



Ontological Analysis

4 - The tools of Formal Ontology

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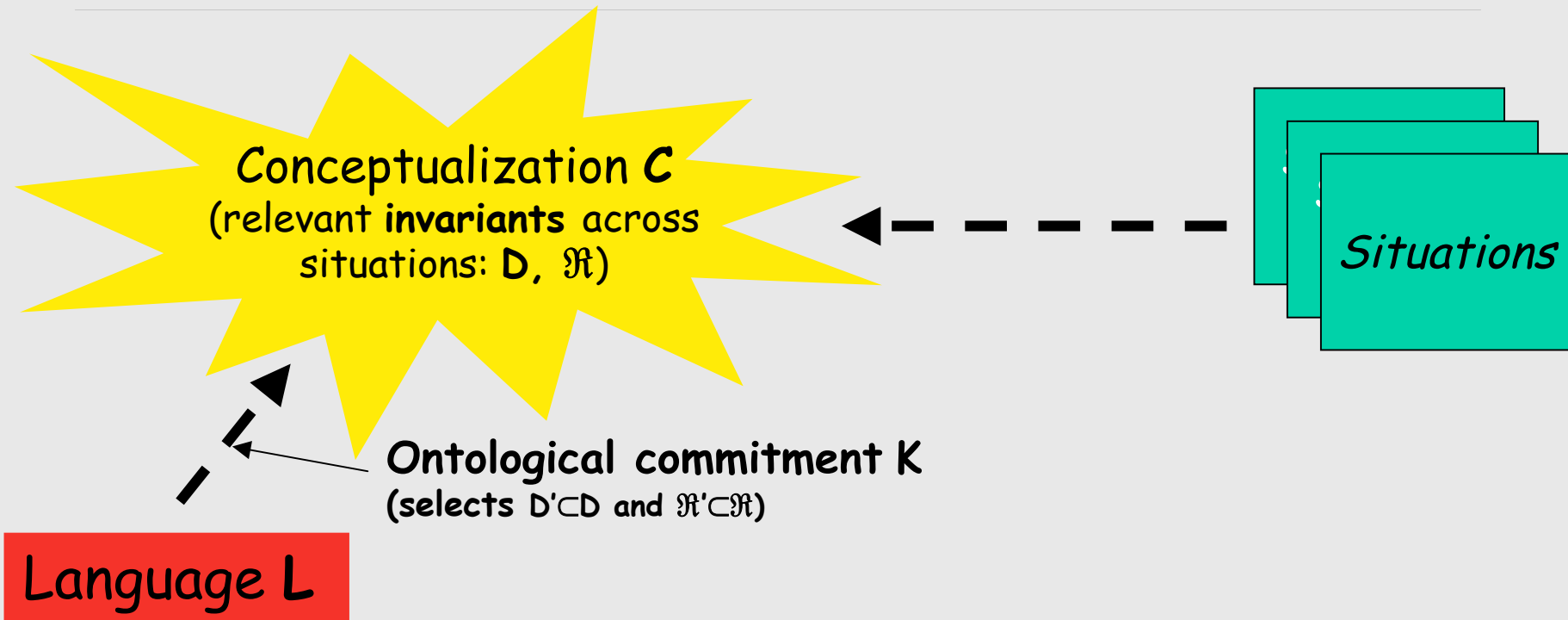
Trento, Italy

Formal Ontology

- Theory of **formal distinctions and connections** within:
 - entities of the world, as we perceive it (**particulars**)
 - categories we use to talk about such entities (**universals**)
- Why **formal**?
 - Two meanings: **rigorous** and **general**
 - Formal logic: connections between truths - neutral wrt **truth**
 - Formal ontology: connections between things - neutral wrt **reality**
- **NOTE:** “represented in a formal language” is not enough for being formal in the above sense!
- (**Analytic ontology** may be a better term to avoid this confusion)



The first steps of ontological analysis



- Be clear about the *domain of discourse* (existence...)
- Choose the relevant *concepts and conceptual relations*
- Choose the *primitive relations*
- Choose meaningful *names* for these

Mereology as an example of formal ontological analysis

- Primitive: **proper part-of** relation (PP)
 - asymmetric
 - transitive
 - $Pxy =_{\text{def}} PPxy \vee x=y$
 - $Oxy =_{\text{def}} \exists z (Pzx \wedge Pzy)$
- Axioms:

supplementation: $PPxy \rightarrow \exists z (PPzy \wedge \neg Ozx)$

principle of sum: $\exists z \forall w (Owz \leftrightarrow (Owx \vee Owy))$

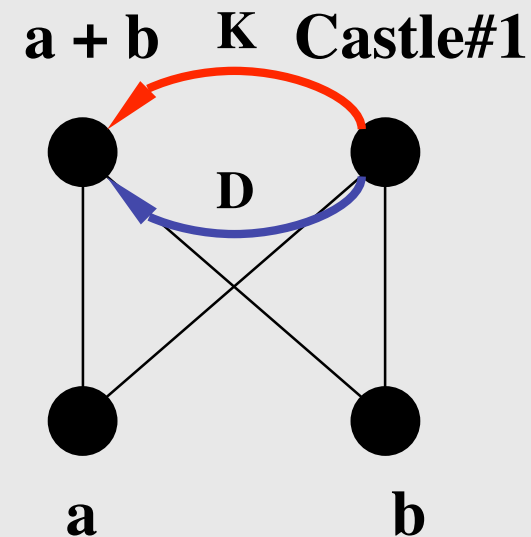
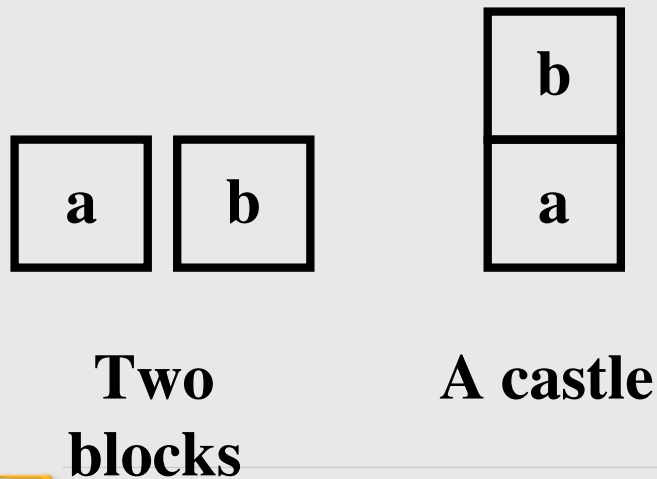
extensionality: $x = y \leftrightarrow \forall w (Pwx \leftrightarrow Pwy)$

Excluded models:



Part, Constitution, and Identity

- *Parts not enough to make the whole*: structure changes identity
- *Mereological extensionality* is lost
- *Constitution* links the two entities
- *Constitution* is asymmetric (implies *dependence*)



Essential properties

- For an individual
 - John must have a brain
 - John must be a human
 - John must be alive
- For a type
 - All human beings must have a brain
 - All human beings must be “a whole” (all of a piece)



Essential properties and rigidity

- Certain entities **must** have some properties in order to *exist*
 - John must have a brain
 - John must be a person.
- Certain properties are essential to **all** their instances (***being a person*** vs. ***being hard***).
- These properties are **rigid** - Their extension is the same in all possible worlds. If an entity is ever an instance of a rigid property, it must necessarily be such.
- By the way, what's the meaning of *exist*?
 - Being an element of the domain of discourse
 - Being present *at a certain time* (or in a certain world...)



Formal Rigidity

- ϕ is rigid (+R): $\forall x (\text{pos } \phi(x) \rightarrow \text{nec } \phi(x))$
 - e.g. Person, Apple
- ϕ is non-rigid (-R): $\exists x (\text{pos } \phi(x) \wedge \neg \text{nec } \phi(x))$
 - e.g. Red, Male
- ϕ is anti-rigid (\sim R): $\forall x (\text{pos } \phi(x) \rightarrow \neg \text{nec } \phi(x))$ e.g. Student, Agent

Meta-properties



Formal rigidity - variations

- Taking time and actual existence into account:

$$\mathbf{nec}\forall xt((E(x,t) \wedge \phi(x,t)) \rightarrow \mathbf{nec}\forall t'(E(x,t') \rightarrow \phi(x)))$$

- *Welty, C. and Andersen, W. Towards OntoClean 2.0: A framework for rigidity (Applied Ontology 1(1), 2006)*

Carrying essential properties

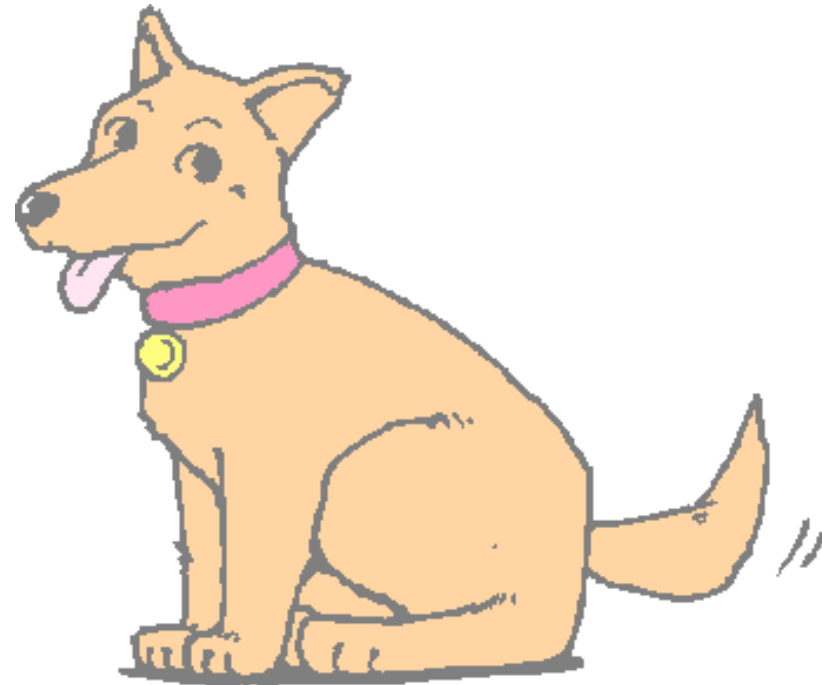
- A property P *carries* a *relevant* essential property Q (different from P) iff Q is essential to all instances of P, and still Q is not rigid:
 - Every person must have a brain.
- Compare with:
 - Every person must be a mammal.

Informative essential properties are those which are NOT rigid



Unity, Identity, and Essence

- **Unity**: is the collar part of my dog?
 - *Being a whole* (of a certain kind) is also a (relevant) essential property
- **Identity**: is this my dog?
 - Essential properties of *dogs*
 - Essential properties of *my dog*



Defining unity

- A tentative formulation: x ***is a whole*** under a unifying relation U iff U is an ***equivalence relation*** that binds together all the parts of x , such that, ***necessarily***,

$$P(y,x) \rightarrow (P(z,x) \Leftrightarrow U(y,z))$$

but not

$$U(y,z) \Leftrightarrow \exists x(P(y,x) \wedge P(z,x))$$

- P is the ***part-of*** relation
- U can be seen as a ***generalized indirect connection***



Unity Refined

$$\delta_U(x) =_{\text{df}} U(x, x)$$

(x belongs to the domain of U)

$$\mathbf{U}_U(x) =_{\text{df}} \Sigma_{\delta_U}(x) \wedge \forall y, z ((\delta_U(y) \wedge \delta_U(z) \wedge P(y, x) \wedge P(z, x)) \rightarrow U(y, z))$$

(x is unified by U)

$$\mathbf{W}_U(x) =_{\text{df}} \text{Max}_{\mathbf{U}_U}(x)$$

(x is a whole under U)

$$\Sigma_{\phi}(x) =_{\text{df}} \forall y (P(y, x) \rightarrow \exists z (\phi(z) \wedge P(z, x) \wedge O(z, y)))$$

(sum of ϕ s)



Kinds of Whole

- Depending on the **nature of the *unifying relation***, we can distinguish:
 - ***Topological wholes*** (a piece of coal, a heap of coal)
 - ***Morphological wholes*** (a constellation)
 - ***Functional wholes*** (a hammer, a bikini)
 - ***Social wholes*** (a population)
- * a whole can have ***parts that are themselves wholes*** (with a different unifying relation)



Unity and Plurality

- *Ordinary objects: **wholes or sums of wholes***
 - *Singular: **no wholes as proper parts***
 - *Plural: **sums of wholes***
 - *Plural wholes (the sum is **also a whole**)*
 - *Collections (the sum is not a whole)*

