Qualities in Possible Worlds

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Abstract. The paper analyzes how and under which assumptions it is possible to compare (in a relationist setting and relatively to qualities) entities living in different worlds. We begin with a standard technique to construct quality kinds via an abstraction process. In the first case, the process is applied across all the possible worlds and we show that the resulting quality system has problematic consequences. Then, we focus on the alternatives that arise when the abstraction process is applied within each single world independently, i.e., assuming similarity judgments make sense only when referring to entities living in the same world. This situation leads to worlds with unrelated quality systems and we look at the problem of quality comparison across worlds. We analyze under which assumptions this comparison is possible and discuss its limits by considering the structural information that one can infer from the elements shared by (two or more) overlapping worlds. Exploiting the use of such information and comparing this situation with the construction of time in branching worlds, it becomes possible to relate and (in a sense to be explained) to 'tune' the quality systems of different worlds.

Motivations for this work come from epistemological considerations. Consider a possible world as a context or an information system. The framework we develop helps to understand whether the quality systems of the two contexts (information systems) can be related and, if so, it provides a basic methodology to formally link them.

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Introduction

There are two traditional and alternative views about time and they are often identified with the two philosophers that most contributed to them, namely, Newton and Leibniz. The Newtonian position or *substantiavalism* claims that time *flows equably without relation to anything external*, that is, time is a container-like manifold and what happens occupies it only *contingently*. Leibniz contrasted this position for its ontological import and pushed forward what is also known as *relationism*, i.e., the view that time is derived from relationships between events. Analogous distinctions arise when dealing with the notion of space since the view of space as an independent *container* and the view of space as a conceptual *construction* are both consistent and philosophically sound.

One can establish a parallelism between this philosophical contraposition about time and the main philosophical alternatives about *properties*: universalism, trope theory, and resemblance nominalism (see [2] for a good overview). *Universalism* assumes properties (called *universals*) as primitive entities of which *particulars* (specific events, objects, etc.) are instances. Since the nature of universals and their relationships are independent from the instances, universalism mirrors the substantivalist approach. *Trope theory* assumes properties as classes of *exactly resembling* tropes, the latter being individualized properties that inhere in particulars. That is, properties are constructed from tropes by recognizing that some particulars possess resembling tropes. Finally, *resemblance nominalism* rejects the existence both of tropes and of universals and constructs properties as classes of *resembling* particulars. In this case, only one resemblance relation is admitted and, it is well known, in this approach co-extensional properties are identified (attempts to overcome this consequence make the construction of properties quite complex [11]).

Both trope theory and resemblance nominalism are interesting theories for comparing particulars in a relationist setting. However, to directly connect our approach to the relationist view of time and space where tropes are not considered, here we take a weaker position than trope theory but, to avoid complicated constructions, stronger than resemblance nominalism. More specifically, we assume that particulars are comparable only with respect to a fixed number of properties. Because we are interested in lengths, weights, volumes, masses, shapes, colors, etc., we call these properties *quality kinds*. In addition, we say that two particulars share the same *quality* if they are indistinguishable with respect to a quality kind, for example, they have exactly the same weight. In terms of trope theory, this presupposes a system of type $\langle O, T^1, \ldots, T^n, \mathsf{i}, \equiv \rangle$, where O is the set of particulars (also called objects), $T^1, \ldots, T^n$ are disjoint sets of tropes corresponding to the quality kinds, i is the inherence relation between tropes and particulars, and $\equiv$ is the exact resemblance relation holding only between tropes of the same kind. In this system, the fact that two particulars x and y share a quality of kind i can be stated by $\exists t, s \in T^i\, (\mathsf{i}(t, x) \wedge \mathsf{i}(s, y) \wedge t \equiv s)$, i.e. qualities can be understood as equivalence classes of exactly resembling tropes. To avoid tropes we associate to each quality kind a resemblance relation directly holding between particulars, i.e. we consider a system of type $\langle O, \equiv^1_O, \ldots, \equiv^n_O \rangle$. The sharing of a quality of kind i is here represented by $x \equiv^i_O y$, i.e. qualities can be associated to equivalence classes of exactly resembling particulars. This system is stronger than resemblance nominalism because of the presence of n different resemblance relations. On the other hand, it is weaker than trope theory because tropes themselves cannot be reconstructed in it. Note that, those committing to tropes are not left out. They can rephrase our formalization adopting the definition of the i-resemblance between particulars proposed above in the system of tropes.

When modeling time (and space) it is standard to introduce *structural constraints*. For example, a precedence relation can force time to be linear or branching, a congruence relation can constrain the metric, etc. Even though less usual, these structural constraints are not uncommon for quality kinds. For example, a RGB structure can be assumed for colors, and weights are usually linearly arranged. Clearly different quality kinds have different structures, therefore, in general, we will apply structural constraints separately for each quality kind.

The paper starts (section 1) with a basic structure in a single world and develops to consider more complex structures as well as different possible worlds. The idea is to look at the construction of time from an event structure as a guideline and to extend it to possible worlds. The second part (section 2) looks at the different ways to introduce quality kinds in possible worlds and to relate quality systems of different worlds. Our motivation is twofold: on the one hand we want to see what needs to be assumed (and for which reason) in a relationist framework for qualities. On the other hand we want to understand how and when epistemological considerations should enter to complete the overall system. If a possible world is seen as a context or an information system, one can rephrase our work as a theoretical study on how these contexts or systems can be formally related with a strong emphasis on the principles that may help in setting their relationships.

1. From events to time, from objects to quality kinds

Following the relationist approach, this section begins defining an *abstraction process* and looks at the construction of time in an event structure to later implement it for the construction of quality kinds. Toward the end of the section, we see how to deal with structural constraints for quality kinds.

1.1. Abstraction process

Fix a generic structure $\mathcal{S} = \langle D, \equiv \rangle$ where D is a non-empty set (the domain) and $\equiv$ is a reflexive, symmetric and transitive binary relation (an equivalence relation) on D. One obtains a new structure $\mathcal{S}^e = \langle D^e, =^e \rangle$ where D^e is the set of (non-empty) equivalence classes[1] of D and $=^e$ is the equality on D^e. The process that leads to the new structure $\mathcal{S}^e$ is quite standard and is known as *abstraction*. In the case of time, the above structure is called an event structure $\mathcal{E} = \langle E, \equiv_E \rangle$: E is a non-empty set of *events* and $\equiv_E$ is the *temporal coincidence* relation between them. The abstract structure of $\mathcal{E}$, called *time structure*, is $\mathcal{T} = \langle T, =^e \rangle$ where T is informally the set of *times* (temporally indiscernible events)[2]. The intuition behind this construction is that different events can be temporally co-localized, 'they happen at the same time' one would say. Times, then, are the result of abstracting from events by considering their temporal aspect only.

Theories of properties constructed from the *resemblence* relation are the result of a similar process applied on other aspects of the entities in the domain. First, these entities are grouped via the relation of *exact resemblance* which is associated to a quality kind. For example, the property 'being scarlet' is abstracted from entities that resemble with respect to the color quality kind. Formally, given $\mathcal{O} = \langle O, \equiv_O^c \rangle$ (where O is a non-empty set of *objects* and $\equiv_O^c$ is the equivalence relation of *color exact resemblance in O*), it is possible to build the color abstract structure $\mathcal{C} = \langle C, =_C \rangle$ where C is the set of equivalence classes representing *color properties*. In presence of different quality kinds, we consider several resemblance relations, one for each quality kind. Therefore, the general structure has form $\mathcal{O} = \langle O, \equiv_O^1, \ldots, \equiv_O^n \rangle$.[3] From this, we can abstract n different structures: $\langle D^1, =_{D^1} \rangle, \ldots, \langle D^n, =_{D^n} \rangle$ where D^i is the equivalence class of objects resembling each other with respect to $\equiv_O^i$ (i-resembling, for short), i.e. qualities of kind i.

[1] That is, $x^e \in D^e$ if (*i*) $x^e \subseteq D$ is non-empty and (*ii*) whenever $a \in x^e$, then $b \in x^e$ if and only if $a \equiv b$.

[2] Following standard practice, 'times' is used as generic term. Here the events can be extended or punctual. The construction is exactly the same in both cases although extended events generate extended times, while punctual events generate punctual times.

[3] In practice, some $\equiv_O^i$ could be defined on a subset of O only. This is quite common in knowledge representation and can be captured introducing these subsets of O as separate domains or introducing sorts. These technicalities are largely irrelevant: they make the formalism more complicated without affecting the general argument. We disregard them here.

1.2. Structuring

Our next goal is to introduce structural information on the system(s) obtained by abstraction. We do this by considering further relations in the structures and studying the constraints they impose on the quality kinds. Let us go back to the general structure $\mathcal{S} = \langle D, \equiv \rangle$ and its abstraction structure $\mathcal{S}^e = \langle D^e, =^e \rangle$. Structuring relations can be introduced directly at the level of the domain (the set of events in the case of times, the set of objects in the case of qualities). Let us add the structuring relation R to $\mathcal{S}$ and put $\mathcal{S}' = \langle D, \equiv, R \rangle$. On the basis of R, a new relation R^e (the abstraction of R) can be defined in the abstraction structure. Let R be binary, then we put: $x^e\, R^e\, y^e \triangleq \exists a \in x^e, b \in y^e\; a\, R\, b$. The abstraction structure of $\mathcal{S}'$ is then $\mathcal{S}'^e = \langle D^e, =^e, R^e \rangle$.

Given an event structure, one can induce an ordering on the abstract structure of times by using a precedence relation $\lhd_E$ (asymmetric and transitive, i.e., a strict order) for R. Let $\mathcal{E}_{pre} = \langle E, \equiv_E, \lhd_E \rangle$ be the event structure $\langle E, \equiv_E \rangle$ augmented with the new relation. The idea is to use $\lhd_E$ to further *structure* times. The structure we obtain with the technique described earlier is an *ordered time structure* $\mathcal{T}_{pre} = \langle T, \equiv_T, \lhd_T \rangle$ where $\langle T, =^e \rangle$ is a time structure and $\lhd_T$ is the abstraction of $\lhd_E$. Further constraints arise by considering a quaternary relation between events: $e_1 e_2 \preceq_E e_3 e_4$ stands for "the distance between e_1 and e_2 is less or equal to the distance between e_3 and e_4". This relation induces metric constraints on events and, indirectly, on times. Let $\mathcal{E}_{cg} = \langle E, \equiv_E, \lhd_E, \preceq_E \rangle$ be the previous event structure augmented with $\preceq_E$. The associated time structure is $\mathcal{T}_{cg} = \langle T, =^e, \lhd_T, \preceq_T \rangle$ where $\preceq_T$ is the abstraction of $\preceq_E$.

All the constructions we have just implemented on event structures can be applied to structures for qualities $\mathcal{O} = \langle O, \equiv^1_O, \ldots, \equiv^n_O \rangle$ when expanded with structuring relations. For example, quality kind i can be enriched with m structuring relations $R^i_1, \ldots R^i_m$ on objects that induce corresponding abstraction relations $R^{i,e}_1, \ldots R^{i,e}_m$ on qualities.

Digression 1. Instead of $\mathcal{S} = \langle D, \equiv \rangle$, one can start from a structure $\mathcal{S}_p = \langle D, \sqsubseteq \rangle$ where $\sqsubseteq$ is a reflexive and transitive binary relation (a pre-order) on D. A new relation $\approx$ on D is defined by $x \approx y \triangleq x \sqsubseteq y \wedge y \sqsubseteq x$, which turns out to be an equivalence relation. At this point, one applies the previous abstraction process on $\mathcal{S}' = \langle D, \approx \rangle$. The relation $\sqsubseteq^e$ (the abstraction of $\sqsubseteq$) turns out to be an order. One sees that relations $\sqsubseteq$ and $\sqsubseteq^e$ are more expressive than $\equiv$ and $=^e$, they include some (weak) structuring constraint. In temporal structures with *extended* events, the relation $\sqsubseteq_E$ is usually interpreted as *temporal inclusion*. In the case of qualities, the relation $\sqsubseteq_O$ can be the *ordered inexact resemblance* that induces a relation of *specialization* between qualities. When we pair $\sqsubseteq_E$ with a precedence relation as in structure $\langle E, \sqsubseteq_E, \lhd_E \rangle$, a different abstraction process, known as *filtering*, can be defined (see [7,14]). The abstraction process based on filters is stronger than the one based on equivalence relations since it may generate times that do not correspond to (the temporal extension of) events in the domain E. We do not discuss further this alternative since the specific abstraction methods are marginal to our goal. Also, note that the number of primitive relations in the structures is not very informative since it may be reduced without loss of expressivity. For example, both a pre-order and a precedence relations can be defined in a structure $\langle E, || \rangle$, where $||$ is the *meets* relation between extended events as defined in [1].

Digression 2. The abstraction R^e of a relation R has been defined using an existential quantifier: $x^e\, R^e\, y^e$ was defined by $\exists a \in x^e, b \in y^e$ such that $a\, R\, b$. It follows that there might be entities $c \in x^e$ and $d \in y^e$ that are not in relation R, i.e., c is equivalent to a, d is equivalent to b, $a\,R\,b$ and $\neg c\,R\,d$. It may look strange that the same relationship can be true for some entities and false for others since these very entities are all indistinguishable with respect to the quality kind related to that relationship. The situation does not change if R^e is defined using a universal quantification: the same problem arises considering 'negative' statements. One can force a sort of homogeneity by adding specific constraints on relations R like, e.g., $x \equiv y \rightarrow \forall z(z\,R\,x \leftrightarrow z\,R\,y)$.

We have seen that *both* times (qualities) and their structure are built from events (objects, respectively) and their relations; nothing else is needed. Technically, it is possible to introduce the structuring relations directly in the abstract structure and to define the corresponding relations on events or objects. For example, let us introduce the relation R^e in $\mathcal{S}^e$ (the abstraction structure of $\mathcal{S}$). Given R^e, in $\mathcal{S}$ it is possible to define R as: $a\,R\,b$ iff there exist $x^e, y^e \in D^e$ such that $x^e\,R^e\,y^e$ and $a \in x^e, b \in y^e$. Once the abstraction process is fixed, it is possible to introduce the structuring relations in $\mathcal{S}^e$ or in $\mathcal{S}$. External motivations may drive this choice, like the objective vs. the subjective *nature* of these relations. The philosophical construction of time from events seems to commit to the ontological nature of relations, but this is hardly the case for qualities like color or shape. For example, it is widely accepted that colors can be structured in different ways (e.g., RGB, CMYK, HSB), that is, the colors themselves do not isolate a unique structure. In [9], it has been shown that, given a quality kind, a single *ontological* exact resemblance relation can generate *all* the qualities (called *qualia* in [9]) relative to this kind. In such approach, structuring relations are introduced directly on qualia and different structural constraints can be applied to the same set of qualia.

In this paper, we assume a weaker position (that gives also a direct parallelism with the construction of time) and introduce relations directly on the elements of the domain. This does not prevent us from considering them as ontological or as epistemological relations, and allows for a direct connection with the approach in [9]. Given this assumption and the analogy between the construction of time and the construction of quality kinds, in the next section we proceed by looking at *quality structures* $\mathcal{QS}$ of form $\langle D, \equiv^1, \ldots, \equiv^n, R_1^1, \ldots, R_{m_1}^1, \ldots, R_1^n, \ldots, R_{m_n}^n \rangle$. Note that we concentrate on sets D of generic objects. Thus, D may contain both objects and events, so that some classes represent the temporal quality, others physical qualities, etc. However, we look at qualities in general and the analysis of the particular commonsense relationship between different types of qualities is out of the scope of this paper.

2. Quality Change through Worlds

In the previous sections we motivated the use of a unique quality structure $\mathcal{QS}$ and explained the role of the relations in generating and structuring qualities. In particular, we have seen that the *exact* i-resemblance for an entity $a \in D$ is given by the equivalence class built from $\equiv^i$ to which a belongs. *Similarity* notions, generally of qualitative or metrical nature, can be expressed via (combinations of) the structural relations $R_1^i, \ldots, R_{m_i}^i$.

In this section, we study how to compare entities living in *different* worlds. In a sense, $\mathcal{QS}$ gives us all the tools to compare entities living in the same world, since the quality structure encodes all the types of comparisons we are interested in. Things get more complicated when there are different worlds. Let us add to $\mathcal{QS}$ a set of possible worlds W and the relation *being in a world*. Writing $a{\downarrow_w}$ for "$a \in D$ is in the world w", we consider $\mathcal{QS} = \langle D, W, \downarrow, \equiv^1, \ldots, \equiv^n, R^1_1, \ldots, R^1_{m_1}, \ldots, R^n_1, \ldots, R^n_{m_n} \rangle$ and use this structure to analyze different technical and philosophical positions.

A first approach is characterized by *cross-world* equivalence which consists in assuming that the $\equiv^i$ relations are independent from $\downarrow$, i.e., they apply to entities no matter in which world they live. Although this setting seems to arise naturally from the one-world case, it suffers from a puzzling problem. Note first that it must be possible that some entity changes (at least some of) its qualities in different worlds. (If not, all the worlds would look the same and we loose the reason for introducing them.) Now, if entity a 'persists' through worlds (i.e. a exists in different worlds) and a's quality of kind i changes in two worlds where a persists, then to which equivalent class of $\equiv^i$ does a belong? For example, let $w \neq w'$ with $a{\downarrow_w}$, $a{\downarrow_{w'}}$ and assume a is red in w and yellow in w'. Assuming $\equiv^c$ is a cross-world relation for the color quality kind, we get a contradiction if we include a in the class of the red entities as well as if we put it in the class of the yellow ones. Different solutions have been provided to this problem (and to the analogous problem arising in the case of change through time). Here we sketch the more relevant and focus on the *endurantist* solution.

On one side of the spectrum we find David Lewis [8] who claims, analogously to the *stage theory* [5], that entities cannot be in different worlds, they must be *world bound*. Lewis introduces a new relation, *counterpart* (C), to link similar entities in different worlds and uses it to formally interpret the modal operators. Going back to the previous example: if $a{\downarrow_w}$, then not $a{\downarrow_{w'}}$ but there is a counterpart entity a', i.e. an entity for which $\mathsf{C}(a', a)$, such that $a'{\downarrow_{w'}}$ and a is (say) red while a' is yellow. Therefore, with respect to $\equiv^c$, a belongs to the class of red entities while a' belongs to the class of yellow entities. The weakness of this solution is that the counterpart relation must be taken as primitive and why a' (and not, say, a'') is the counterpart of a is not explained by the theory.

The *intensional* interpretation of the modality [3] matches up with the *perdurantist* position with respect to change through time [13]. Here an entity a has a *world stage* $a_{/w}$ in each world w to which it belongs, also $a_{/w}{\downarrow_{w'}}$ is false whenever $w \neq w'$. To formalize the color example, one now has three entities, a, $a_{/w}$, and $a_{/w'}$, and the $\equiv^c$ applies to world stages only: $a_{/w}$ belongs to the class of red stages while $a_{/w'}$ belongs to the class of yellow stages. Then, we can say that a is red *at* w, because it has a world stage $a_{/w}$ that it is red (for criticisms of this reduction of predication to a world see [12,15]).

The *endurantist* solution [16,10] to cross-world change requires the introduction of a world argument in the properties: entity x is not red in general, it can be red relatively to a world which must be specified. The principal criticism to this solution regards the *de facto* negation of *intrinsic properties*: all the properties become relations with the worlds.

Our approach stems from this latter position. Assume the set of quality kinds is fixed across possible worlds,[4] and that equivalence and structuring relations are *localized* to single worlds, i.e. qualities are word dependent. Let a, b be both in w, then $a \equiv^i_w b$ means that a i-resembles b in the world w. Maybe, in another world, a and b do not resemble each other with respect to the quality kind i. This means that the equivalence classes themselves are local: the class of entities that are red in w could be completely different from the class of entities that are red in w'. This approach weakens the standard endurantist position. While endurantists are able to link the class of (say) red objects

[4] This assumption is not necessary but simplifies the comparison.

in w with the class of red objects in w' (because they use a single red property with an additional world-index), this move is not possible in our approach. All we know is which quality kind the localized qualities refer to. Instead, the correspondence between a quality in w and a quality in w' is not given to us: given the class of red objects in one world, one has no way to infer which class corresponds to it in a different world. A new equivalence at the level of the qualities in the two worlds is needed. One way to obtain it is by iterating the abstraction process. Alternatively, one could introduce an additional primitive playing a role similar to the counterpart relation (here over qualities). However, we are interested in understanding whether and on which assumptions such a link can be derived without additional primitives. Once again a look at the construction of time becomes helpful.

2.1. The construction of times in branching-worlds

Graeme Forbes [4] describes the construction of time-series from event-series via an abstraction process. In his approach, the construction is localized to worlds: in each world he constructs a separate set of times on the basis of the events present in that world. Similarly to the case of qualities, Forbes has to face the problem of relating across worlds the result of the localized abstractions. Before analyzing the solution proposed by Forbes, let us clarify the framework he adopts. The only entities he considers are events, in particular punctual events.[5] The notion of *branching worlds* plays a central role in the construction and he characterizes it in this way: "two worlds may share an initial segment of their courses of history, diverging from each other only after a certain point. Such worlds are called branching worlds." ([4]. p.86). Two worlds share those events that lay in the shared course of history while events in different branches (that is, after the worlds separate) are necessarily different. Worlds that do not share an initial segment of their course of history are totally apart and Forbes argues that no interrelationship between them should be sought. In this setting, a single equivalence relation on events is enough to construct (localized) times by independently applying it to the set of events present in each world. Additional *precedence* ($\lhd_T$) and *distance* (d) relations between events are introduced, we will see below their roles.

Forbes' proposal consists in reiterating the abstraction process. Once localized times are obtained, he focuses on the branching relationship among worlds. Since this is an equivalence relation, one can apply the abstraction process to worlds to obtain equivalence classes of branching worlds: worlds are in the same branching class if and only if they share some initial segment of their courses of history. The time relationship across worlds is obtained by a new abstraction process that applies to the (localized) times of all the worlds belonging to the same branching class. (Times in worlds belonging to different branching class remain unrelated.) The abstraction that leads to equivalence classes among localized times becomes possible because of the order and the distance relationships and because any pair of worlds in the same branching class share a segment, i.e., at least two punctual *times*. Two times t_1 and t_2 in two different branches are said to be equivalent if they have same distance from a time t shared by the two worlds: $t_1 \equiv_T t_2$ iff

[5] According to Forbes, punctual events are considered because they simplify the construction.

$d_T(t_1, t) = d_T(t_2, t)$[6] and $t \lhd_T t_1, t \lhd_T t_2$. The class of times given by $\equiv_T$ is the time-series for the whole branching class. Before proposing a generalization of this construction for qualities, some observations are needed.

As we have seen, the assumption of a 'shared segment' of events in two branching worlds plays a key role in the construction since it grants the alignment of the measurement systems. Since most quality kinds are organised neither in one dimension nor linearly, the reference system needed to relate qualities in different worlds might be much more complex than that used by Forbes. As a consequence, the required shared component between two worlds might be much richer if it has to ensure that the quality kinds can be compared and aligned. Also, we must be clear on the meaning of the term 'shared segment'. One might mean to say that there is (at least) an event that is in both worlds. A stronger reading would imply that the worlds share (at least) one time, i.e., an equivalence class of events. In Forbes' work these two notions collapse because the equivalence relation is defined in a whole branching class. However, when localized equivalences are adopted, the two readings have different imports. Furthermore, in our localized environment if we use x, for which $x{\downarrow_w}$ and $x{\downarrow_{w'}}$, to align worlds w and w' on quality i, how can we ensure that x remains 'the same' in the two worlds at least with respect to the quality kind i? We face a sort of circularity: we need a common reference system to compare entities in different worlds and we need common entities to establish a shared reference system. Forbes solution goes even further. He does not assume that there are different segments sharing all the entities they contain. He actually claims that exactly *the same* segment is in the two worlds. With this stronger assumption, Forbes forces all the other relations between entities to be shared as well.

2.2. Tuning the System with Epistemological Considerations

Our final goal is to find a way to align qualities in different possible worlds. The goal is trivial if one has some external condition relating the qualities in one world to the qualities in the others. Following Forbes, a more interesting case arises if one assumes that some objects are invariant across worlds so that these furnish exact correspondences between the equivalence classes to which they belong. We believe it is instructive (and in some cases necessary) to start from a much weaker position. We will see that the overall procedure we develop, which we dub *tuning*, often requires considerations that go beyond the information contained in the given world structures. In these cases, we discuss epistemological considerations that allow the procedure to go through. For this reason, we call this an *epistemological tuning*.

In general, we make the overall assumption that there is a one-to-one correspondence between the quality kinds in the worlds and that, for each i, each world has the same number of i-qualities.[7] This correspondence is given beforehand. We illustrate the basic idea by considering two worlds and a single quality kind. The abstraction process we apply is local, that is, $x \equiv_w y \rightarrow (x{\downarrow_w} \wedge y{\downarrow_w})$. Let w, w' be the worlds and $\equiv_w, \equiv_{w'}$ the two relations. The tuning process consists in establishing a (motivated) correspon-

[6] Here the distance between times is not directly induced by the distance between events. Forbes assumes that in the shared segment of the worlds there exist at least two events e_1 and e_2 and uses these to fix a common *origin* and *unit* of measure. Thus, one can define a unique d_T for the two worlds.

[7] An interesting complication (useful in applications) arises if we drop this assumption taking into account worlds where quality kinds have different granularities.

dence between the qualities in w and the qualities in w' which, in turn, makes possible a comparison (with respect to the given quality kind) of objects in the two worlds. We write $q_w \multimap q_{w'}$ to state that quality q_w (an equivalence class in w) corresponds to quality $q_{w'}$ (an equivalence class in w'). For instance, if q_w is the class of red objects in w, then $q_w \multimap q_{w'}$ tells us that q_w is the class of red objects in w'.

We have seen that Forbes uses the 'shared segment' between two worlds w and w' to synchronize the events occurring in these branches. This segment designates what w and w' have in common, their overlapping part so to speak. Our goal is to extend this notion of 'shared segment' to general worlds (that is, to worlds where a branching relationship is not defined) to make it applicable for objects and qualities. Given a quality, we want to collect the elements the two worlds have in common with respect to this quality. In other words, we want to isolate what remains constant through w and w' with respect to the given quality.

We start from the objects that exist in both w and w'. Since the objects can change through worlds, we do not know if two entities equivalent in w are equivalent in w' also; it is possible that an entity is red in both w and w' while another changes from red in w to, say, brown in w', or that all entities are red in w and brown in w'. This situation is expressed formally by saying that from $x \equiv_w y \wedge x{\downarrow_{w'}} \wedge y{\downarrow_{w'}}$ one can infer neither $x \equiv_{w'} y$ nor $q_w \multimap q_{w'}$. To gather more information, we now investigate the information conveyed by the relations compared in the structure. Let D_w be the set of entities in w, R_w the relation R restricted to D_w, and $\varepsilon(R)$ the extension of R. Formally, $D_w = \{x \in D \,|\, x{\downarrow_w}\}$, $R_w(x_1, \ldots, x_n)$ if $R(x_1, \ldots, x_n)$ and $x_j{\downarrow_w}$ for all relevant j, and $\varepsilon(R) = \{\langle x_1, \ldots, x_n\rangle \,|\, R(x_1, \ldots, x_n)\}$. We write $F^i_{w,w'}$ for the set of common facts in w and w' with respect to a quality kind i, that is,

$$F^i_{w,w'} = \bigcup_{j=1}^{n_i} (\varepsilon(R^i_{j,w}) \cap \varepsilon(R^i_{j,w'})).$$

In our simplified case, the set of facts reduces to $F_{w,w'} = \varepsilon(\equiv_w) \cap \varepsilon(\equiv_{w'})$.

We now look at particular cases to exemplify how one can use this information to tune qualities and to show when there is need for further constraints.

Let $F_{w,w'} = \{\langle a_1, a_2\rangle, \langle a_2, a_3\rangle, \langle a_1, a_3\rangle\}$.[8] Then, the three objects a_1, a_2, and a_3 are indiscernible in both the worlds w and w'. Let q^1_w be the quality in w to which a_1, a_2, and a_3 belong, and similarly let $q^1_{w'}$ be the quality in w'. Now, are q^1_w and $q^1_{w'}$ corresponding qualities? To all effects, it is compatible with the available information to assume that the three objects have not changed (with respect the given quality kind) in the two worlds so that one can posit the equivalence $q^1_w \multimap q^1_{w'}$. This choice is depicted in figure 1.a where an equivalence class is graphically represented by a closed line (with its elements listed inside) and the segment connecting the two classes shows that they correspond. Still, it is possible that all three object changes "in the same way". Things may be more complex. Consider the situation depicted in figure 1.b in which $F_{w,w'} = \{\langle a_1, a_2\rangle\}$, $a_3 \equiv_w a_1$, and $\neg a_3 \equiv_{w'} a_1$. Here only a_1 and a_2 are equivalent in both w and w'. This figure represents the choice of taking $q^1_w \multimap q^1_{w'}$ and a_3 as changing object. However, we do not have enough information to rule out other cases. For instance, it might be that both a_1 and a_2 actually change in the same way while $q^1_w \multimap q^2_{w'}$. Finally, nothing stops us from the more radical reading: all the objects actually change, two of which change in the same

[8]To simplify the notation, we omit all pairs $\langle a_i, a_i\rangle$ (reflexivity) as well $\langle a_j, a_i\rangle$ if $\langle a_i, a_j\rangle$ is listed (symmetry).

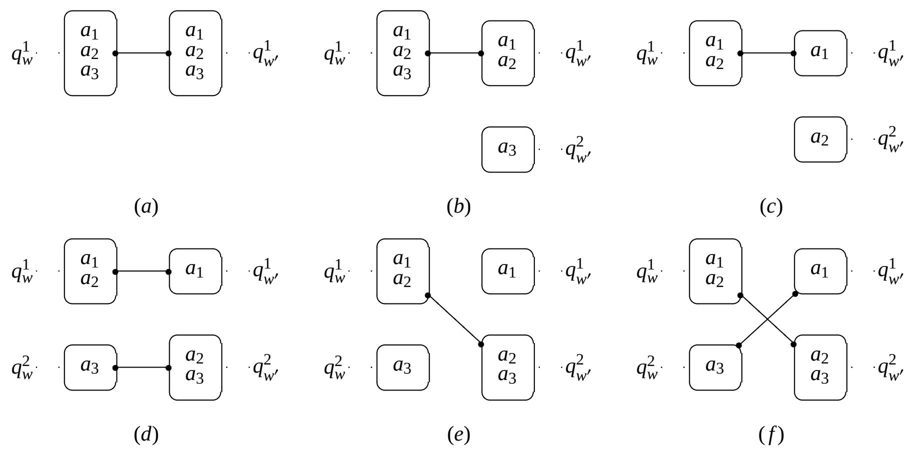

Figure 1. Possible equivalences between qualities in local abstraction processes.

way. These considerations show that additional assumptions need to be taken to justify the correspondence between qualities.

In applications, one can often determine if the two worlds are really apart. For example, consider two satellites and suppose we want to tune their color measurement systems. Let us say that we are given two sets of data about the same piece of land. In tuning the systems, there is a spectrum of options to choose from. If we can assume that the piece of land did not change to a considerable point during the interval in which the data were collected (e.g., if the two sets are collected instantaneously and at exactly the same time), then it is sensible to apply the *minimal object change hypothesis* (mOCH), that is, the systems should be tuned taking the reading that forces the minimal number of changes in the objects as illustrated in figures 1.a, 1.b, and 1.d. Figure 1.c shows that this hypothesis might not be enough. In this figure, one object must change and yet we have two alternatives: $q_w^1 \multimap q_{w'}^1$ (i.e., a_2 changes) and $q_w^1 \multimap q_{w'}^2$ (i.e., a_1 changes).

Suppose now that the satellites are two equivalent copies of the same satellite model. The measurement systems they use are similar and, although they may not give exactly the same values, they are *qualitatively* compatible. Cases like this push us to consider a condition here called *minimal structural change hypothesis* (mSCH). In this reading, structural relations become relevant because a strong structural similarity can be assumed. If a local precedence relation $\lhd_w$ is available, the number of possible correspondences between qualities may decrease considerably. We show this with an example. Consider figure 1.c and add a new object (namely, a_3) to obtain figure 2.a. Both the situations in the figures 2.b and 2.c are compatible with mOCH. In the case of figure 2.c both a_2 and a_3 do not change, but while $a_3 \lhd_w a_2$, we have $a_2 \lhd_{w'} a_3$. Instead, the case in figure 2.b gives us two unchanging objects (a_1 and a_3) while preserving their order as well ($a_3 \lhd_w a_1$ and $a_3 \lhd_{w'} a_1$). Therefore, if we have reasons to assume a strong correspondence between $\lhd_w$ and $\lhd_{w'}$, we have a criterion to prefer the tuning of figure 2.b to that of figure 2.c.

It is easy to construct some other example in which mOCH and mSCH do not individuate a unique correspondence between qualities. But the real issue is that the two hypotheses are independent: their interaction may lead to inconsistent results and they

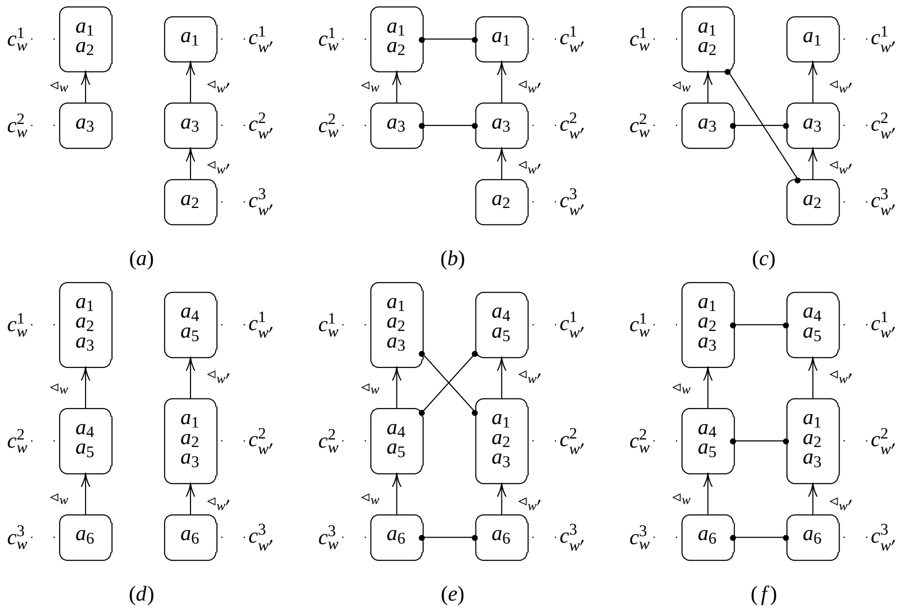

Figure 2. Possible equivalences between local qualities in the presence of local structuring relations.

must be used with some care. Take the situation in figure 2.d. A possible correspondence is shown in figure 2.e where no change in objects is presupposed. However, here the precedence relationships in the two worlds are incompatible. The correspondence in figure 2.f complies with the precedence constraints but presupposes the change of two (or more) objects. One can verify that there is no way to satisfy both mOCH and mSCH. If a choice has to be made (among which we include the choice of adopting weakened versions of the two hypotheses), it should be based on further information. For example, if we know that there is just one satellite that collected the two sets of data about a piece of land with an interval of six months, then hypothesis mSCH becomes more reliable and we can favor the correspondence of figure 2.f over that of figure 2.e.

Once we have reached a motivated correspondence between qualites in w and qualities in w', we should be able to say how and in which cases the correspondence can be used to compare objects in the two worlds. From the abstraction processes, one can define a cross-world equivalence relation between objects in w and objects in w':

$$x \equiv_{w,w'} y \triangleq x \in q_w \wedge y \in q_{w'} \wedge q_w \multimapboth q_{w'}$$

This allows us to compare those objects that belong to qualities connected by the correspondence and the information we have gathered so far is not enough to relate other objects or qualities. The situation changes when structural relations are added. Let us go back to the familiar case of the precedence relation. Suppose that we have motivated (or inferred) a correspondence for which $q^1_w \multimapboth q^1_{w'}$ and $q^3_w \multimapboth q^3_{w'}$. Also, let us say that $q^1_w \lhd_w q^2_w \lhd_w q^3_w$, and $q^1_{w'} \lhd_{w'} q^2_{w'} \lhd_{w'} q^3_{w'}$ and no direct correspondence has been posited between q^2_w and $q^2_{w'}$. If there is only one quality between classes q^1_w and q^3_w, and so between classes $q^1_{w'}$ and $q^3_{w'}$, then statements that hold for objects in q^2_w can be rephrased for objects in $q^2_{w'}$ and vice versa. We can then extend the correspondence between qualia in the two worlds with $q^2_w \multimap\!\circ\, q^2_{w'}$. Furthermore, if the two systems are based on the same metric (the strategy adopted by Forbes), all we need is a minimal correspondence that fixes the reference system and the measurement unit in the two worlds.[9] From this information, the correspondence extends naturally to all the qualities in these worlds.

Finally, what can we say if $F^i_{w,w'} = \emptyset$? We can look for a sequence of worlds $w, w_1, \ldots, w_n, w'$ such that $F^i_{w,w_1} \neq \emptyset$, $F^i_{w_n,w'} \neq \emptyset$, and $F^i_{w_r,w_{r+1}} \neq \emptyset$, for $1 \leq r < n$, and obtain an indirect tuning of w and w'. If such a sequence does not exists, our procedure cannot provide a correspondence and further assumptions must be taken into account.

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[9] It would be useful to analyze the minimal initial correspondence (at least for important sets of structural relations) needed to formally derive an overall correspondence between worlds.