



## 04. Description Logics

*Primitive and Defined Concepts*

*Specialization/Generalization*

*Concept and Role Constructors*

*Reasoning with Concepts and Individuals*

*Classic and SHIQ*

*Description Logics and UML, ER and CORBA*





# Description Logics

- A *precise* notation for representing “noun phrases”.
- An Information Management system that uses these to make assertions, and answer questions based on *inferences* - a logic.
- Fundamental ontology: the application domain is populated by *individuals*, related by binary relationships, called *roles/attributes*, grouped into *classes* (*concepts*).
- First Order Logic (FOL) would be fine for describing these, but it is intractable (semi-decidable, to be exact)
- Looking for a restricted subset, which allows us to reason with it while still representing useful things.

*Datalog is another such useful subset of FOL*



# Primitive and Defined Concepts

- ↪ In addition to *primitive* concepts (*natural kinds*), such as PERSON, CHAIR, ... there are *defined* concepts.
- ↪ Defined concepts have names:
  - ✓ "person with gender=M and no object related to it by hasSpouse" ==> "BACHELOR"
  - ✓ "person with age between 13 and 17" ==> "TEENAGER"
  - ✓ "person who eats only non-meat foods" ==> "VEGETARIAN"
- ↪ Such concepts can be described in terms of relative clauses or compound nouns:
  - ✓ "person who has at least 3 children"
  - ✓ "towns located in MA or NH or VT,..." (NE\_TOWNS)



## *Specialization/Generalization*

- ↳ Both primitive and defined concepts can have additional assertions made about them, representing necessary conditions. A standard way to make such assertions is to use is-a-subconcept-of/is-subsumed-by/is-a-kind-of ( $:<$ ).
- ↳ For example,
  - ✓ PERSON  $:<$  ANIMATE ,
  - ✓ TEENAGER  $:<$  LIKES-FRIES
- ↳ Note (AB): Liking French fries is not part of the definition of TEENAGER, even though all teenagers have this property.



# *A Language for Defining Concepts*

↳ We need a language for defining concepts, based on experiences with what has been useful in many applications:

- ✓ Atomic/primitive concepts: PERSON, COURSE, BOOK
- ✓ Boolean combinations thereof:
  - ✓ ANIMAL and HERBIVORE;
  - ✓ not ANIMATE;
  - ✓ PERSON or CORPORATION;
  - ✓ PERSON and (not MALE).



## ...But Also...

- ↳ Concepts defined by enumeration of individuals: {M,F}
- ↳ Concepts from “concrete domains” (numbers, Programming Language values)
- ↳ Sets of objects satisfying restrictions on their role fillers (for this, we need some atomic/primitive roles: graduateOf, locatedIn, likes, hasPart )
  - ✓ Objects all of whose locatedIn values are in NE\_TOWNS
  - ✓ Objects some of whose graduateOf values are in UNIVERSITY
  - ✓ Objects with at least 3 graduateOf fillers
  - ✓ Objects related to the Rutgers object by graduateOf (= Objects whose graduateOf role includes Rutgers as filler).



# Concept Constructors

ANIMAL and HERBIVORE

not ANIMATE

PERSON or CORP

PERSON and (not MALE)

{M,F}

*Objects with locatedIn values  
in NE\_TOWN*

*Objects with some graduateOf  
values in UNIVERSITY*

*Objects with 3+ graduateOf fillers*

*Objects with graduateOf fillers that  
include Rutgers*

...

(and ANIMAL HERBIVORE)

(not ANIMATE)

(or PERSON CORP)

(and PERSON (not MALE))

(one-of M F)

(all locatedIn NE\_TOWN)

(some graduateOf UNIVERSITY)

(at-least 3 graduateOf)

(fills graduateOf Rutgers)



## More Concept Constructors

- ⇒ (at-least 3 children DOCTOR)  
*//contrast this with (and (at-least 3 children)  
(all children DOCTOR))*
- ⇒ (domain graduateOf)  
*//objects having a filler for graduateOf role*
- ⇒ (range graduateOf)  
*//objects which are fillers of graduateOf role*
- ⇒ (same-as (firstName) (lastName))  
*// objects for which the firstName and lastName  
values are identical*
- ⇒ (subsetOf (friends) (co-workers))  
*// objects whose co-workers include all their friends*





# Syntax

↪ Can describe concepts of arbitrary complexity by nesting;  
e.g., *"Courses taken by 60 to 90 students, who are all  
undergrads, and taught by a CS professor"*

(and

COURSE

(at-least 60 takers)

(at-most 90 takers)

(all takers (and STUDENT

(all inYear (one-of 1 2 3 4))))

(exactly 1 taughtBy)

(all taughtBy (and PROFESSOR

(fills inDepartment "CS"))))



## ...Different Syntax...

*"Persons who eat only non-meat"*

↪  $(:and \text{ PERSON } (:all \text{ eats } (:not \text{ MEAT})))$

↪  $and(PERSON, all(eats, not(MEAT)))$

↪  $PERSON \sqcap \forall \text{ eats. } \neg \text{MEAT}$

↪  $\langle \text{concept} \rangle \langle \text{and} \rangle$

$\langle \text{primitive name} = \text{"PERSON"} \rangle$

$\langle \text{all} \rangle$

$\langle \text{primrole name} = \text{"eats"} \rangle$

$\langle \text{not} \rangle \langle \text{primitive name} = \text{"MEAT"} \rangle$

$\langle /not \rangle \langle /all \rangle \langle /and \rangle \langle /concept \rangle$



# OWL Syntax

```
✚ <owl:intersectionOf rdf:parseType="Collection">  
  <owl:Class rdf:about="#PERSON" />  
  <owl:Restriction>  
    <owl:onProperty rdf:resource="#eats"/>  
    <owl:allValuesFrom>  
      <owl:complementOf rdf:resource="#MEAT"  
    />  
  </owl:allValuesFrom>  
  </owl:Restriction>  
</owl:intersectionOf>
```



## Roles

- ↳ Fundamental observation: Relationships are like concepts!  
Hence they can also be defined, using *role constructors*.
- ↳ *childOf* is the inverse of *hasChildren*  
(inverse hasChildren)
- ↳ *descendantOf* is the transitive closure of *childOf*  
(trans childOf)
- ↳ *sonOf* is the restriction of *childOf* so that its range of values is MALE  
(restriction childOf MALE)
- ↳ *nephewOf* is the composition of *sonOf* and *siblingOf*  
(compose sonOf siblingOf)



## Summary

- Descriptions are composite, variable-free *terms*, which can be built up from primitive symbols, using *constructors*.
- There are constructors for both concepts and roles (binary relationships.)
- There is a collection of constructors that have been found useful over the years.
- Except for transitive closure of roles, all other constructors can be expressed in FOL -- see some examples later. (In fact, you only need 3 variables, if you can reuse them when nesting)



## Standard Reasoning

- ↳ Does concept *C* **subsume** concept *D*?  $D :< C$ 
  - ✓ (and PERSON MALE) :< PERSON
  - ✓ (at-least 3 hasChildren) :< (at-least 1 hasChildren)
  - ✓ (at-most 2 parts) :< (at-most 10 parts)
- ↳ Suppose now hasSons :< hasChildren
  - ✓ (all hasSons STUDENT) :<  
(all hasChildren PERSON)
  - ✓ (fills hasSons Adam) :< (at-least 1 hasChildren)



# Incoherence

⇒ Is concept *C* incoherent?

(and PERSON

(at-least 3 hasDegree)

(all hasDegree (one-of "BA" "BS" ) )

⇒ Another way of putting it: Is *C* :< NOTHING?

⇒ Problem: reasoning with the complete set of concept constructors we encountered is still as hard as for all of FOL!

⇒ Solution: Description Logics research has been about finding subsets of constructors and characterizing the computational complexity of reasoning with them.



# The Classic Description Logic

primitive concept

$C(.)$

role

$r(.,.)$

attribute (role with  $\leq 1$  filler)

$f(.,.)$

(and  $C$   $D$ )

$y \mid C(y) \wedge D(y)$

(all  $p$   $C$ )

$y \mid \forall z. p(y,z) \Rightarrow C(z)$

(at-least  $n$   $p$ )

$y \mid \exists^{\geq n} z. p(y,z)$

(at-most  $m$   $p$ )

$y \mid \exists^{\leq m} z. p(y,z)$

(one-of ( $e_1$   $e_2$  ...))

$y \mid y=e_1 \vee y=e_2 \vee \dots$

(fills  $p$   $e$ )

$y \mid p(y,e)$

(same-as ( $f_1 \dots f_n$ ) ( $g_1 \dots g_k$ ))

$y \mid f_n(\dots(f_1(y))) = g_k(\dots(g_1(y)))$

THING, NOTHING

$y \mid \text{true} \qquad y \mid \text{false}$

H-THING, NUMBER

(min  $n$ )

$w \mid w \geq n$

(max  $m$ )

$w \mid w \leq m$

(inverse  $r$ )

$(x,y) \mid r(y,x)$

(test  $fn$   $arg_1$  ...  $arg_k$ )

$fn(arg_1, arg_2, \dots)$





## SHIQ DL -- FaCT Reasoner

primitive concept

$C(.)$

role

$r(.,.)$

(and  $C$   $D$ )

$y \mid C(y) \wedge D(y)$

(or  $C$   $D$ )

$y \mid C(y) \vee D(y)$

(not  $C$ )

$y \mid \neg C(y)$

(all  $p$   $C$ )

$y \mid \forall z. p(y,z) \Rightarrow C(z)$

(some  $p$   $C$ )

$y \mid \exists z. p(y,z) \wedge C(z)$

(at-least  $n$   $p$   $C$ )

$y \mid \exists^{\geq n} z. p(y,z) \wedge C(z)$

(at-most  $m$   $p$   $C$ )

$y \mid \exists^{\leq m} z. p(y,z) \wedge C(z)$

(inverse  $r$ )

$(x,y) \mid r(y,x)$

$r$  is transitive

$(y,w) \mid \forall z. r(y,z) \wedge r(z,w) \Rightarrow r(y,w)$

[Horrocks]



# DLs as Information Managers



- ↪ Descriptions are used in *all* these languages!
- ↪ Variety of *ASK* operations about concepts:
  - ✓  $\text{concept-subsumes?}(C,D)$ : Boolean
  - ✓  $\text{incoherent?}(C)$ : Boolean
  - ✓  $\text{concept-disjoint?}(C,D)$ : Boolean

*The rest of the slides use Classic*



# Constraints

✚ Provide necessary conditions (*axioms*) for primitive concepts and roles of the form *name* :< *D*, *name* :< *r*

✓ (define-primitive-concept PERSON

(and THING (all age INTEGER)) )

✓ (define-primitive-role wifeOf

:is-a spouseOf, :inverse husbandOf, :attrib True)

✓ (define-disjoint-primitive-concept BIRD ANIMATE  
genus)

ops provided by IM;  
in Classic, have prefix cl-

has at most one value



## Definitions

➤ Definitions of concepts (name =<sub>def</sub> C)

✓ (*define-concept* TEENAGER

(and PERSON (all age (and (min 13) (max 17))))))

➤ Subsumption constraints between complex concepts  
("general axioms")

"graduate courses are taught by tenured professors"

(and COURSE (all crsNumber (min 500))) :<

(all taughtBy (and PROFESSOR

(all title (one-of 'associate 'full))))

➤ A collection of such axioms is known as a *T-box*  
(Terminological box)



## Reasoning with Axioms

- ↪ Non-recursive axioms about primitives can be expanded: if you have axiom  $N \prec C$ , replace all occurrences of  $N$  by  $(\text{and } N \ C)$  -- Classic only supports this for concepts and for roles
- ↪ General axioms are much harder to reason with (Exponential Time Complete problem for SHIQ -- seems to be ok, however, for practical KBs)
- ↪ FaCT supports this for concepts; also  $r1 \prec r2$  for roles



## Recursive Axioms

↪ Recursive axioms, e.g.,  $\text{PERSON} :< (\text{all parents PERSON})$

↪ What does this mean?

Suppose  $\text{FOO} =_{\text{def}} (\text{all parents FOO})$

and  $\text{BAR} =_{\text{def}} (\text{all parents BAR})$ ;

Does this mean that FOO and BAR are the same concept?

↪ Recursive concept definitions also make reasoning hard --  
FaCT supports these too.



## Managing Definitions

What kinds of services might an Information Manager offer for definition management?

- ↪ Detect inconsistent concepts;
- ↪ Find concepts that mean the same thing, and let the user know of the alias;
- ↪ Automatically organize definitions into a subconcept hierarchy by finding for each concept, most specific existing subsumers and most general subsumees.



## Reasoning with Concepts in Classic

- Primitive concepts: INSTRUCTOR , COURSE
- Primitive roles: takers, attributes: teaches, taughtBy  
$$\text{COURSE} <: (\text{and } (\text{all taughtBy INSTR})$$
$$(\text{same-as (self)(taughtBy teaches ))})$$
- "Desirable classes are taught by lucky instructors"  
$$\text{DESIRABLE} =_{\text{def}} (\text{and COURSE (all taughtBy LUCKY)})$$
- "Lucky instructors have classes with small enrolments"  
$$\text{LUCKY} =_{\text{def}} (\text{and INSTR (all teaches (atmost 8 takers))})$$
- "Small courses have fewer than 8 takers."  
$$\text{SMALL\_COURSE} =_{\text{def}} (\text{and COURSE (atmost 8 takers)})$$
  
$$\underline{\text{DESIRABLE == SMALL\_COURSE}}$$

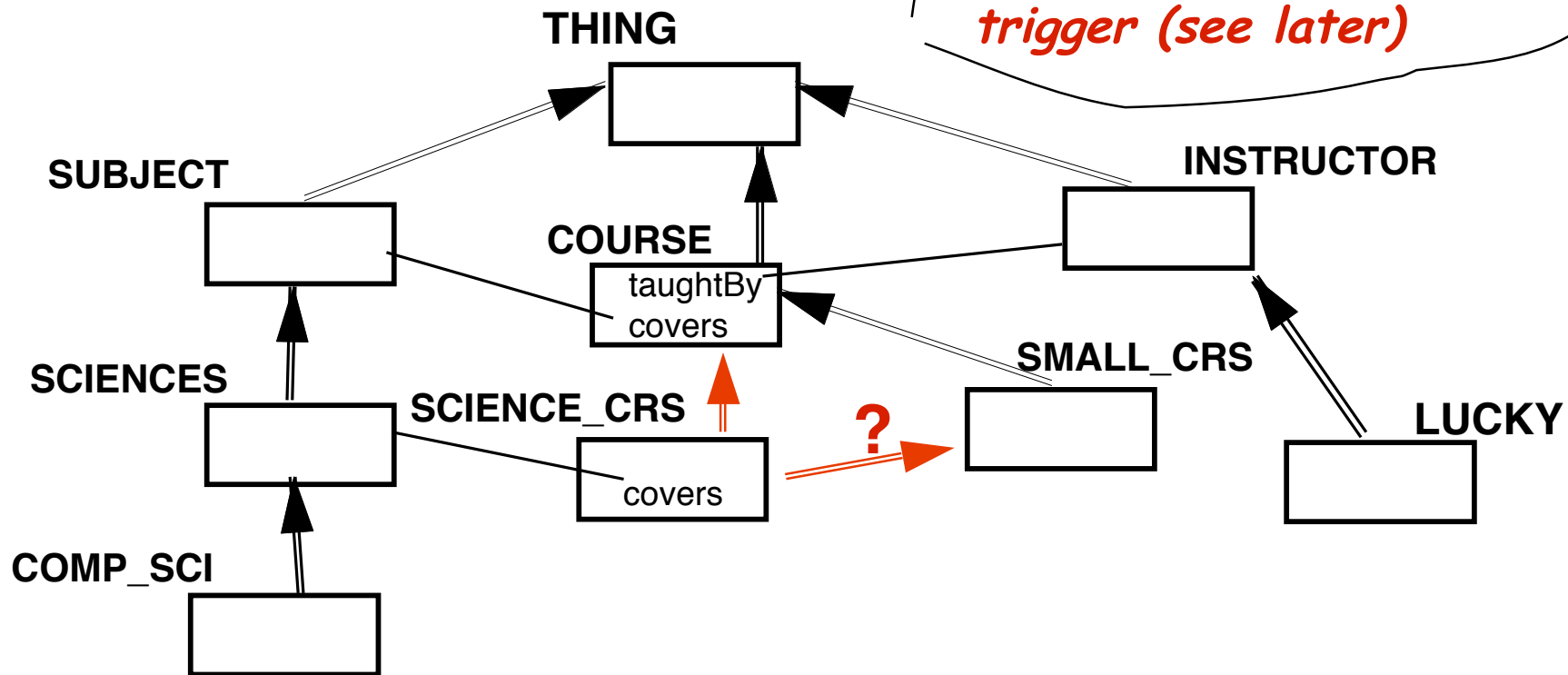


# Classification

**SCIENCE\_CRS** =<sub>def</sub> (and **COURSE** (all covers **SCIENCES**))

**SCIENCE\_CRS :< (and (atmost 7 takers)(all takers (fills year 4)))**

*In Classic this is only a trigger (see later)*





## Classic Rules

- Classic *rules* express general axioms like  $C :< D$ , for named concepts  $C$ , even if  $C$  is a defined concept. But these are *not* used in subsumption reasoning. So the inference  $\text{SCIENCE\_CRS} :< \text{SMALL\_CRS}$  would not be made in Classic.
- Rules act like triggers: when an *individual* becomes an instance of  $C$ , it is also added to  $D$ . This means that even for individuals, reasoning does not work backwards: from *not*  $D$  one cannot infer *not*  $C$ .



## Test Concepts

- You can think of test-concepts as placeholders where one can put constructors not available in Classic.
- So for example, you can say  
(test-c some child Doctor)  
where presumably you will eventually define a 3 place function called *some*  
(some <role> <Concept> <individual>)  
that checks if the third argument has a <role> filler that is an instance of <Concept>.
- But this definition of *some* cannot be used in subsumption reasoning -- it will be treated as a black box.



## *The Assertional Box (A-Box)*

↪ The A-Box provides operations for manipulating individuals, and relationships between them:

- ✓ Create individuals

*(ind-create cs430)*

- ✓ Inter-relate them

*(ind-add-fillers cs430 covers Databases)*

*(ind-add-fillers cs430 taughtBy Gabrielle)*

- ✓ Assert them to be instances of concepts

*(ind-add cs430 COURSE)*



## *Capturing Incomplete Information*

↪ A full DL can be used as part of the A-Box Tell language:

✓ *(ind-add cs323 (all taughtBy (fills dept "CS")))*

*"We do not know who teaches cs323, but it will be from CS dept"*

✓ *(ind-add mahdi (all hasDegree (one-of BA BS)) )*

*"Mahdi's degree is either BA or BS"*

↪ This feature can be used to represent *incomplete* information.



## Reasoning with Individuals

↪ Individuals can be asserted to satisfy descriptions, e.g.,

Calvin : PERSON

Calvin : (all friendOf (the age (and (min 5) (max 7))))

↪ Consistency checking: From friendOf(Calvin, Susie) verify that Susie's age is not known to be under 5 or over 7

↪ Propagation: If Susie's age is not known, then infer partial information

Susie : (the age (and (min 5) (max 7)))

↪ Individual Classification: In either case, if

CHILD =<sub>def</sub> (the age (and (min 0) (max 12)))

then Susie is inferred to be a child

Susie : CHILD



# Open World Reasoning

➤ Suppose you have been told the following

(ind-create Calvin), (ind-create Susie)

(ind-add-filler Calvin friendOf Susie)

➤ From

{friendOf(Calvin, Susie), Susie:FEMALE}

one cannot conclude

Calvin : (all friendOf FEMALE)

because not all friends might be known at this time -- one might find more in the future.



## Closed World Reasoning

- ↪ The way to say that all friends are known is to add an atmost bound equal to the current number of fillers:

*(ind-add Calvin (atmost 1 friendOf)).*

This “closes” the role friendOf on Calvin, and allows all restrictions to be checked on role friendOf for Calvin.

- ↪ Classic will do the counting and add the at-most automatically, if you just say

*(ind-close-role Calvin friendOf)*

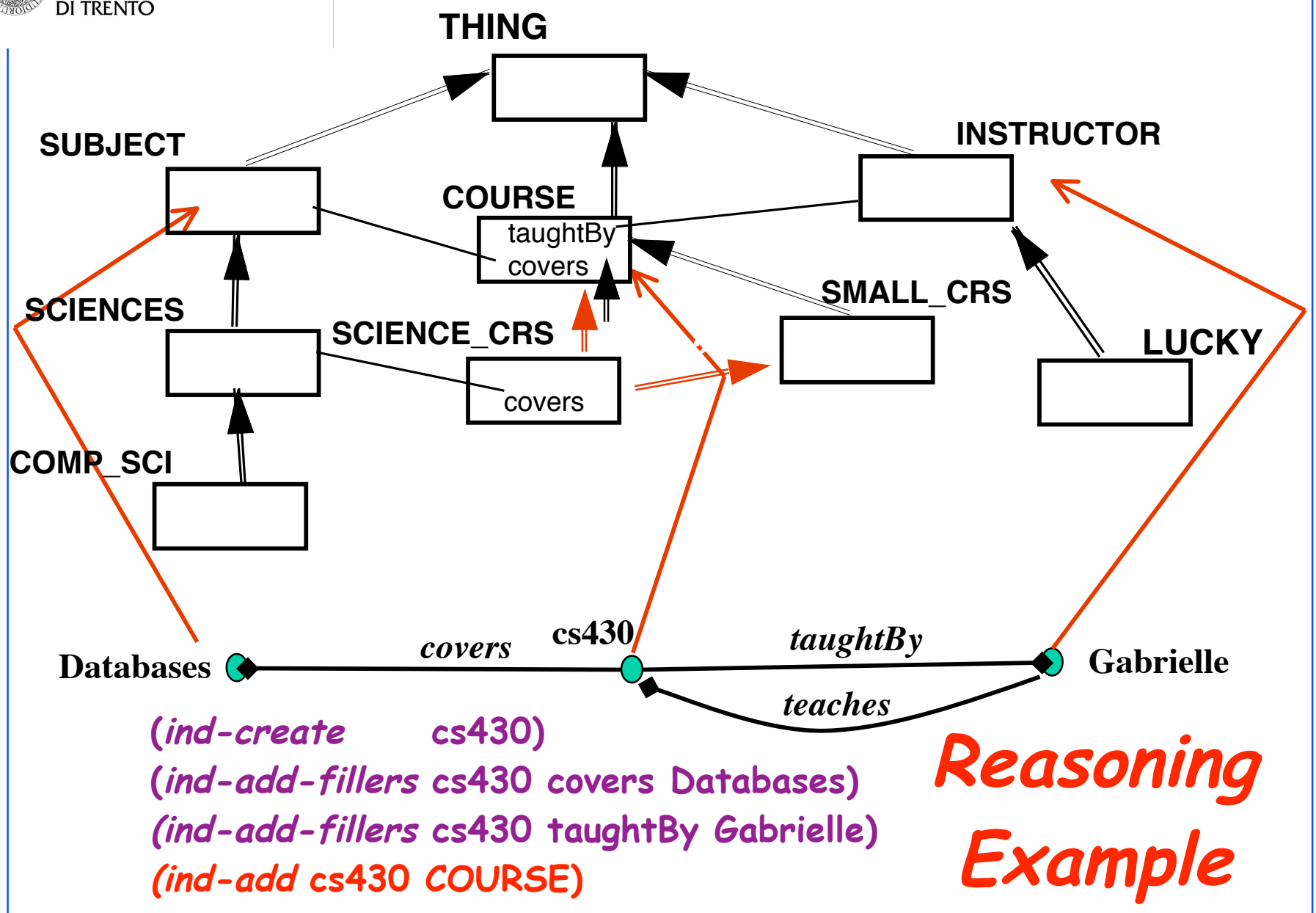


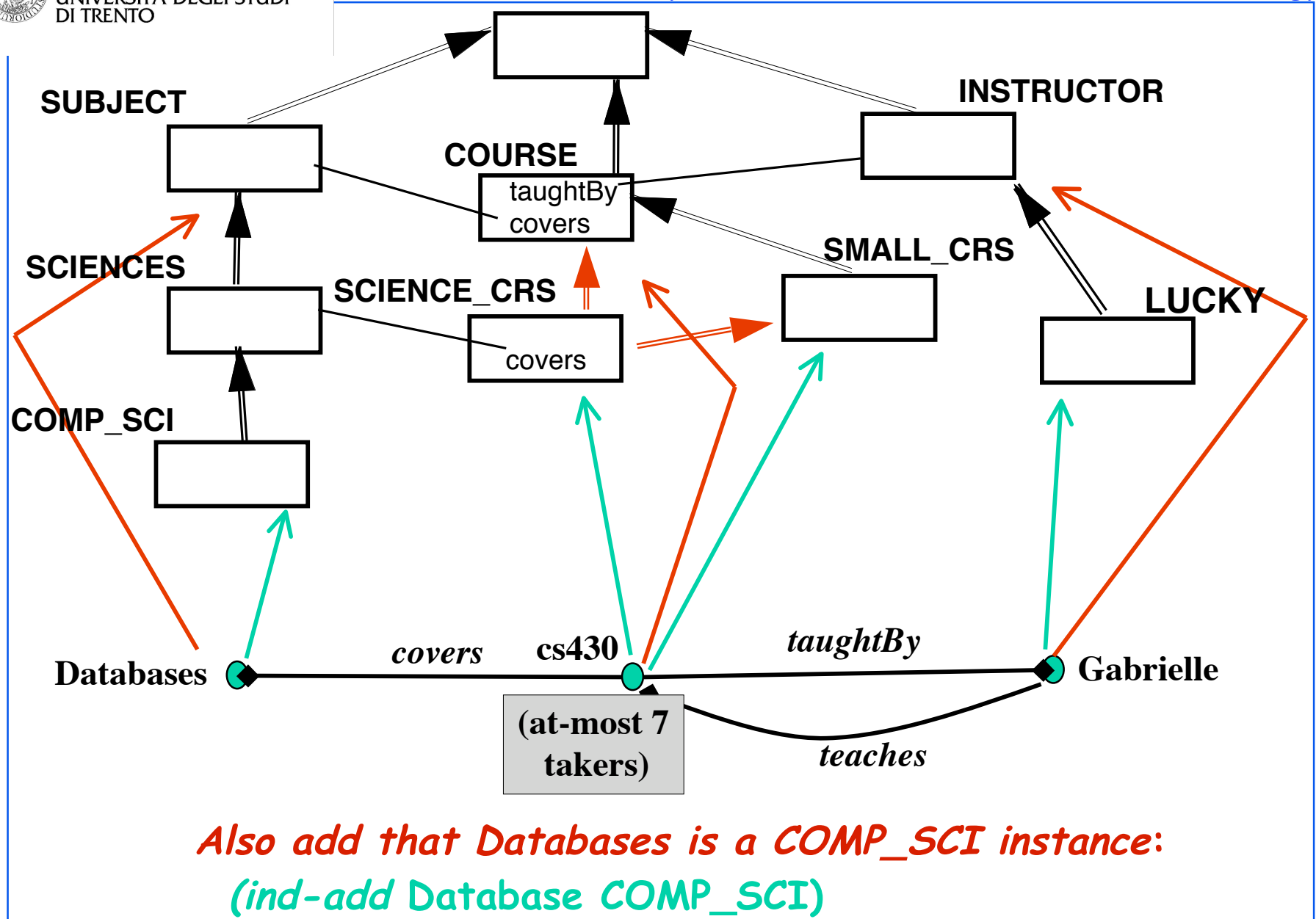


COURSE <: (and (all taughtBy INSTRUCTOR)  
(all covers SUBJECT)  
(same-as (self) (teaches taughtBy)) )

SCIENCE\_CRS =<sub>def</sub> (and COURSE (all covers SCIENCES) )

```
SCIENCE_CRS  :< (and (at-most 7 takers)
                     (all takers (fills year 4)))
```







## Questions about Individuals

- ⇒ *(instance? <individual e> <concept C> )* -- test for membership.
- ⇒ *(instances <concept Q>)*: -- what are the instances classified under Q; i.e., any concept can act as query.
- ⇒ *(ind-parents/ancestors <individual e>)* -- what named concepts is e an instance of.
- ⇒ *(fillers <individual e> <role r>)* -- fillers of role r for e.
- ⇒ *(ind-expr <individual e>)* -- description of e.
- ⇒ *(cl-all <individual e>)* -- everything known about e.
  - ✓ "Everything about Susie's age": *(cl-all Susie age)*  
would return *(and (min 5) (max 7))*
  - ✓ "Everything about SCIENCE\_CRS takers?":  
*(cl-all cs430 takers) ==> (fills year 4)*



# *Description Logics for Information Management*

- Define the schema: define-concept      *L* - Define
- Define views: define-concept      *L* - Define
- Describe individuals partly : assert-ind      *L* - Tell
- State queries: ask-instances      *L* - Ask
- Intensional answers: concept-aspect      *L* - Answer
- Set up simple triggers: assert-rule      *L* - Tell



# *The Meaning of Links Revisited*



- ↪ FOO = ITALIAN\_FLAG: "Among others, FOOs have green color"
- ↪ FOO=GREEN\_PARROT: "All FOO instances have color green"
- ↪ FOO=FROG: "The default color of FOOs is green"
- ↪ FOO=PHYSICAL\_OBJECT: "only FOO's can have property hasColor with value green"



# The Meaning of Links Revisited

How does this help in disambiguating:



↪ FOO :< (all hasColor GREEN)

-- *GREEN is a set of values*

↪ FOO :< (some hasColor GREEN)

↪ FOO :< (fills hasColor GREEN)

-- *GREEN is a value*

↪ (some hasColor GREEN) :< FOO



## Clarifying Is-A

↪ PERSON :< (and (the age INT)

(the gender (oneof M F))

(the wt INT)))

↪ 49'ER :< (and (fills age 49) (fills gender M) (the wt INT))

49'ER is not a subconcept of PERSON!

Did you mean (and PERSON (**atmost 1 age**) ... )?

↪ MAN = (and PERSON (the gender (oneof M)) (the wt INT))

↪ Mahdi : (and PERSON (fills age 49)(fills gender M)

(fills wt 145)(the wife PERSON)





## The Arch Example

*roles:* subpart , lintel, upright

lintel :< subpart, upright :<subpart

PHYSICAL-OBJECT :< THING

BLOCK :< PHYSICAL-OBJECT

COMPOSITE-OBJECT =<sub>def</sub>

(and PHYSICAL-OBJECT (at-least 1 subpart)

? :< ? =<sub>def</sub> ? (all subpart PHYSICAL-OBJECT)

ARCH =<sub>def</sub>

(and COMPOSITE-OBJECT (the lintel BLOCK)

(all upright BLOCK) (at-least 2 upright)

(all materials (one-of Marble Brick Steel)) )

(domain lintel) :< ARCH



## UML in SHIQ

MANAGER :< EMPLOYEE, TOP\_MANAGER :< MANAGER

AREA\_MANAGER :< MANAGER

MANAGER :< (or TOP\_MANAGER AREA\_MANAGER)

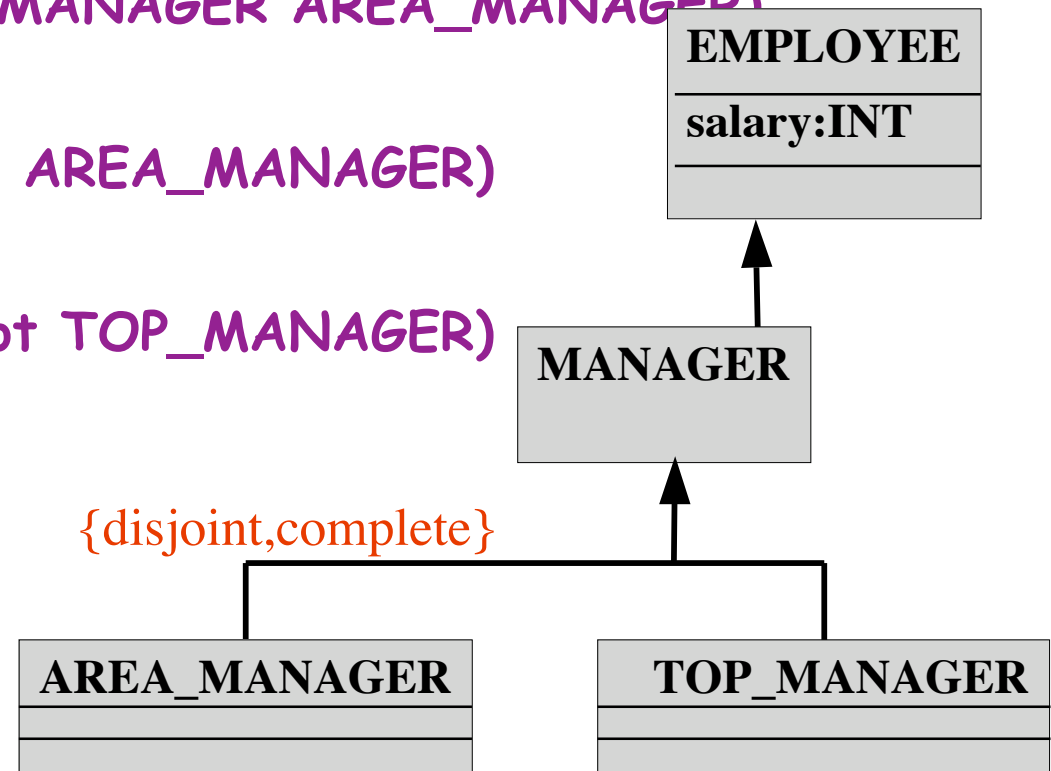
*//complete*

TOP\_MANAGER :< (not AREA\_MANAGER)

*//disjoint*

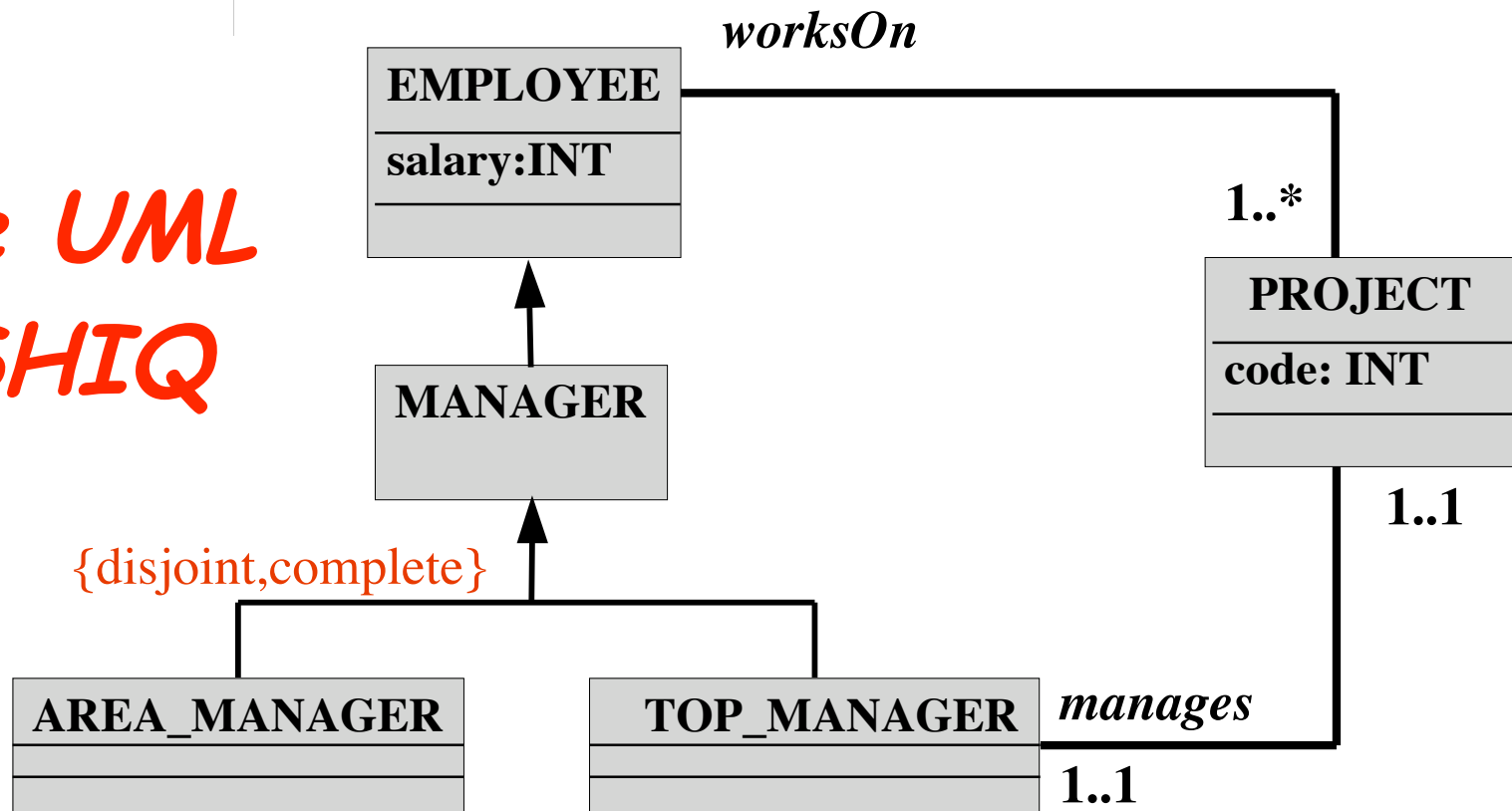
AREA\_MANAGER :< (not TOP\_MANAGER)

*//disjoint*





## More UML in SHIQ

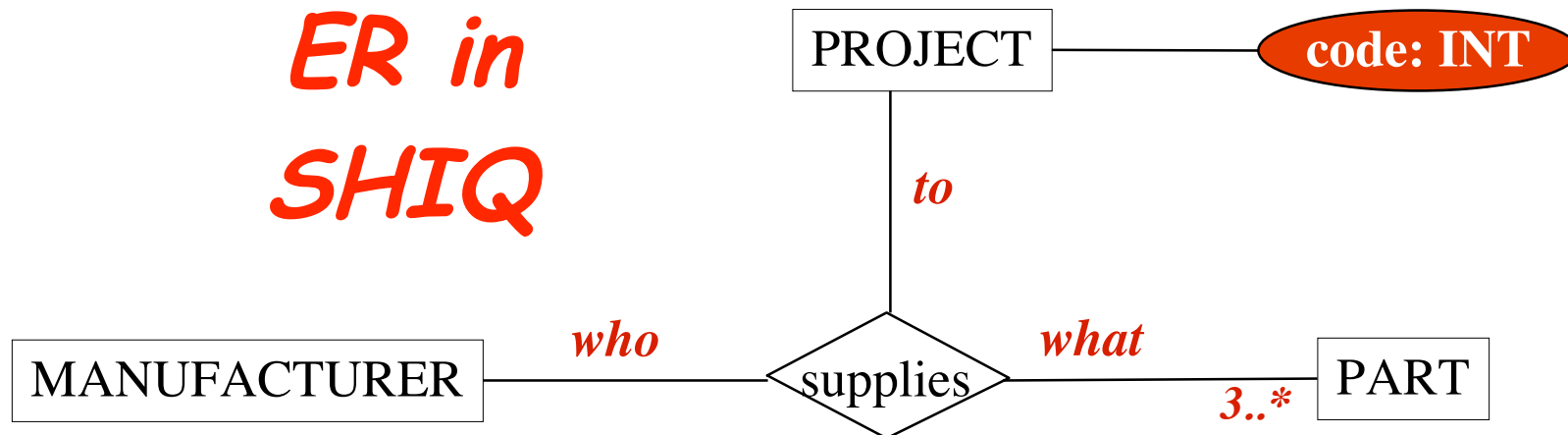


TOP\_MANAGER :< (all manages PROJECT) (exactly 1 manages)

PROJECT :< (all (inverse manages) TOP\_MANAGER)  
(exactly 1 (inverse manages))



## ER in SHIQ



Relationships are “reified” into concepts:

- ✓ SUPPLIES :< (the who MANUF) (the what PART) (the to PROJECT)
- ✓ MANUF :< (all (inverse who) SUPPLIES)
- ✓ PROJECT :< (all (inverse to) SUPPLIES)
- ✓ PART :< (and (all (inverse what) SUPPLIES) (at-least 3 (inverse what)) )



## Representing Keys

- ✓ (at-most 1 (inverse code)) -- describes all integers that have an most one associated project (through the code role)
- ✓ PROJECT :< (and (the code INT)  
(all code (at-most 1 (inverse code)))) // key  
constraint
- ↳ Note: We can't express that if there are two instances of SUPPLIES with the same *who*, *to*, and *what* fillers, then they are the same. To express this, you need predicates with more than two variables.



# Modeling CORBA Services

```
interface CAR{  
  attrib CAR-MODEL model;  
  attrib OWNER ownedBy;  
  attrib MANUFACT madeBy;  
  ...  
  deliver( in MANUFACT src,  
           in DEALER dest,  
           in DATE time) signals  
    (BadDealer);  
  sell(...);  
  destroy(...);
```

```
DELIVER :< (and ACTION  
            (the this CAR)  
            (the src MANUFACT)  
            (the dest DEALER)  
            (the time DATE)
```



## Modeling a CORBA Interface

```
CAR :<  
  (the model    CAR_MODELS)  
  (the ownedBY  OWNER)  
  (the madeBy   MANUFACT)  
    (the deliver DELIVER)  
      //preconds include  
      (same-as  madeBy (deliver src))  
      //postconds include  
      (same-as  ownedBy (deliver dest))  
      //exception BadDealer signalled when  
      (not (overlaps src (dest represents)))
```



# *Medical Ontologies*

↪ Check out

`saussure.irmkant.rm.cnr.it/onto/`

for a philosophically well-thought out and detailed medical ontology





# Some Complexity Results

| Constructors                                    | T box       |          |        | Subsumes?        | Member?  |
|---|-------------|----------|--------|------------------|----------|
|   | (prim :< D) | (D :< C) | cyclic |                  |          |
| <i>AL (and, all)</i>                            | -           | -        | -      | $O(n^2)$         |          |
| <i>AL</i>                                       | +           | -        | -      | co-NP-complete   |          |
| <i>Classic with host individuals</i>            | +           | -        | -      | $O(n^3)$         |          |
| <i>ALE (and, all, some)</i>                     |             |          |        | NP-compl.        | PSPACE   |
| <i>ALC (and, all, not)</i>                      | -           |          |        | PSPACE-complete  |          |
| <i>ALC (and, all, not)</i>                      | +           | +        |        | EXPTIME-complete |          |
| <i>ALCNR (r-and, nrs)</i>                       | -           |          |        | PSPACE           | PSPACE   |
| <i>ALCNR, SHIQ</i>                              | +           | +        | +      | NEXPTIME         | NEXPTIME |
| <i>muALCQ, ALCN+complex roles but not r-and</i> |             |          |        | EXPTIME-complete |          |
| <i>AL &amp; role same-as</i>                    | -           | -        | -      | undecidable      |          |
| <i>AL &amp; attrib. same-as</i>                 |             |          | +      | undecidable      |          |



# Implementation Strategies

- ⇒ Translate to other logics, use existing theorem provers.
- ⇒ Normalize and compare approach: Find normal form which makes *explicit* facts implied by a description  
e.g.,  $\text{atmost}(0,p) \implies \text{atmost}(0,p) \ \& \ \text{all}(p,\text{NOTHING})$ 
  - ✓ usually relatively fast; used in most widely distributed systems (Classic, Loom, Back)
  - ✓ often incomplete; has problems with disjunction, case-by-case reasoning, reasoning by contradiction



# Implementation Strategies

- Tableaux-like calculus: show  $C :< D$  by showing (and  $C$  (not  $D$ )) inconsistent;
  - ✓ Prove this by *contradiction*: try to construct an individual object that can be an instance of (and  $C$  (not  $D$ ))
  - ✓ Use "completion rules", e.g., for  $\{y:\text{all}(P,C), P(y,w)\}$  add  $\{w:C\}$ ;
  - ✓ Usually complete, so termination of the algorithm is the big issue.