*
- ▶ Other approach: combine modal logics on the basis of operator of STIT & PDL-like operators  
[Troquard & Vieu, 2006]

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# STIT Language

The language of STIT ( $L_S$ ):

$$\phi \triangleq p \mid a = b \mid \neg\phi \mid \phi \wedge \phi \mid \Box\phi \mid [a \text{ cstit}: \phi]$$

- ▶  $p \in \mathcal{A}tm$ , the set of atomic propositions
- ▶  $a, b \in \mathcal{A}gt$ , the set of agents
- ▶  $[a \text{ cstit}: \phi]$ : agent  $a$  sees to it that  $\phi$  is true (agentive operator)
- ▶  $\Box\phi$ :  $\phi$  is true however things might turn up in the future (historical necessity operator)

# Models of STIT (1) – Branching Time

$\mathcal{M} = \langle Mom, <, Agt, Choice, v \rangle$  where:

1  $\langle Mom, < \rangle$  is a *tree-like branching time structure*.

- ▶  $Mom$  is a non empty set of moments.
  - ▶  $<$  is a strict partial order relation: transitive, irreflexive
  - ▶ no backward branching (past is determined)
  - ▶ every two moments have a common past moment (historical connectedness)
- ▶ Forward branching: indeterminacy of the future



# Branching Time

- Branching Time: basis for the very possibility of agency

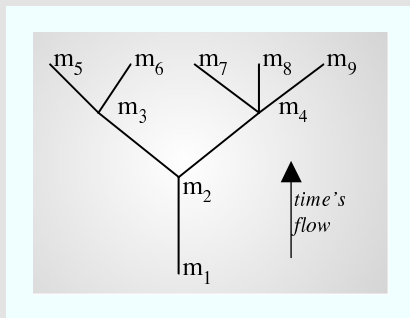


Figure: Branching Time

# A few definitions – Histories

- ▶ Chain

subset of  $Mom$  s.t. every pair of its members is comparable (linear order)

- ▶ History, noted  $h$

maximal chain in  $Mom$

a complete temporal evolution of the world, one possible course of the world

- ▶  $H_m = \{h | m \in h\}$

set of histories *passing through moment*  $m$ , i.e., those histories in which  $m$  occurs.

- ▶ *Hist*: the set of all histories.

# Histories

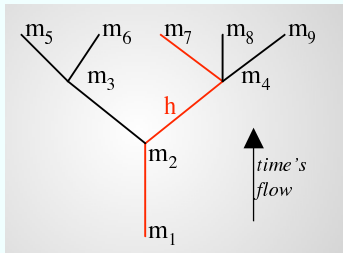


Figure: History  $h$

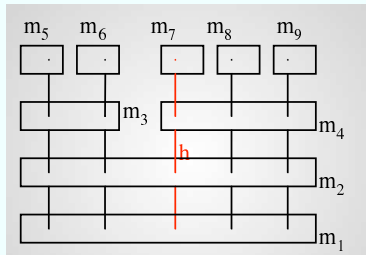


Figure: History  $h$

## Models of STIT (2) – Agents and Choice

$\mathcal{M} = \langle Mom, <, Agt, Choice, v \rangle$  where:

- 2  $Agt$  is a non empty set of agents
- 3  $Choice : Agt \times Mom \mapsto 2^{2^{Hist}}$ , values noted  $Choice_a^m$   
s.t.  $Choice_a^m$  is a partition into equivalence classes of  $H_m$
- 4 Choices are effective: no choice between undivided histories
  - ▶ two histories are undivided at a moment  $m$  iff they do not branch at  $m$  (but later)
- 5 Agents are independent: there is at least a common history for any combination of choices
- 6  $v$  is a valuation function  $v : Atm \mapsto 2^{Mom \times Hist}$

# Choices and STIT

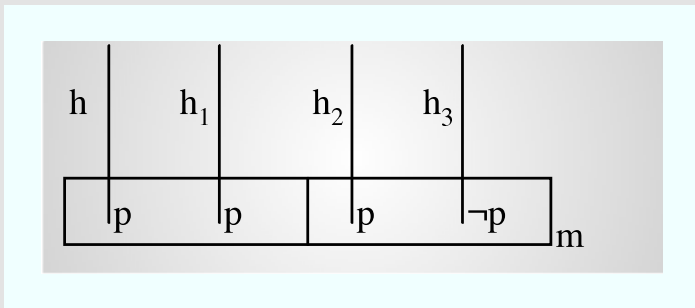


Figure: Two Choices at  $m$ .

$$Choice_a^m = \{\{h, h_1\}, \{h_2, h_3\}\}$$

$$\mathcal{M}, m/h_1 \models [a \text{ cstit}: p]$$

$$\mathcal{M}, m/h_2 \models \neg[a \text{ cstit}: p]$$

# STIT Semantics

Formulas are evaluated in a model, **at an index  $m/h$** : a pair of a moment  $m$  in  $Mom$  and a history  $h$  from  $H_m$

- ▶  $\mathcal{M}, m/h \models p \iff m/h \in v(p)$
- ▶ standard semantics for Boolean operators
- ▶  $\mathcal{M}, m/h \models \Box\phi \iff \mathcal{M}, m/h' \models \phi$  for all  $h' \in H_m$
- ▶  $\mathcal{M}, m/h \models [acstit: \phi] \iff Choice_a^m(h) \subseteq |\phi|_m^{\mathcal{M}}$   
where  $|\phi|_m^{\mathcal{M}} = \{h \in H_m \mid \mathcal{M}, m/h \models \phi\}$

# The logic STIT

## Axiomatics for (Chellas's) STIT:

- ▶ Axioms for propositional logic
- ▶ S5 axioms for both  $\Box$  and  $[_\text{cstit}: \_]$ ;  $\Box\phi \rightarrow [a \text{ cstit}: \phi]$
- ▶ Axioms for standard identity in  $\mathcal{Agt}$
- ▶ Independence of agents:  
 $\text{diff}(a_0, \dots, a_k) \wedge \Diamond[a_0 \text{ cstit}: p_0] \wedge \dots \wedge \Diamond[a_k \text{ cstit}: p_k] \rightarrow$   
 $\Diamond([a_0 \text{ cstit}: p_0] \wedge \dots \wedge [a_k \text{ cstit}: p_k]), \text{ for all } k \geq 1$

Rules of inference: *modus ponens*, *necessitation* for  $\Box$

STIT is sound and complete wrt the class of models  $\mathcal{M}$  (Belnap et al.)

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- ▶ **OntoSTIT, a first-order equivalent**
- ▶ Limitations of STIT and OntoSTIT
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# OntoSTIT - Language

- ▶ First-order logic with identity, with a special set of propositional constants
- ▶ Unsorted domain of particulars: agents, moments and histories
- ▶ Language  $L_O$ , with symbol conventions:
  - ▶  $\Omega$ , set of variables ranging on particulars:  $x_1, \dots, x_n$  ( $x, y, z, \dots$ )
  - ▶  $\Lambda$ , set of constants denoting particulars: **a**, **h**, **m**, **y**, **z**, ...
  - ▶  $\Pi$ , set of constants denoting states of affairs: **p**, **p'**, **p''**, ...
  - ▶  $\Delta$ , set of primitive predicates:
    - AG*, is an agent; *MO*, is a moment; *HT*, is a history
    - IN*, a moment is in a history
    - PRE*, a moment precedes another
    - HOLDS*, at a moment and history, a proposition is true
    - PO*, being in the same “possible outcomes”

# OntoSTIT - Axiomatics

- ▶ Argument restriction axioms
- ▶ Existential axioms
- ▶ Temporal order on moments
  - ▶ *PRE* is a strict partial order
  - ▶ Any two moments in the past of a third one are ordered
  - ▶ Any two moments have a “common root”
- ▶ Histories
  - ▶ Histories are maximal linear orders of moments (based on *IN* and *PRE*)

# OntoSTIT - Axiomatics

## ► Agency (*PO* or Possible Outcomes)

- For agent  $a$  and moment  $m$  fixed,  $PO(a, m, h_1, h_2)$  is an equivalence relation on histories passing through  $m$ .
- Histories undivided at  $m$  belong to the same equivalence class (effectiveness of choice)
- There is a common history to any possible combination of equivalence classes of different agents (independence of agents)

## Possible Outcomes

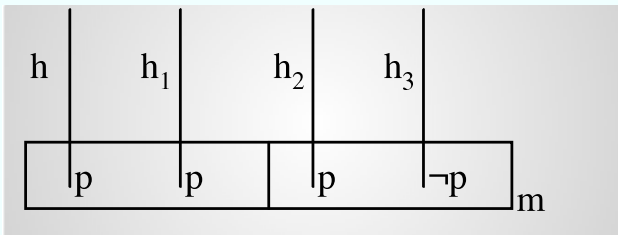
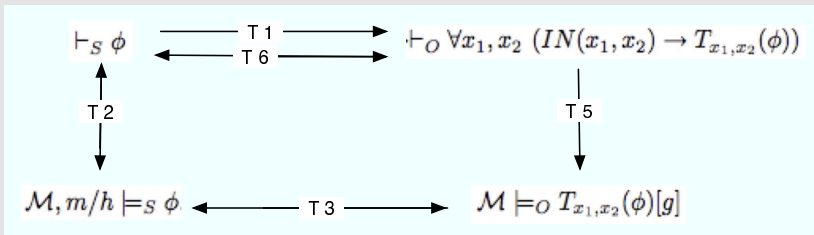


Figure: Two Possible Outcomes at  $m$ .

$PO(\mathbf{a}, \mathbf{m}, \mathbf{h}, \mathbf{h}_1), PO(\mathbf{a}, \mathbf{m}, \mathbf{h}_2, \mathbf{h}_3), \neg PO(\mathbf{a}, \mathbf{m}, \mathbf{h}_1, \mathbf{h}_2),$   
 $\neg PO(\mathbf{a}, \mathbf{m}, \mathbf{h}_1, \mathbf{h}_3), \neg PO(\mathbf{a}, \mathbf{m}, \mathbf{h}, \mathbf{h}_2)$  and  $\neg PO(\mathbf{a}, \mathbf{m}, \mathbf{h}, \mathbf{h}_3),$   
 $HOLDS(\mathbf{m}, \mathbf{h}, \mathbf{p}), \neg HOLDS(\mathbf{m}, \mathbf{h}_3, \mathbf{p})$

# Equivalence theorem

- ▶ Direct equivalence between STIT and OntoSTIT theorems impossible
- ▶ Standard Translation technique (Blackburn, van Benthem, Wajszczyk):  
Equivalence between STIT and the subtheory of OntoSTIT limited to a *translation* of STIT's formulas (T6)
- ▶ For all  $\phi \in L_S$ :  $\vdash_S \phi \leftrightarrow \vdash_O \forall x_1, x_2 (IN(x_1, x_2) \rightarrow T_{x_1, x_2}(\phi))$



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# Expressing agency, not action

- ▶ 'Robert switches off the light':
  - ▶  $\forall h(PO(\mathbf{Robert}, \mathbf{n}, \mathbf{h}, h) \rightarrow HOLDS(\mathbf{n}, h, \text{Light-is-off}))$
- ▶ No action, no distinction between the ways of seeing to it that some proposition holds
- ▶ This formula corresponds as well to:  
'Robert unscrews the bulb'  
in fact, it is just:  
'Robert makes sure that the light is off'

## Robert switches off the light

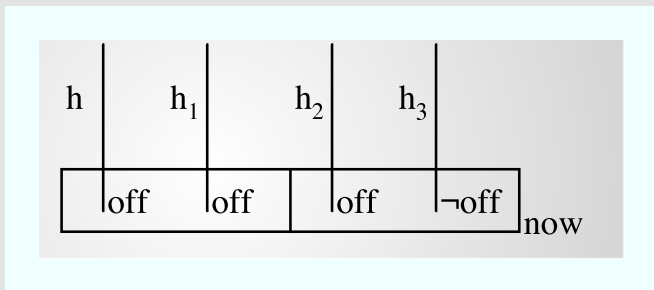


Figure:  $\forall h (PO(\mathbf{Robert}, \mathbf{n}, \mathbf{h}, h) \rightarrow HOLDS(\mathbf{n}, h, \text{Light-is-off}))$



# Duration of underlying actions

- ▶ Considering non-instantaneous actions is possible:
- ▶ ‘Robert drives from Trento to Lublin’
- ▶ ‘Booth kills Lincoln’
  - ▶  $\forall h(PO(\mathbf{Booth}, \mathbf{n}, \mathbf{h}, h) \rightarrow$   
 $\exists m(PRE(\mathbf{n}, m) \wedge HOLDS(m, h, \textit{Lincoln-is-dead}))$
  - ▶ In STIT, we should consider the extension with  
Prior-Thomason’s tense operators (future operator **F**)

# Booth kills Lincoln

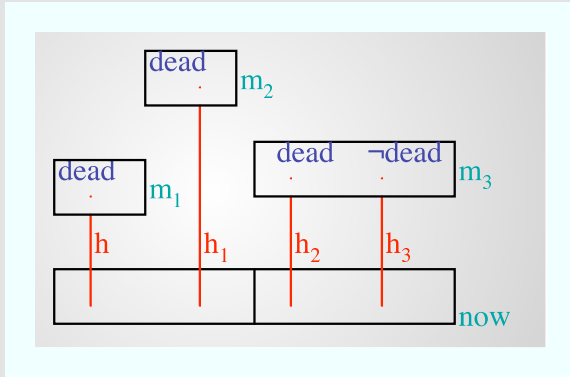


Figure:  $\forall h(PO(\mathbf{Booth}, \mathbf{n}, \mathbf{h}, \mathbf{h}) \rightarrow \exists m(PRE(\mathbf{n}, m) \wedge HOLDS(m, h, Lincoln-is-dead)))$

## Agentive and causal gaps

In (Onto)STIT, actions are seen *ex post acto*  
Choice fully determines the outcome

- ▶ Once Robert decides and start driving from Trento to Lublin, he will reach Lublin in any case, i.e., in all histories of the same “possible outcome”  
he can’t change his mind,  
his car cannot break down...
- ▶ Once Booth has pulled the bullet,  
nothing can stop the course of events of the bullet flying,  
the bullet entering Lincoln’s body, and Lincoln dying

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## Extension of the domain

- ▶ In a non-deterministic world, a single action individual can unfold into different “action courses” on different histories
  - ▶ Action tokens + action courses
  - ▶ Action courses are the actual perdurants here
- ▶ Action courses may have a duration
  - ▶ Intervals (including degenerate ones)

# Extension of the language

## ► New primitive predicates

- *ACT*, is an action token
- *Act*, is an action course
- *CO*, is a course of an action token
- *INT*, is an interval
- *INI*, a moment is in an interval
- *RT*, an action course runs through an interval
- *AGO*, is the agent of an action course

# Extension of the axiomatics

- ▶ Argument restrictions and existential axioms
- ▶ Intervals and moments
  - ▶ Intervals (*IT*) are convex linear stretches of moments with a beginning and an ending moments (based on *INI* and *PRE*)
- ▶ Actions
  - ▶ Each action course (*Act*) runs through (*RT*) a unique interval (*IT*) and is the course of (*CO*) a unique action token (*ACT*)
  - ▶ All the courses of the same action token share the same agent (*AGO*) and the same starting moment. — Their ending moments may differ.
- ▶ Actions and Choices
  - ▶ The courses of a same action token lie on histories that belong to the same equivalence class of possible outcomes

# Expressing actions and filling the agentic gap

- ▶ 'Robert switches off the light':

- ▶  $\exists x (ACT(x) \wedge \textit{switching-off-the-light}(x) \wedge$   
 $\forall y, h (CO(y, x) \wedge LON(y, h) \rightarrow$   
 $AGO(\mathbf{r}, y) \wedge BAct(\mathbf{n}, y) \wedge EAct(\mathbf{n}, y) \wedge HOLDS(\mathbf{n}, h, \textit{Light-is-off})))$

- ▶ 'Booth kills Lincoln':

- ▶  $\exists x (ACT(x) \wedge \textit{killing-Lincoln}(x) \wedge$   
 $\forall y, z (CO(y, x) \wedge LON(y, \mathbf{h}) \wedge EAct(z, y) \rightarrow$   
 $AGO(\mathbf{b}, y) \wedge BAct(\mathbf{n}, y) \wedge HOLDS(z, \mathbf{h}, \textit{Lincoln-is-dead})))$

- ▶ Integrating actions into the picture helps also dealing correctly with agency in a non-deterministic world, where agents are independent



# Booth kills Lincoln

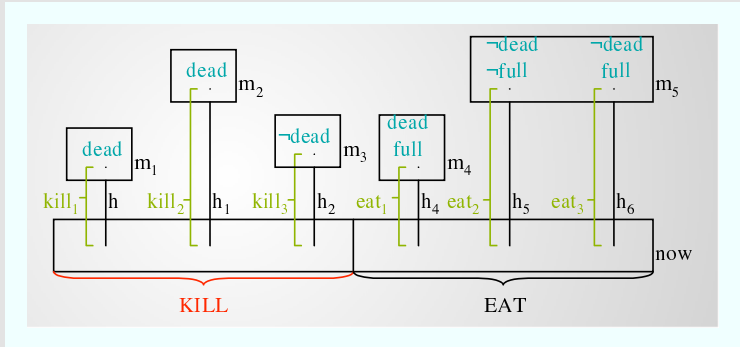


Figure:  $\exists x(ACT(x) \wedge$

$killing-Lincoln(x) \wedge \forall y, z(CO(y, x) \wedge LON(y, \mathbf{h}) \wedge EAct(z, y) \rightarrow$   
 $AGO(\mathbf{b}, y) \wedge BAct(\mathbf{n}, y) \wedge HOLDS(z, \mathbf{h}, Lincoln-is-dead)))$

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## Where we are

- ▶ Integrated action and agency into a first-order theory
  - ▶ Allowed for actions with duration in a non-deterministic context, i.e.,
  - ▶ Allowed for actions that may: abort, last longer or less
- 
- ▶ Showed how ontology studies can benefit from work done in modal logics

## Further improvements and extensions

- ▶ refine the temporal theory, add causality into the picture
- ▶ define categories of action, starting with their temporal patterns
- ▶ introduce a notion of expected (intended) outcome to define finer-grained categories
- ▶ substitute reified propositions for full atomic formulas (eliminate *HOLDS*, adding a moment and a history as arguments to all “regular” predicates)
- ▶ back to logic: work out a modal equivalent; find a decidable fragment...
- ▶ and... check if this is a methodology to apply elsewhere

## Another approach – a modal logic of agency and actions with duration

- ▶ alternative to OntoSTIT+: modal logic
  - ▶ actions with *duration*, explicit *continuation*, and *control* over them

$$\varphi =_{def} \perp \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \mathcal{S} \varphi \mid \varphi \mathcal{U} \varphi \mid \\ \Box\varphi \mid [\beta:a]\varphi \mid [\overline{\alpha:a}]\varphi \mid Stit_A\varphi$$

- ▶ PDL-like operator of action =  $[\beta:a]\varphi$ 
  - ▶ action  $\beta$  is being triggered by  $a$ , and  $\varphi$  will be true after
- ▶ Conversely:  $[\overline{\alpha:a}]\varphi$ 
  - ▶ action  $\alpha$  performed by  $a$  ends, and  $\varphi$  was true before
- ▶ explicit ‘continuation of an action’  $\alpha : [\alpha_\lambda:a]\varphi$

# Similarities

## ► courses of actions are linear

- $\models [\beta:a]\varphi \leftrightarrow \langle\beta:a\rangle\varphi$
- $\models [\overline{\alpha:a}]\varphi \leftrightarrow \langle\overline{\alpha:a}\rangle\varphi$
- $\models [\alpha:a][\overline{\alpha:a}]\varphi \rightarrow \varphi$

## ► relations between action and choice

- $\models \neg\Diamond Stit_i\langle\alpha:j\rangle\top$   $i \neq j$
- $\models \langle\alpha:a\rangle\top \leftrightarrow Stit_a\langle\alpha:a\rangle\top$
- $\not\models [\alpha:a]\varphi \rightarrow Stit_a[\alpha:a]\varphi$

# Differences

- ▶ actions are under control till their end (Searle)
  - ▶  $\models \langle \alpha : a \rangle \top \rightarrow \langle \alpha_\lambda : a \rangle \top \mathcal{U} \langle \overline{\alpha : a} \rangle \top$
  - ▶ it is not expressed yet in OntoSTIT+
- ▶ actions are *types* (without hierarchy)
  - ▶ no *tokens*, while they are present in OntoSTIT+

## Some References

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# OntoSTIT - Axiomatics (1/3)

## ► Temporal order on moments

(As1)  $PRE(x, y) \rightarrow MO(x) \wedge MO(y)$

(As2)  $PRE(x, y) \wedge PRE(y, z) \rightarrow PRE(x, z)$

(As3)  $\neg PRE(x, x)$

(As4)  $PRE(x, z) \wedge PRE(y, z) \rightarrow x = y \vee PRE(x, y) \vee PRE(y, x)$

(As5)  $\forall x, y \exists z ((PRE(z, x) \vee z = x) \wedge (PRE(z, y) \vee z = y))$

## ► Agents

(As6)  $\exists x AG(x)$

## OntoSTIT - Axiomatics (2/3)

### ► Moments and histories

$$(As7) \exists x MO(x)$$

$$(As8) IN(x, y) \rightarrow MO(x) \wedge HT(y)$$

$$(As9) MO(x) \rightarrow \exists y IN(x, y)$$

$$(As10) HOLDS(x, y, \mathbf{p}) \rightarrow IN(x, y), \text{ for each constant } \mathbf{p} \text{ in } \Pi$$

### ► Histories

$$(Ds1) LO(z) \triangleq \forall x, y (IN(x, z) \wedge IN(y, z) \rightarrow x = y \vee PRE(x, y) \vee PRE(y, x))$$

$$(Ds2) MLO(x) \triangleq LO(x) \wedge \neg \exists y (LO(y) \wedge x \neq y \wedge \forall z (IN(z, x) \rightarrow IN(z, y)))$$

$$(As11) HT(x) \rightarrow MLO(x)$$

$$(Ds3) UD(x, y, z) \triangleq \exists t (PRE(z, t) \wedge IN(t, x) \wedge IN(t, y))$$

# OntoSTIT - Axiomatics (3/3)

## ► Agency (Possible Outcomes)

(As12)  $PO(\mathbf{a}, y, z, t) \rightarrow MO(y) \wedge HT(z) \wedge HT(t)$ ,  
for each constant  $\mathbf{a}$  in  $\Lambda$

(As13)  $PO(\mathbf{a}, y, z, t) \rightarrow IN(y, t)$ ,

(As14)  $IN(y, z) \rightarrow PO(\mathbf{a}, y, z, z)$ ,

(As15)  $PO(\mathbf{a}, y, s, t) \wedge PO(\mathbf{a}, y, t, z) \rightarrow PO(\mathbf{a}, y, s, z)$ ,

(As16)  $PO(\mathbf{a}, y, z, t) \rightarrow PO(\mathbf{a}, y, t, z)$ ,

(As17)  $PO(\mathbf{a}, y, t, t') \wedge UD(t', t'', y) \rightarrow PO(\mathbf{a}, y, t, t'')$ ,

(As18)  $PO(\mathbf{a}_1, y, z_1, t_1) \wedge \dots \wedge PO(\mathbf{a}_k, y, z_k, t_k) \rightarrow$   
 $\exists t (PO(\mathbf{a}_1, y, z_1, t) \wedge \dots \wedge PO(\mathbf{a}_k, y, z_k, t))$ ,  
for different constants  $\mathbf{a}_1, \dots, \mathbf{a}_k$  in  $\Lambda$ , for any  $k > 1$

# Extension of the axiomatics (1/2)

## ► Intervals and moments

$$(Ap1) \quad INI(x, y) \rightarrow MO(x) \wedge INT(y)$$

$$(Ap2) \quad INT(x) \rightarrow \forall x, y (INI(x, z) \wedge INI(y, z) \rightarrow x = y \vee PRE(x, y) \vee PRE(y, x))$$

$$(Dp1) \quad BEG(x, y) \triangleq INI(x, y) \wedge \forall z (PRE(z, x) \rightarrow \neg INI(z, y))$$

$$(Dp2) \quad END(x, y) \triangleq INI(x, y) \wedge \forall z (PRE(x, z) \rightarrow \neg INI(z, y))$$

$$(Ap3) \quad INT(x) \rightarrow \exists y, z (BEG(y, x) \wedge END(z, x))$$

$$(Ap4) \quad INT(x) \wedge INI(k, x) \wedge INI(l, x) \wedge PRE(k, y) \wedge PRE(y, l) \rightarrow INI(y, x)$$

$$(Dp3) \quad TP(x, y) \triangleq \forall z (INI(z, x) \rightarrow IN(z, y))$$

$$(Ap5) \quad INT(x) \rightarrow \exists y (TP(x, y))$$

## Extension of the axiomatics (2/2)

### ► Actions

$$(Ap6) \quad RT(x, y) \rightarrow Act(x) \wedge INT(y)$$

$$(Ap7) \quad Act(x) \rightarrow \exists!y(RT(x, y))$$

$$(Ap8) \quad CO(x, y) \rightarrow Act(x) \wedge ACT(y)$$

$$(Ap9) \quad Act(x) \rightarrow \exists!y(CO(x, y))$$

$$(Ap10) \quad ACT(x) \rightarrow \exists y(CO(y, x))$$

$$(Ap11) \quad ACT(x) \rightarrow \exists!y(AG(y) \wedge \forall z(CO(z, x) \rightarrow AGO(y, z)))$$

$$(Dp4) \quad BAct(x, y) \triangleq \exists s(RT(y, s) \wedge BEG(x, s))$$

$$(Dp5) \quad EAct(x, y) \triangleq \exists s(RT(y, s) \wedge END(x, s))$$

$$(Ap12) \quad CO(x, z) \wedge CO(y, z) \rightarrow \exists t(BAct(t, x) \wedge BAct(t, y))$$

$$(Dp7) \quad LON(x, y) \triangleq \exists s(RT(x, s) \wedge TP(s, y))$$

### ► Actions and Choices

$$(Ap13) \quad CO(x, z) \wedge CO(y, z) \wedge AGO(u, x) \wedge BAct(w, x) \wedge LON(x, k) \wedge LON(y, l) \rightarrow PO(u, w, k, l)$$

## Preconditions of “action” can be expressed in OntoSTIT

‘Robert switches off the light’ only if the light was on

- ▶  $\forall x PRE(x, \mathbf{n}) \rightarrow \exists y (PRE(x, y) \wedge PRE(y, \mathbf{n}) \wedge \neg HOLDS(y, \mathbf{h}, \text{Light is off})) \wedge$   
 $\forall h (PO(\mathbf{Robert}, \mathbf{n}, \mathbf{h}, h) \rightarrow HOLDS(\mathbf{n}, h, \text{Light-is-off}))$