Model-based Auditing with Inequalities

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Abstract. Auditors must rely on evidence of the party being audited, who may have a legitimate or illegitimate interest to overstate an account or understate it or leave it out all together. In this paper, we apply an approach called model-based auditing to use a mathematically precise model of the expected relationships between the flow of money and the flow of goods or services, to analyze potential deficiencies financial reporting. We look in particular at inequalities, derived from the interests of actors involved and from assumptions about the domain, the information system and the control measures. The usefulness of the approach in practice is shown by a case study of a customs warehouse.

Keywords: value modeling, auditing, evidence

1 Introduction

Much legal evidence nowadays is collected and processed in information systems. A particularly interesting aspect of evidence is the burden of proof [1]: which party bears the burden of having to provide the evidence? We follow Christiaanse et al [3] and look at legal evidence from the point of view of auditing theory. In particular, we apply a computational approach to auditing, called model-based auditing [4,13]. The approach makes use of a precise mathematical model of the expected relationships between the flow of money and the flow of goods or services in a transaction (value-cycle). These reconciliation relations can be expressed as a series of audit equations. Given such a model, hypotheses can be generated for testing and requirements about evidence can be formulated.

We start from actor analysis: what are the interests of the actors involved? Do actors have legitimate or illegitimate reasons for overstating or understating some records, or leaving records out altogether? For instance, in a tax declaration, typically businesses want to understate profit, as taxes may be collected as a percentage of profit. Reasons for overstatement or understatement determine the direction of verification. In case of likely overstatement, auditors use positive verification: test accuracy of each of the claims. In case of likely understatement, auditors must use negative verification: compare the completeness of the claims to some other independent source and see whether any records are missing.

In this paper we will further investigate this practice of positive and negative verification using inequalities: relative comparisons between accounts in a
model of the value cycle. To illustrate the approach and show its usefulness in practice, we address a case study of a customs warehouse. A customs warehouse contains so-called bonded goods for which payment duties have been suspended, until a final customs destination is known. To verify completeness of the bonded goods, customs need positive evidence of the free goods, for which duties have already been paid, and independent evidence of the total number of goods in the warehouse. We aim to show that equations expressed in tours better supports an auditor’s line of reasoning. Special focus is on reasoning with inequalities to represent the interests of the actors in the case is instrumental in indicating risks in the internal controls ensuring the reliability of the warehouse management system.

The remainder of the paper is structured as follows. Section 2 contains formal definitions used in our version of model-based auditing. Section 3 develops the idea of using inequalities, rather than equations. Section 4 contains the case study. The paper ends with reflections and directions for future research.

2 Value Nets

Model-based auditing is a computational approach to compliance monitoring and auditing, which is based on a normative meta-model of the value cycle: the movement of money and goods or services [6, 8, 13, 4].

Audit Equations. Economic transactions can be formally modeled as a value cycle which is interlinked to the business model of the enterprise. This view is inspired by accounting models in the owner ordered accounting tradition [12]. When used in computer science, often the purpose of these models is to analyze the representations of actions and events in a business process, and study their well-formedness. Compare for example the use of the REA ontology [11], e3-value [7], or DEMO [5]. For every main type of business (e.g. trade, manufacturing, services, etc.), the expected relationships between indicators representing the flow of money and the flow of goods or services are known. Hence these relationships generate audit equations, upon which hypotheses can be generated for cross-verification purposes.

Value nets. Value cycle models can be formalized as a value net, illustrated in Figure 1. The notation is based on Petri nets, so events are depicted by rectangles and states are modeled by circles, or places, as they are called. In the example the economic events are nothing more than series of buy and sell transactions, representing a value exchange among agents involved. Value nets are process specifications that do not aim at process execution but at process diagnosis and normative analysis. So the generated audit equations represent expected normative relationships representing or expressing transfers of values among agents involved.

Definition 1. A value net is a Petri Net with the following characteristics

− The net is cyclic in the following sense. There is a special place labelled ‘money’ from which it is possible to reach every other node in the model and
that can be reached by every node in the model. So every node in the model is on a path from ‘money’ to ‘money’.

- The net is divided in a money part and a production part, and ‘money’ is in the money part. Those transitions in which the input places are in the money part and the output places in the production part, or vice versa, are called exchanges. All other transitions are called conversions.

- Each place is labeled as positive (+) for assets, or negative (−) for liabilities.

We use vector algebra and matrices to make calculations scalable. Note that a weighted Petri Net can always be expressed by an incidence matrix \( M \), where a cell \( m_{pt} \) contains a weight \( n > 0 \) when there is a link from place \( p \) to transition \( t \).

A dimensioned Petri Net associates units of measurement with each place.

**Definition 2.** A *dimensioned Petri Net* is a tuple \((P,T,F,B,u)\) with \( P \) a finite collection of places, \( T \) a finite collection of transitions, \( F \) and \( B \) each an incidence matrix of the form \( \mathbb{N} \times \mathbb{N} \), and \( u \in \mathbb{U} \) a vector of units.

Here \( F \) refers to forward links (consume) and \( B \) to backward links (produce). Unit vector \( u \) couples a specific unit to each place. A dimensioned marking is a pair \( m \in \mathbb{N} \times \mathbb{U} \). For convenience, flow matrix \( G \) is defined by \( G = B - F \). The effect of a transition vector \( t \) on matrix \( G \) is given by the product \( G \cdot t \).

**Tours.** Next we introduce the notion of a tour. A tour relates the traces of behavior of an enterprise to the cyclic structure of its value net. The cyclic structure is not directly apparent in behavior because transitions can fire at any point in the cycle. In a tour, we group those transactions that make up what Ijiri calls a causal chain of events [9]. Generally such a chain of events is a variation on buying products and resources, producing an end product, and selling the product. The cycle starts with consumption of money and ends with production of money. A formal version of the tour concept was introduced in [6]. It defines a tour as a constellation of events whose total effect is expressed on money only. All other produced tokens are at some point consumed by another step.
Definition 3. A transition vector $t$ with firing result $m = G \cdot t$ is a tour when $m[\text{money}] > 0$ and $m[x] = 0$ for any place $x \neq \text{money}$.

In this case, vector $t$ is the number of times a specific event has to occur per tour. Every time a tour has occurred the amount of money increases by $m[\text{money}]$. This increase is called the value jump. This formal tour concept therefore captures the essence of the notion of value.

Factoring Tours. Tours consist of parts. We can reveal the various parts of a tour by factoring or partitioning tour result $G \cdot t$. Let partition $P = P_{\text{mon}} \cup P_{\text{prod}}$ separate the places in the money and production part. From a partition of places also the transitions can be separated. Transitions in $T_{\text{mon}}$ and $T_{\text{prod}}$ are between places in the money and in the production part respectively. For instance, a $T_{\text{buy}}$ transition consumes value from the money part and produces value in the production part. We use the partitioned transitions to filter the flow matrix’ columns. Let $G_x[i,j] = G[i,j]$ if $j \in T_x$ and zero otherwise.

\[ G = G_{\text{mon}} + G_{\text{buy}} + G_{\text{prod}} + G_{\text{sell}} \]  \hspace{1cm} (1)

If we also split matrix $G$ into $B - F$ to see the difference between production and consumption, we can express connections between the value cycle’s parts.

Let $t$ be a tour. Tour result $G \cdot t$ is factored into a causal chain as follows

\[ (B_{\text{mon}} + B_{\text{buy}} + B_{\text{prod}} - F_{\text{prod}} - F_{\text{sell}}) \cdot t \]  \hspace{1cm} (2)

The various terms in this expression are the input and output of the different steps in a tour.

Grouping the money and production parts generates the following equations.

\[ (B_{\text{buy}} + B_{\text{prod}} - F_{\text{prod}} - F_{\text{sell}}) \cdot t = 0 \]  \hspace{1cm} (3)

\[ (B_{\text{sell}} + B_{\text{mon}} - F_{\text{mon}} - F_{\text{buy}}) \cdot t = G \cdot t \]  \hspace{1cm} (4)

These equations are well known in auditing. Equation (3) is Kirchhoff’s law of preservation for the production process. It shows that in a tour the input and output of production cancel out. In the Dutch accounting tradition, such preservation laws are called BETA-equations, as they relate the Begin, End, Toename (increments) and Afname (decrements) of states [12]. Equation (4) shows the value jump. It expresses the difference between the monetary values produced and consumed.

3 Inequalities

Example. Consider the following example [2]. Under Dutch tax law, usage of a company car is seen as part of the income of an employee. The employer has to add a certain percentage of the catalog value of the car to the income of the employee, over which income tax has to be paid and social premiums are deducted. The percentage lies between 4% and 25%, depending on the CO₂
emissions of the car. Only when the employee can demonstrate that he or she is using the car for business purposes, and at most 500 kilometers per year are driven for private purposes, no such addition to the income has to be made.

The reasoning runs as follows. Every car contains an odometer: a device that measures the distance driven. For the distances driven for business purposes, the employee is supposed to keep a registration. It is clearly specified that such a registration should contain the date, departure place and time, and destination place and time, and distance traveled.

\[
\text{total distance} = \text{distance}_{\text{private}}^{(-)} + \text{distance}_{\text{business}}^{(+)}
\]  

An employee may be motivated to understate \(^{-}\) the distance driven in a year for private purposes, to remain under the 500 threshold. Given that the odometer cannot be manipulated, he or she may be motivated to overstate \(^{+}\) the distance driven for business purposes. What about the inequalities in this case? Suppose the odometer indicates a total distance of 22107\(km\). Suppose we have a registration from the employee that claims a total of 21956\(km\) were driven for business purposes. Clearly the employee claims that no surcharge on his income should be added, as the difference remains under 500\(km\). In other words, the employee states that the normative expectation meets reality. The distance driven for business purposes must have a relationship with some agenda, clients and visiting addresses, which can be checked positively. The odometer cannot be manipulated, so we have two independent sources of evidence and the possibility to re-perform the calculations of the registered distance per business appointment. If the outcome of the re-performance is larger than the registered distance for business purposes we may assert that the registered distance is not overstated so the claim of the employee is correct and contains no material error.

As stated the tour result \(G \cdot t\) can be factored into the input and output of the different steps in a tour, which captures the essence of the notion of value from a future oriented, or normative perspective. In practice these values derived from a factored tour are coined as Critical Performance Indicators (CPI). In case practice differs from expectations an explanation is warranted. Now it is easy to see how these relationships generate audit equations, upon which hypotheses can be generated for auditing purposes. Audit equations are a kind of audit objectives, which serve as normative statements tested with empirical data, giving audit evidence of some condition.

**Actor Analysis.** Based on such considerations, Christiaanse et al [3] developed the following practical approach to identify independent sources of evidence.

1. Identify the **actors** involved and their general interests.
2. For the main actor, identify the **type of business**. Sketch a coarse value net: how are revenues being generated?
3. Identify the **variables** in the value net. Use ‘quasi goods’ (tickets), or side conditions (energy) to make an intangible service measurable. Now write down the **audit equations**, with corresponding **units of measurement**.
4. Determine whether actors have a reason to overstate (+) or rather understate (−) a variable. This will determine positive verification (accuracy), or a negative verification (completeness). Simplify the equations by substitution.

5. Identify reliable data sources (messages; databases) of these variables. Are the sources independent, or from actors with countervailing interests? Try to reduce variables without data to other variables, using the equations.

4 Case: Customs Warehouse

In this section we use the value net approach with inequalities to ensure reliability of a warehouse information system for customs purposes. The warehouse is maintained by ABC, a company manufacturing electronic equipment.

Data Collection. Data for this case was collected by a series of interviews with experts at both ABC and the Netherlands Customs Administration. In addition to interviews, we collected documentation on systems, processes and regulatory objectives. We visited the premises of a logistics service provider to observe how the inbound process is carried out.

Case Description. ABC is a Dutch company that is manufacturing electronic equipment. As part of its supply chain operations ABC is running a Customs Warehouse under license. Goods in such a warehouse are formally under customs supervision and payment of import duties is suspended until a final customs destination is known. To keep its license, ABC must demonstrate to the customs administration that the warehouse maintains a reliable administration of product movements: all products entering and exiting the warehouse are recorded (completeness), and for all products their customs status (in transit, EU, non-EU) is known (accuracy). Every month ABC prepares a so called Electronic Periodic Declaration (EPD): an audit file with evidence of the warehouse movements, so customs can supervise at a distance. Before granting a license, customs IT auditors must make sure that the processes and information systems that feed the EPD can be relied on.

Problem. How to assess the reliability of the warehouse management system and therefore of the EPD?

Case Analysis. We apply the step wise method to identify sources of evidence.

Step 1. The main actors are ABC and customs administration, as well as the freight forwarder and logistics service provider, since these are partly responsible for customs documents and inbound and outbound processes.

Step 2. The essence of the case can be represented in a value net, as shown in Figure 2. This diagram was made on the basis of a sketch of the mental model of a customs warehouse, as used by a customs officer to explain.

Step 3. From the value net the following preservation equations are generated. This selection of equations deals specifically with inward processing.
Fig. 2. Value net of the essential transitions of a customs warehouse

\[ \text{process} = \text{store} + \text{export from iwp} + \text{move iwp to bwh} + \text{import from iwp} \]

\[ \text{process} = \text{store} + \text{move fwh to iwp} + \text{move bwh to iwp} \]

\[ \text{move iwp to bwh} + \text{enter bwh} = \text{export from bwh} + \text{move bwh to iwp} \]

\[ \text{enter fwh} + \text{import into fwh} = \text{move fwh to iwp} + \text{exit fwh} \]

The equations capture the expected behavior. First, we can verify independently what enters and leaves the entire warehouse (WH). This assumption is based on an IT audit of the reliability of the warehouse systems and procedures. In particular, process controls related to the inbound process (e.g., match between purchase order and actual goods received). Second, the warehouse has three compartments: bonded warehouse (BWH), free warehouse (FWH) and inward processing (IWP). At all times \( WH = BWH + IWP + FWP \). The equations above capture the flows to and from IWP. Note that movements into FWH require payment of duties: these classify as import. Third, from the value net, we derive tour equations (Figure 3, which relate these flows to journals of process steps, which are typically expected to be recorded as movements in the warehouse management system (e.g., SAP). The various proportions are noted here as \( \lambda \) parameters. The differences as a result of one step, are shown as \( \Delta \).

Under inward processing, products are being assembled. The outflow from IWP depends on the inflow of parts in a certain proportion, namely the bill of materials. Products typically have a higher value than the value of individual parts added. There is a value jump, because ABC provides a service to the customer. Legal rules determine how to account for this value jump when calculating import duties. Hence the \( \lambda \) in the tour equations represents the number of times a specific event has to occur per tour. Every time a tour has occurred
\[ \Delta \text{Journal of process} = \lambda_1 + \lambda_2 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \]
\[ \Delta \text{Journal of import into fwh} = \lambda_5 + \lambda_{11} + \lambda_{12} + \lambda_{13} \]
\[ \Delta \text{Journal of import from iwp} = \lambda_6 + \lambda_8 + \lambda_{11} \]
\[ \Delta \text{Journal of enter bwh} = \lambda_3 + \lambda_6 + \lambda_7 \]
\[ \Delta \text{Journal of enter fwh} = \lambda_4 + \lambda_8 + \lambda_9 + \lambda_{10} \]
\[ \Delta \text{Journal of move bwh to iwp} = \lambda_2 + \lambda_6 + \lambda_7 \]
\[ \Delta \text{Journal of export from bwh} = \lambda_3 + \lambda_{10} + \lambda_{13} \]
\[ \Delta \text{Journal of exit fwh} = \lambda_4 + \lambda_5 \]
\[ \Delta \text{Journal of move iwp to bwh} = \lambda_2 + \lambda_{10} + \lambda_{13} \]
\[ \Delta \text{Journal of move fwh to iwp} = \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \]
\[ \Delta \text{Journal of export from iwp} = \lambda_7 + \lambda_9 + \lambda_{12} \]
\[ \Delta \text{Journal of store} = \lambda_1 \]

**Fig. 3.** Tour equations for the value net in Figure 2, focusing on inward processing.

the amount of money increases. The issue is for which parts import duties are due to customs, to ensure correctness of the EPD.

The table in Figure 4 shows the combination of records that need to be manipulated, in order to adjust some accounts without being detected. This puts a requirement on segregation of duties. In particular, authorizations for these accounts should not be shared.

The import into fwh paths:
- 5: import into fwh → exit fwh
- 11: import into fwh → move fwh to iwp → process → import from iwp
- 12: import into fwh → move fwh to iwp → process → export from iwp
- 13: import into fwh → move fwh to iwp → process → move iwp to bwh → export from bwh

The import from iwp paths:
- 6: enter bwh → move bwh to iwp → process → import from iwp
- 8: enter fwh → move fwh to iwp → process → import from iwp
- 11: import into fwh → move fwh to iwp → process → import from iwp

Remember the events in the tour form a causal chain that produces and consume an equal amounts of goods to create an increase in money. These kind of computations are of help or can be used to analyse the goods and money flows within the company.

**Step 4.** The company has reason to overstate $FWH^{(+)}$, as for these goods no duties are due. Customs can use positive verification. Conversely, there is reason to understate $BWH^{(-)}$, as for these parts duties are due. Similarly there is reason to understate $IWH^{(-)}$. For these two variables correctness must be established by comparing to some other source: the total $WH$. 
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| process                   | 1       | 1       | 0       | 0       | 0       | 1       | 1       | 1       | 1       | 1         | 1         | 1         | 1         |

**Fig. 4. Toms**

**Step 5.** What are reliable data sources (messages; databases) of these variables? These accounts are maintained in a warehouse management system. All customs relevant movements are logged; these logs are the journals of Figure 3. The customs relevant movements in the warehouse management system are collected and filtered, by a dedicated customs reporting system, which aggregates the accounts and prepares the EPD, and the monthly declarations for import duties.

In this case it is sufficient to test regularly whether the registered goods, i.e., the parts in free warehouse and the bonded warehouse are correct. This is done by stock count and verification against the sales contract and customs documents. In all cases we have at least two independent sources.

## 5 Conclusion

Model-based auditing is a computational approach to compliance monitoring and auditing, which is based on a normative meta-model of the value-cycle: the movement of money and goods or services [6, 8, 13, 4]. We use a matrix algebra for specifying value cycle models as a kind of Petri nets, called value nets.

The first step of any audit is understanding the business. Actor analysis reveals whether parties have reasons to overstate or understate some accounts. If so than sets of *inequalities* can be expected, rather than equations. In this paper we show, that by combining inequalities and underlying assumptions about the domain, process and system, we can arrive at a series of reconciliation relations that together ‘span’ the business model and ensure both accuracy and completeness statements. We use value nets as a means to communicate with business experts. We can automatically derive equations. We annotate them with expected overstated or understatement. This determines the kind of verification: positively against the records or negatively, against a completeness expectation from independent sources.

We have applied this idea to re-construct and explain the existing reasoning in an elaborate case study of a customs warehouse. The use of the value net analysis and the inequalities is instrumental in bringing out specific risks in the internal controls and the set-up of the customs warehouse system. In particular, it shows the crucial importance of (i) a reliable first recording, in this case in the
inbound process, where the actual goods are related to the records of the goods in the warehouse information system, and (ii) the importance of having at least two independent sources to establish correctness and therefore completeness.

Future research aims to relate our ideas on inequalities to the so-called Closed World Assumption, as known in artificial intelligence, e.g., [10]. If some fact isn’t recorded in a system $S$, it is assumed not true: if $S \not\models \varphi$, then $S \models \neg \varphi$.

In practice the closed world assumption can only be made in highly regulated environments, such as court proceedings (if it wasn’t brought to the attention of the court it cannot be used as evidence), absentee lists (if you didn’t sign, you weren’t there), or indeed, accounting information systems that report on the accuracy and completeness of revenues.

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