2 - Representing Time

Modeling in Knowledge Representation: the Parthood Relation
ICT School Doctorate Course

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Introduction
Time

An important philosophical domain of inquiry

A foundational ontology

An essential domain in Physics

An essential domain in Knowledge Representation and Reasoning

A basic ingredient of most linguistic statements
Which time?

Which domain: absolute or relative time?

- **Absolute:** There is a purely temporal “space” in which events are temporally located.
  - Standard physics
- **Relative:** Time is an implicit structure induced by temporal relations over events
  - Leibniz - Newton controversy
  - Commonsense and psychological evidence

Which structure?

- Which temporal relations
- Which axioms
Which primitive temporal entities?

Absolute case

- “instants”, “moments”: points
- “intervals”, “periods”: regions
- both?

Relative case

- *things that happen*: “events”, “eventualities”…
- all concrete entities?

Three main options studied here

- instants
- intervals; periods
- events
Which temporal relations?

- Precedence relation
  - which order for instants?
  - which order for intervals / periods?
  - which order for events?
- Extended temporal entities → mereological structure
  - which mereology?
  - how do precedence and mereology interact?
Which temporal location?

In the absolute case, to link *things that happen* and time

- **Temporal argument in (some) predicates**
  - \( Raining(t) \land n < t \)
  - \( Loves(caesar, cleopatra, t) \land t < n \)

- **Temporal location**
  - Binary relation relating an event and a time:
    \( Raining(e) \land Occurs(e, t) \land n < t \)

- **Matching dimensions**
  - extended events: periods are best suited
  - both instantaneous and extended events: intervals and instants, or simply instants
Temporal location and temporal logics-1

- Reified temporal logics [Allen, 1984; Shoham 1986; Galton 2006]
- “Fluents”, “properties”: eventuality types, not tokens
  - $\text{Holds}(p, t) \iff$ proposition $p$ is true at/during $t$
  - $\text{Holds}(\text{raining}, t) \land n < t$
- Distinctions according to “dissectivity”, “cumulativity”, etc.
Temporal location and temporal logics-2

- Reasoning about time vs. reasoning about propositions in time
- Modal temporal logics [Prior, 1967]
  - Implicit time
  - Past
    - $Hp$: It has always been the case that $p$
    - $Pp$: It has been the case that $p$
  - Future
    - $Gp$: It will always be the case that $p$
    - $Fp$: It will be the case that $p$
  - $F$ raining
Syntax vs. Semantics, Theory vs. Structures

First-order Theory $\mathcal{I}$:
- a language $\mathcal{L}$ (constants, predicates, functions) and
- a set of axioms

Structure $\mathcal{S}$:
- $\langle$ domain $\mathcal{D}$, distinguished elements (constants) of $\mathcal{D}$, relations in $\mathcal{D}$, operators on $\mathcal{D} \rangle$
- where relations and operators are extensionally or intensionally characterized
A very very short Model Theory - 2

▶ Relation between syntax and semantics
  ▶ Language and *signature*: $\mathcal{L}$-structures
  ▶ Interpretation and truth
    ▶ interpretation $\mathcal{I}$: function from $\mathcal{L}$ to the signature of $\mathcal{S}$ (often already included in the structure)
    ▶ assignment $\mathcal{A}$: function from the set of variables to $\mathcal{D}$
    ▶ $\mathcal{S} \models_{\mathcal{I},\mathcal{A}} P(x,y)$ or $\llbracket P(x,y) \rrbracket_{\mathcal{I},\mathcal{A}}^\mathcal{S} = \top$ iff $\langle \mathcal{A}(x), \mathcal{A}(y) \rangle \in \mathcal{I}(P)$
  ▶ *Models* of $\mathcal{T}$ are structure-interpretation pairs that make all theorems of $\mathcal{T}$ true for any assignment: $\mathcal{S} \models \phi$
Soundness and completeness

A theory $\mathcal{T}$ is sound iff it has a model

A theory $\mathcal{T}$ in $\mathcal{L}$ is semantically complete with respect to the class of $\mathcal{L}$-structures $\mathcal{C}$ or $\mathcal{T}$ axiomatizes the class $\mathcal{C}$ iff $\mathcal{C}$ is the class of all $\mathcal{L}$-models of $\mathcal{T}$, which is equivalent to $\mathcal{T} \vdash \phi \iff$ for all $S$ in $\mathcal{C}$, $S \models \phi$

NB1: this exploits the completeness of first-order logic. A logic is sound and complete iff all theorems are valid formulas (formulas true for any interpretation) and reciprocally: $\vdash \phi \iff \models \phi$

NB2: a theory $\mathcal{T}$ in $\mathcal{L}$ is syntactically complete iff for any closed $\mathcal{L}$-sentence $\phi$ either $\mathcal{T} \vdash \phi$ or $\mathcal{T} \vdash \neg \phi$
A very very short Model Theory - 4

- **Relation between models**
  - Two $\mathcal{L}$-structures are *$\mathcal{L}$-equivalent* iff they make true the same formulas of $\mathcal{L}$. The models of a syntactically complete $\mathcal{L}$-theory are all $\mathcal{L}$-equivalent.
  - Two $\mathcal{L}$-structures are *isomorphic* iff there is an isomorphism between them, i.e., an injective and surjective function from one domain to the other that preserves constants, relations and operators. Isomorphic $\mathcal{L}$-structures are $\mathcal{L}$-equivalent.
  - Löwenheim-Skolem theorem: if a theory has an infinite model, then for any infinite cardinal $\lambda$ it has a model of cardinality $\lambda$
  - A theory is $\lambda$-*categorical* iff all its models of cardinality $\lambda$ are isomorphic
  - A theory is *categorical* iff for any infinite cardinal $\lambda$, it is $\lambda$-categorical
Instant theories and structures

- **Primitive relation**
  - precedence: strict order

- **Which order?**
  - total (linear) or partial (branching)
    - parallel times: alternative worlds
    - linear to the left, branching future: planning
    - linear to the right, branching past: diagnostic
  - unbounded (?)
  - dense or discrete
    - commonsense?
    - Achille and the turtle
    - computers are discrete but calculus assume real time
  - continuity: the dividing instant. Not first-order definable!

- The theory of total unbounded dense order is syntactically complete and $\omega_0$-categorical (countably categorical):
  $\langle \mathbb{Q}, < \rangle$
Interval / period theories and structures

- Temporal precedence: still an order
- Extended entities
  - parthood: period theories
  - adjacency: “interval” theories
Period theories and structures [van Benthem, 83]

- Convex stretches of time
- Two order relations: a precedence $\prec$ and a mereological parthood $\sqsubseteq$
- A variety of period theories, depending on the axioms on $\prec$, $\sqsubseteq$, and those linking the two
- We present here the period theory axiomatizing the structure consisting of intervals in $\mathbb{Q}$

- $\prec$ is an unbounded strict order, “discrete”:
  - NEIGH $\forall x, y \ (x \prec y \rightarrow (\exists z_1 \ (x \prec z_1 \land \neg \exists u \ (x \prec u \prec z_1)) \land \exists z_2 \ (z_2 \prec y \land \neg \exists u \ (z_2 \prec u \prec y))))$
Period theories and structures - 2

- $\sqsubseteq$ is reflexive, transitive, antisymmetric, and such that
  - strong supplementation, called FREE by van Benthem
    \[
    \forall x, y (\forall z (z \sqsubseteq x \rightarrow zOy) \rightarrow x \sqsubseteq y)
    \]
    where $xOy \triangleq \exists z (z \sqsubseteq x \land z \sqsubseteq y)$
  - existence of the product, called CONJ here
    \[
    \forall x, y (xOy \rightarrow \exists z(z \sqsubseteq x \land z \sqsubseteq y \land \forall u((u \sqsubseteq x \land u \sqsubseteq y) \rightarrow u \sqsubseteq z)))
    \]
    $z$ is noted $x \sqcap y$
  - existence of the “convex sum”, called DISJ
    \[
    \forall x, y (xUy \rightarrow \exists z(x \sqsubseteq z \land y \sqsubseteq z \land \forall u((x \sqsubseteq u \land y \sqsubseteq u) \rightarrow z \sqsubseteq u)))
    \]
    where $xUy \triangleq \exists z (x \sqsubseteq z \land y \sqsubseteq z)$; $z$ is noted $x \sqcup y$
  - DIR \quad $\forall x, y \exists z (x \sqsubseteq z \land y \sqsubseteq z)$
  - DENS \quad $\forall x \exists y, z(y \prec z \land x = y + z)$
    where $x = y + z$ iff $x = y \sqcup z \land \forall w (w \sqsubseteq x \rightarrow (wOy \lor wOz))$
Period theories and Structures - 3, Links between $\prec$ and $\sqsubseteq$

- **MON** \(\forall x, y \ (x \prec y \rightarrow \forall z ((z \sqsubseteq x \rightarrow z < y) \land (z \sqsubseteq y \rightarrow x < z)))\)

- **MOND** \(\forall x, y \ (x \prec y \rightarrow (\forall z (z \prec y \rightarrow (x \sqcup z) \prec y) \land \forall z (y \prec x \rightarrow y < (x \sqcup z))))\)

- **CONV** \(\forall x, y, z \ ((x \prec y \land y \prec z) \rightarrow \forall u ((x \sqsubseteq u \land z \sqsubseteq u) \rightarrow y \sqsubseteq u)\)

- **LIN** \(\forall x, y (x \prec y \lor y \prec x \lor \exists z (z \sqsubseteq x \land z \sqsubseteq y))\)

- **ORI** \(\forall x, y \ (x O y \rightarrow (x = y \lor (x \sqsubseteq y \land \exists z (x < z \land y = x + z)) \lor \exists z_1, z_2 \ (z_1 < x < z_2 \land y = (z_1 + x) + z_2)) \lor (y \sqsubseteq x \land (\exists z (y < z \land x = y + z)) \lor \exists z_1, z_2 \ (z_1 < y < z_2 \land x = (z_1 + y) + z_2)) \lor \exists z_1, z_2 \ (z_1 < z_2 \land x = z_1 + z_2 \land z_1 < y \land z_2 \sqsubseteq y \land \exists z_3 (y = z_2 + z_3 \land x < z_3)) \lor \exists z_1, z_2 \ (z_1 < z_2 \land y = z_1 + z_2 \land z_1 < y \land z_2 \sqsubseteq x \land \exists z_3 (x = z_2 + z_3 \land y < z_3))))\)
From points to periods and vice-versa

Let $\langle \mathbb{Q}, \prec \rangle$ be a linear, unbounded, dense strict order structure, then

$\langle I, \prec, \sqsubseteq \rangle$ s.t.

$I = \{(q_1, q_2) \mid q_1, q_2 \in \mathbb{Q} \text{ and } q_1 < q_2\}$

where $(q_1, q_2) = \{q \in \mathbb{Q} \mid q_1 < q < q_2\}$

$(q_1, q_2) \prec (q_3, q_4)$ iff $q_2 \leq q_3$

$(q_1, q_2) \sqsubseteq (q_3, q_4)$ iff $q_3 \leq q_1 < q_2 \leq q_4$

is a model of the period theory (is a period structure)

Period theory is countably categorical, i.e., all models of cardinality $\omega_0$ are isomorphic to $\langle I, \prec, \sqsubseteq \rangle$

Let $\langle I, \prec, \sqsubseteq \rangle$ be a period structure, then

$\langle P, \prec \rangle$ s.t.

$P = \{F \subseteq I \mid (\forall x \in F)(\forall s \in I)(x \sqsubseteq s \rightarrow s \in F) \text{ and } (\forall x, y \in F)(x \sqcap y \in F)\}$

$F_1 < F_2$ iff $(\exists x \in F_1)(\exists y \in F_2)(x < y)$.

is a linear, unbounded, dense strict order structure
Interval theories and structures - 1

- Convex “intervals”
- Allen’s relations [Allen 83, 84; Allen & Hayes 85, 89]
- 13 possible relations between any ordered pair of intervals

Before \[ \frac{x}{y} \]

Meets \[ \frac{x}{y} \]

Overlaps \[ \frac{x}{y} \]

Starts \[ \frac{x}{y} \]

During \[ \frac{x}{y} \]

Finishes \[ \frac{x}{y} \]

Equals \[ \frac{x}{y} \]

- Inverse relations: After, Met-by, Overlapped-by, Started-by, Contains, Finished-by

\[ After(x, y) \leftrightarrow Before(y, x) \]
Allen & Hayes’s theory [85, 89]

- based on a unique primitive || (meets)
- combines order and adjacence

\[ Before(x, y) \triangleq \exists z (x || z \land z || y) \]
\[ Equals(x, y) \triangleq \exists z, t (z || x \land x || t \land z || y \land y || t) \]
\[ Overlaps(x, y) \triangleq \exists z, t, u_1, u_2, u_3 (z || x \land z \land u_1 \land u_1 || y \land u_1 || u_2 \land u_2 || u_3 \land u_3 || t \land y || t) \]

- unbounded, “continuous” and linear time
Alternative axiomatizations (Ladkin, Galton, Hajnicz)

UNI \[ \forall x, y, z, v ((x\parallel z \land x\parallel v \land y\parallel z) \rightarrow y\parallel v) \]

UEND \[ \forall x, y, z, v ((x\parallel z\parallel y \land x\parallel v\parallel y) \rightarrow z = v) \]

LIN \[ \forall x, y, z, v ((x\parallel y \land y\parallel z\parallel v) \rightarrow (x\parallel v \lor \exists s(x\parallel s\parallel v) \lor \exists s(z\parallel s\parallel y))) \]

UNB \[ \forall x \exists y, z (y\parallel x\parallel z) \]

SUM \[ \forall x, y (x\parallel y \rightarrow \exists z, v, s(z\parallel x\parallel y\parallel v \land z\parallel s\parallel v)) \]

DENSIM \[ \forall x, y, z, u (P<(x, y, z, u) \rightarrow \exists v, w (P<(x, y, v, w) \land P<(v, w, z, u))) \]

with \[ P<(x, y, z, u) \triangleq x\parallel y \land z\parallel u \land \exists w(x\parallel w\parallel u) \]
Interval theories and structures - 4

Theorems

IRREF \( \forall x (\neg(x\parallel x)) \)

ASYM \( \forall x, y (x\parallel y \rightarrow \neg(y\parallel x)) \)

ANTRA \( \forall x, y (x\parallel y \rightarrow \neg\exists z (x\parallel z \land z\parallel y)) \)
From point to interval structures and vice-versa

- Let $\langle \mathbb{Q}, < \rangle$ be a linear, unbounded, dense strict order structure, then
  - $\langle I, \parallel \rangle$ s.t.
    - $I = \{(q_1, q_2) \mid q_1, q_2 \in \mathbb{Q} \text{ and } q_1 < q_2\}$
      where $(q_1, q_2) = \{q \in \mathbb{Q} \mid q_1 < q < q_2\}$
    - $(q_1, q_2) \parallel (q_3, q_4)$ iff $q_2 = q_3$
  - is a model of interval theory (an interval structure)

- Interval theory is countably categorical, i.e., all models of cardinality $\omega_0$ are isomorphic to $\langle I, \parallel \rangle$

- Let $\langle I, \parallel \rangle$ be an interval structure, then
  - $\langle P, < \rangle$ s.t.
    - $P = \{[x, y] \mid x, y \in I \text{ and } x \parallel y\}$
      where $[x, y] = \{(z, v) \mid z, v \in I \text{ and } x \parallel v \text{ and } z \parallel y\}$
    - $[x, y] < [z, v]$ iff $\exists w (x \parallel w \parallel v)$.
  - is a linear, unbounded, dense strict order structure
Events

- Relative approach: no time, only temporal relations
- Simultaneity is not identity
- Properties like density, unboundedness make little sense
The need for events

- Causal reasoning and planning are based on events, esp. actions
- Linguistic evidence: event names, event anaphora, verb modification...
- But: identity criteria for events are not obvious
  - co-localization
    - spatio-temporal, not temporal: simultaneity does not entail identity
    - distinction object / event: myself and my life
    - distinction between events: the spinning of the ball and the warming up of the ball
  - causal equivalence, logical equivalence...
Temporal theory based on events - 1 [Kamp 1979]

- Precedence $\prec$: a strict partial order
- Overlap $o$: a reflexive and symmetric relation
- Mixed axioms
  - $\forall x, y (x \prec y \rightarrow \neg xoy)$
  - $\forall x, y, z, t ((x \prec y \land yoz \land z \prec t) \rightarrow x \prec t)$
  - $\forall x, y (x \prec y \lor xoy \lor y \prec x)$
- Construction of an instant structure from an event structure (Russell-Wiener)
  - $\langle E, \prec, o \rangle$: event structure
  - instants are maximal sets of two by two overlapping events
    $I \subseteq 2^E$ s.t. for any $i \in I \ \forall x, y \in i \ xoy$ and
    $\forall x \in E (x \notin i \rightarrow \exists y (y \in i \land \neg xoy))$
  - for all $i, j \in I$, $i \prec j$ iff $\exists x, y (x \in i \land y \in j \land x \prec y)$
  - $\langle I, \prec \rangle$: instant structure
Temporal theory based on events - 2

- An interval structure can be built on top of the instant structure
- Not an isomorphism between the original event structure and this interval structure
  - No sum and no product existence within events: more intervals than events
  - Atomic events generate instants: degenerate intervals are needed
- Hypothesis that only “events” (accomplishments and achievements) contribute to time structure
  - every state and activity is started and ended by a (change of state) event