2 - Representing Time

Modeling in Knowledge Representation: the Parthood Relation
ICT School Doctorate Course

March 7th, 2007

Introduction

Time

An important philosophical domain of inquiry

A foundational ontology

An essential domain in Physics

An essential domain in Knowledge Representation and Reasoning

A basic ingredient of most linguistic statements

Which time?

Which domain: absolute or relative time?

- Absolute: There is a purely temporal “space” in which events are temporally located.
  - Standard physics
- Relative: Time is an implicit structure induced by temporal relations over events
  - Leibniz - Newton controversy
  - commonsense and psychological evidence

Which structure?

- Which temporal relations
- Which axioms

Which primitive temporal entities?

Absolute case

- “instants”, “moments”: points
- “intervals”, “periods”: regions
- both?

Relative case

- *things that happen*: “events”, “eventualities”...
- all concrete entities?

Three main options studied here

- instants
- intervals; periods
- events

Which temporal relations?

- Precedence relation
  - which order for instants?
  - which order for intervals / periods?
  - which order for events?
- Extended temporal entities → mereological structure
  - which mereology?
  - how do precedence and mereology interact?

Which temporal location?

In the absolute case, to link *things that happen* and time

- Temporal argument in (some) predicates
  - $Raining(t) \wedge \mathbf{n} < t$
  - $Loves(caesar, cleopatra, t) \wedge t < \mathbf{n}$
- Temporal location
  - Binary relation relating an event and a time: $Raining(e) \wedge Occurs(e, t) \wedge \mathbf{n} < t$
- Matching dimensions
  - extended events: periods are best suited
  - both instantaneous and extended events: intervals and instants, or simply instants

Temporal location and temporal logics-1

- Reified temporal logics [Allen, 1984; Shoham 1986; Galton 2006]
- "Fluents", "properties": eventuality *types*, not tokens
  - $Holds(p,t)$ iff proposition p is true at/during t
  - $Holds(raining,t) \wedge \mathbf{n} < t$
- Distinctions according to "dissectivity", "cumulativity", etc.

Temporal location and temporal logics-2

- Reasoning about time vs.
  reasoning about propositions in time
- Modal temporal logics [Prior, 1967]
  - Implicit time
  - Past
    - Hp: It has always been the case that p
    - Pp: It has been the case that p
  - Future
    - Gp: It will always be the case that p
    - Fp: It will be the case that p
  - $F\ raining$

A very very short Model Theory - 1

- Syntax vs. Semantics, Theory vs. Structures
  - First-order Theory $\mathcal{T}$:
    a language $\mathcal{L}$ (constants, predicates, functions) and
    a set of axioms
  - Structure $\mathcal{S}$:
    $\langle$ domain $\mathcal{D}$, distinguished elements (constants) of $\mathcal{D}$,
    relations in $\mathcal{D}$, operators on $\mathcal{D}$ $\rangle$
    where relations and operators are extensionally or
    intensionally characterized

A very very short Model Theory - 2

- Relation between syntax and semantics
  - Language and *signature*: $\mathcal{L}$-structures
  - Interpretation and truth
    - interpretation $\mathcal{I}$: function from $\mathcal{L}$ to the signature of $\mathcal{S}$ (often already included in the structure)
    - assignment $\mathcal{A}$: function from the set of variables to $\mathcal{D}$
    - $\mathcal{S} \models_{\mathcal{I},\mathcal{A}} P(x, y)$ or $[\![P(x, y)]\!]^{\mathcal{S}}_{\mathcal{I},\mathcal{A}} = \top$ iff $\langle \mathcal{A}(x), \mathcal{A}(y) \rangle \in \mathcal{I}(P)$
  - *Models* of $\mathcal{T}$ are structure-interpretation pairs that make all theorems of $\mathcal{T}$ true for any assignment: $\mathcal{S} \models \phi$

A very very short Model Theory - 3

- Soundness and completeness
  - A theory $\mathcal{T}$ is sound *iff* it has a model
  - A theory $\mathcal{T}$ in $\mathcal{L}$ is semantically complete with respect to the class of $\mathcal{L}$-structures $\mathcal{C}$ *or* $\mathcal{T}$ axiomatizes the class $\mathcal{C}$ *iff* $\mathcal{C}$ is the class of all $\mathcal{L}$-models of $\mathcal{T}$, which is equivalent to $\mathcal{T} \vdash \phi \Leftrightarrow$ for all $\mathcal{S}$ in $\mathcal{C}$, $\mathcal{S} \models \phi$

  - NB1: this exploits the completeness of first-order logic. A *logic* is sound and complete *iff* all theorems are valid formulas (formulas true for any interpretation) and reciprocally: $\vdash \phi \Leftrightarrow \models \phi$
  - NB2: a theory $\mathcal{T}$ in $\mathcal{L}$ is *syntactically* complete *iff* for any closed $\mathcal{L}$-sentence ϕ either $\mathcal{T} \vdash \phi$ or $\mathcal{T} \vdash \neg\phi$

A very very short Model Theory - 4

- Relation between models
  - Two $\mathcal{L}$-structures are $\mathcal{L}$-*equivalent* iff they make true the same formulas of $\mathcal{L}$. The models of a syntactically complete $\mathcal{L}$-theory are all $\mathcal{L}$-equivalent.
  - Two $\mathcal{L}$-structures are *isomorphic* iff there is an isomorphism between them, i.e., an injective an surjective function from one domain to the other that preserves constants, relations and operators. Isomorphic $\mathcal{L}$-structures are $\mathcal{L}$-equivalent.
  - Löwenheim-Skolem theorem: if a theory has an infinite model, then for any infinite cardinal λ it has a model of cardinality λ
  - A theory is λ-*categorical* iff all its models of cardinality λ are isomorphic
  - A theory is *categorical* iff for any infinite cardinal λ, it is λ-categorical

Instant theories and structures

- Primitive relation
  - precedence: strict order
- Which order?
  - total (linear) or partial (branching)
    - parallel times: alternative worlds
    - linear to the left, branching future: planning
    - linear to the right, branching past: diagnostic
  - unbounded (?)
  - dense or discrete
    - commonsense?
    - Achille and the turtle
    - computers are discrete but calculus assume real time
  - continuity: the dividing instant. Not first-order definable!
- The theory of total unbounded dense order is syntactically complete and ω_0-categorical (countably categorical): $\langle \mathbb{Q}, < \rangle$

Interval / period theories and structures

- Temporal precedence: still an order
- Extended entities
  - parthood: period theories
  - adjacence: “interval” theories

Period theories and structures [van Benthem, 83]

- *Convex* stretches of time
- Two order relations: a precedence $\prec$ and a mereological parthood $\sqsubseteq$
- A variety of period theories, depending on the axioms on $\prec$, $\sqsubseteq$, and those linking the two
- We present here the period theory axiomatizing the structure consisting of intervals in $\mathbb{Q}$

- $\prec$ is an unbounded strict order, "discrete":
  - NEIGH $\quad \forall x, y\, (x \prec y \to (\exists z_1\, (x \prec z_1 \wedge \neg \exists u\, (x \prec u \prec z_1)) \wedge \exists z_2\, (z_2 \prec y \wedge \neg \exists u\, (z_2 \prec u \prec y))))$

Period theories and structures - 2

- $\sqsubseteq$ is reflexive, transitive, antisymmetric, and such that
  - strong supplementation, called FREE by van Benthem
    $\forall x, y\ (\forall z\ (z \sqsubseteq x \rightarrow z\mathsf{O}y) \rightarrow x \sqsubseteq y)$
    where $x\mathsf{O}y \triangleq \exists z\ (z \sqsubseteq x \wedge z \sqsubseteq y)$
  - existence of the product, called CONJ here
    $\forall x, y\ (x\mathsf{O}y \rightarrow \exists z(z \sqsubseteq x \wedge z \sqsubseteq y \wedge \forall u((u \sqsubseteq x \wedge u \sqsubseteq y) \rightarrow u \sqsubseteq z)))$
    z is noted $x \sqcap y$
  - existence of the “convex sum”, called DISJ
    $\forall x, y\ (x\mathsf{U}y \rightarrow \exists z\ (x \sqsubseteq z \wedge y \sqsubseteq z \wedge \forall u((x \sqsubseteq u \wedge y \sqsubseteq u) \rightarrow z \sqsubseteq u)))$
    where $x\mathsf{U}y \triangleq \exists z\ (x \sqsubseteq z \wedge y \sqsubseteq z)$; z is noted $x \sqcup y$
  - DIR $\quad \forall x, y\ \exists z\ (x \sqsubseteq z \wedge y \sqsubseteq z)$
  - DENS $\quad \forall x \exists y, z(y \prec z \wedge x = y + z)$
    where $x = y + z$ iff $x = y \sqcup z \wedge \forall w\ (w \sqsubseteq x \rightarrow (w\mathsf{O}y \vee w\mathsf{O}z))$

Period theories and Structures - 3, Links between $\prec$ and $\sqsubseteq$

- MON $\forall x, y\ (x \prec y \rightarrow \forall z((z \sqsubseteq x \rightarrow z \prec y) \wedge (z \sqsubseteq y \rightarrow x \prec z)))$
- MOND $\forall x, y\ (x \prec y \rightarrow (\forall z\ (z \prec y \rightarrow (x \sqcup z) \prec y)$
  $\wedge \forall z\ (y \prec x \rightarrow y \prec (x \sqcup z))))$
- CONV
  $\forall x, y, z\ ((x \prec y \wedge y \prec z) \rightarrow \forall u\ ((x \sqsubseteq u \wedge z \sqsubseteq u) \rightarrow y \sqsubseteq u))$
- LIN $\forall x, y(x \prec y \vee y \prec x \vee \exists z(z \sqsubseteq x \wedge z \sqsubseteq y))$
- ORI $\forall x, y\ (x\mathrm{O}y \rightarrow (x = y \vee$
  $(x \sqsubseteq y \wedge (\exists z\ (x \prec z \wedge y = x + z) \vee \exists z\ (z \prec x \wedge y = x + z) \vee$
  $\exists z_1, z_2\ (z_1 \prec x \prec z_2 \wedge y = (z_1 + x) + z_2)))$
  $(y \sqsubseteq x \wedge (\exists z\ (y \prec z \wedge x = y + z) \vee \exists z\ (z \prec y \wedge x = y + z) \vee$
  $\exists z_1, z_2\ (z_1 \prec y \prec z_2 \wedge x = (z_1 + y) + z_2))) \vee$
  $\exists z_1, z_2\ (z_1 \prec z_2 \wedge x = z_1 + z_2 \wedge z_1 \prec y \wedge z_2 \sqsubseteq y \wedge$
  $\exists z_3 (y = z_2 + z_3 \wedge x \prec z_3))) \vee$
  $\exists z_1, z_2\ (z_1 \prec z_2 \wedge y = z_1 + z_2 \wedge z_1 \prec y \wedge z_2 \sqsubseteq x \wedge$
  $\exists z_3\ (x = z_2 + z_3 \wedge y \prec z_3)))))$

From points to periods and vice-versa

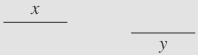
- Let $\langle \mathbb{Q}, < \rangle$ be a linear, unbounded, dense strict order structure, then
  - $\langle I, \prec, \sqsubseteq \rangle$ s.t.
    - $I = \{(q_1, q_2) \mid q_1, q_2 \in \mathbb{Q}$ and $q_1 < q_2\}$ where $(q_1, q_2) = \{q \in \mathbb{Q} \mid q_1 < q < q_2\}$
    - $(q_1, q_2) \prec (q_3, q_4)$ iff $q_2 \leq q_3$
    - $(q_1, q_2) \sqsubseteq (q_3, q_4)$ iff $q_3 \leqslant q_1 < q_2 \leqslant q_4$

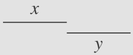
    is a model of the period theory (is a period structure)
  - Period theory is countably categorical, i.e., all models of cardinality ω_0 are isomorphic to $\langle I, \prec, \sqsubseteq \rangle$
- Let $\langle I, \prec, \sqsubseteq \rangle$ be a period structure, then
  - $\langle P, < \rangle$ s.t.
    - $P = \{F \subseteq I | (\forall x \in F)(\forall s \in I)(x \sqsubseteq s \rightarrow s \in F)$ and $(\forall x, y \in F)(x \sqcap y \in F)\}$;
    - $F_1 < F_2$ iff $(\exists x \in F_1)(\exists y \in F_2)(x \prec y)$.

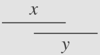
    is a linear, unbounded, dense strict order structure

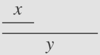
Interval theories and structures - 1

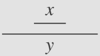
- Convex "intervals"
- Allen's relations [Allen 83, 84; Allen & Hayes 85, 89]
- 13 possible relations between any ordered pair of intervals

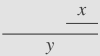
Before 

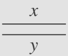
Meets 

Overlaps 

Starts 

During 

Finishes 

Equals 

  - Inverse relations: After, Met-by, Overlapped-by, Started-by, Contains, Finished-by
    $After(x, y) \leftrightarrow Before(y, x)$

Interval theories and structures - 2

- Allen & Hayes's theory [85, 89]
  - based on a unique primitive $\|$ (meets)
  - combines order and adjacence
  - $Before(x, y) \triangleq \exists z(x\|z \wedge z\|y)$
  - $Equals(x, y) \triangleq \exists z, t(z\|x \wedge x\|t \wedge z\|y \wedge y\|t)$
  - $Overlaps(x, y) \triangleq$ $\exists z, t, u_1, u_2, u_3(z\|x \wedge z\|u_1 \wedge u_1\|y \wedge u_1\|u_2 \wedge u_2\|u_3 \wedge u_3\|t \wedge y\|t)$
  - unbounded, "continuous" and linear time

Interval theories and structures - 3

- Alternative axiomatizations (Ladkin, Galton, **Hajnicz**)

UNI	$\forall x, y, z, v\ ((x\|z \wedge x\|v \wedge y\|z) \rightarrow y\|v)$
UEND	$\forall x, y, z, v\ ((x\|z\|y \wedge x\|v\|y) \rightarrow z = v)$
LIN	$\forall x, y, z, v\ ((x\|y \wedge z\|v) \rightarrow (x\|v\ \nabla\ \exists s(x\|s\|v)\ \nabla\ \exists s(z\|s\|y)))$
UNB	$\forall x\ \exists y, z\ (y\|x\|z)$
SUM	$\forall x, y\ (x\|y \rightarrow \exists z, v, s(z\|x\|y\|v \wedge z\|s\|v))$
DENSM	$\forall x, y, z, u\ (\mathsf{P}_<(x, y, z, u) \rightarrow$
	$\exists v, w\ (\mathsf{P}_<(x, y, v, w) \wedge \mathsf{P}_<(v, w, z, u)))$

with $\mathsf{P}_<(x, y, z, u) \triangleq x\|y \wedge z\|u \wedge \exists w(x\|w\|u)$

Interval theories and structures - 4

- Theorems

IRREF	$\forall x\ (\neg(x \| x))$
ASYM	$\forall x, y\ (x \| y \rightarrow \neg(y \| x))$
ANTRA	$\forall x, y\ (x \| y \rightarrow \neg\exists z\ (x \| z \wedge z \| y))$

From point to interval structures and vice-versa

- Let $\langle \mathbb{Q}, < \rangle$ be a linear, unbounded, dense strict order structure, then
  - $\langle I, \| \rangle$ s.t.
    - $I = \{(q_1, q_2) \mid q_1, q_2 \in \mathbb{Q}$ and $q_1 < q_2\}$
      where $(q_1, q_2) = \{q \in \mathbb{Q} \mid q_1 < q < q_2\}$
    - $(q_1, q_2) \| (q_3, q_4)$ iff $q_2 = q_3$

    is a model of interval theory (an interval structure)
  - Interval theory is countably categorical, i.e., all models of cardinality ω_0 are isomorphic to $\langle I, \| \rangle$
- Let $\langle I, \| \rangle$ be an interval structure, then
  - $\langle P, < \rangle$ s.t.
    - $P = \{[x, y] \mid x, y \in I$ and $x \| y\}$
      where $[x, y] = \{(z, v) \mid z, v \in I$ and $x \| v$ and $z \| y\}$
    - $[x, y] < [z, v]$ iff $\exists w\ (x \| w \| v)$.

    is a linear, unbounded, dense strict order structure

Events

- Relative approach: no time, only temporal relations
- Simultaneity is not identity
- Properties like density, unboundedness make little sense

The need for events

- Causal reasoning and planning are based on events, esp. actions
- Linguistic evidence: event names, event anaphora, verb modification...
- But: identity criteria for events are not obvious
  - co-localization
    - spatio-temporal, not temporal: simultaneity does not entail identity
    - distinction object / event: myself and my life
    - distinction between events: the spinning of the ball and the warming up of the ball
  - causal equivalence, logical equivalence...

Temporal theory based on events - 1 [Kamp 1979]

- Precedence $\prec$: a strict partial order
- Overlap o: a reflexive and symmetric relation
- Mixed axioms
  - $\forall x, y\ (x \prec y \rightarrow \neg xoy)$
  - $\forall x, y, z, t\ ((x \prec y \wedge yoz \wedge z \prec t) \rightarrow x \prec t)$
  - $\forall x, y\ (x \prec y \vee xoy \vee y \prec x)$
- Construction of an instant structure from an event structure (Russell-Wiener)
  - $\langle E, \prec, o \rangle$: event structure
  - instants are maximal sets of two by two overlapping events $I \subseteq 2^E$ s.t. for any $i \in I\ \forall x, y \in i\ xoy$ and $\forall x \in E\ (x \notin i \rightarrow \exists y\ (y \in i \wedge \neg xoy))$
  - for all $i, j \in I$, $i < j$ iff $\exists x, y\ (x \in i \wedge y \in j \wedge x \prec y)$
  - $\langle I, < \rangle$: instant structure

Temporal theory based on events - 2

- An interval structure can be built on top of the instant structure
- Not an isomorphism between the original event structure and this interval structure
  - No sum and no product existence within events: more intervals than events
  - Atomic events generate instants: degenerate intervals are needed
- Hypothesis that only “events” (accomplishments and achievements) contribute to time structure
  - every state and activity is started and ended by a (change of state) event