Modeling in Knowledge Representation: the Parthood Relation

Mereogeometries (Lect. 1 of 2)

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Material covered in these lectures

Lecture 1: Weak Mereogeometries

(1) Lines of sight (Galton)
(2) Occlusion Calculus (Randell et al.)
(3) Convex Hull operator (Cohn)

Lecture 2: Full Mereogeometries
Geometrical primitives we will consider

(4) Sphere
(5) Congruence
(6) Conjugate
(7) Can Connect
(8) Closer

Modeling occlusion - 1

Motivations

To describe the relative position of objects from an observer's perspective. Such a system will help the observer to:

(1) detect object's boundary,
(2) explain why objects cannot be seen,
(3) gather information to plan an action (e.g. to navigate)
(4) decide how to move to make an object (or the observer) totally visible/invisible

Global picture

(a) the observer is fixed and the objects are free to move
(b) the objects are fixed and the observer is free to move

Modeling Occlusion - 2

Idealization

Think of the three-dimensional scene as a sphere:

> *the observer is at the center and the objects are projected to the surface of the sphere.*

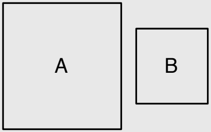
The real distance of the objects from the observer is unknown as well as their real size. The only clue comes from occlusion or lack of it: an object blocks (totally or in part) the view of another one depending on their relative position wrt the observer.

Lines of Sight
A. Galton, 1994

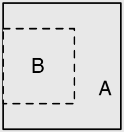
Further restrictions:

(1) The position of the observer is a point in space
(2) The objects are convex and non-rigid
(3) Objects cannot overlap
(4) Shape is irrelevant (e.g. take all objects to be spheres)

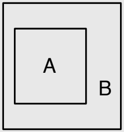
Lines of Sight: relations 1-6



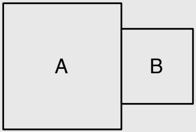
A is clear of B
C(A,B)



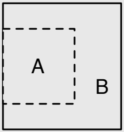
A just hides B
JH(A,B)



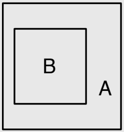
A is in front of B
F(A,B)



A is just clear of B
JC(A,B)

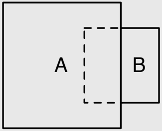


A is just hidden by B
JHI(A,B)

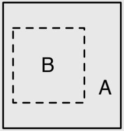


A has B in front of it
FI(A,B)

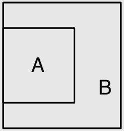
Lines of Sight: relations 7-12



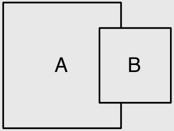
A partially hides B
PH(A,B)



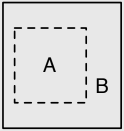
A hides B
H(A,B)



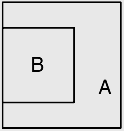
A is just in front of B
JF(A,B)



A is partially hidden by B
PHI(A,B)



A is hidden by B
HI(A,B)

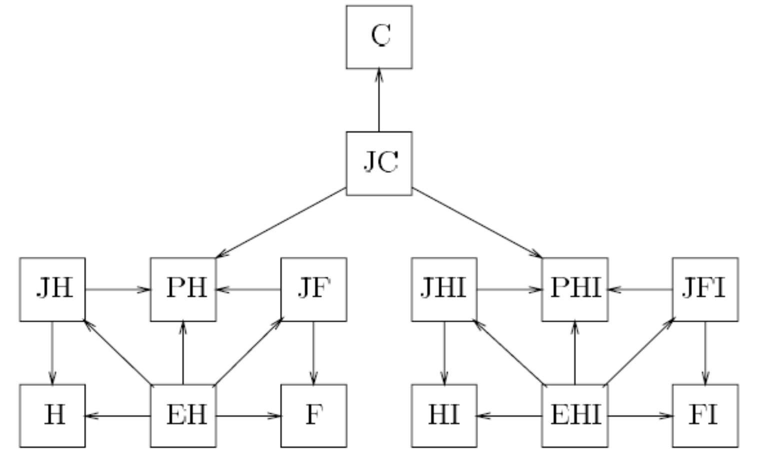


A has B just in front of it
JFI(A,B)

There are two more relations:

“A exactly hides B, `EH(A,B)`” and “A is exactly hidden by B, `EHI(A,B)`”

Conceptual neighborhood



Composing relations

Recall the composition tables of mereotopology presented by Carola.

We can apply the same idea.
E.g. $\mathtt{H} \circ \mathtt{PH} = (\mathtt{PH}, \mathtt{JH}, \mathtt{H})$

<u>Observations</u>:

(1) Some entry of the composition table is not a *conceptual neighborhood*, i.e., a connected subset of the neighborhood diagram.
E.g. $\mathtt{HI} \circ \mathtt{EH} = (\mathtt{F}, \mathtt{HI})$

Combining subtables - 1

Let us focus on a simplified set of relations only, namely:

$$\hat{C} = (\mathtt{C}, \mathtt{JC}),$$
$$\hat{O} = \mathtt{PH},$$
$$\hat{F} = (\mathtt{F}, \mathtt{JF}),$$
$$\hat{H} = (\mathtt{H}, \mathtt{JH}, \mathtt{EH}),$$
$$\hat{O}I = \mathtt{PHI},$$
$$\hat{F}I = (\mathtt{FI}, \mathtt{JFI}),$$
$$\hat{H}I = (\mathtt{HI}, \mathtt{JHI}, \mathtt{EHI})$$

We show that one can build the composition table of these relations from the composition table of smaller sets of relations.

Combining subtables - 2

Take the topological relations on the projected images (these are *new* relations):

- D (disjoint), O (overlap), P (part-of), PI (has-as-part), E (equals).
- and the relative distance relations: N (nearer), NI (further).

Observe that each ^-relation corresponds to a pair of these: $\hat{O}$ = [N,O]; $\hat{O}I$ = [NI,O]; $\hat{F}$ = [N,P]; $\hat{H}I$ = [NI,P]; ...

Some relations need complex pairs, e.g.: $\hat{C}$ = [(N,NI), D]; $\hat{H}$ = [N, (PI,E)]

Combining subtables - 3

	D	O	P	PI	E
D	any	D,O,P	D,O,P	D	D
O	D,O,PI	any	O,P	D,O,PI	O
P	D	D,O,P	P	any	P
PI	B,O,PI	O,PI	O,P,PI,E	PI	PI
E	D	O	P	PI	E

	N	NI
N	N	N, NI
NI	N, NI	NI

One computes the full table by combining these two tables.

E.g. $\hat{O}\hat{I} \circ \hat{F} =$

$[\mathtt{NI}, \mathtt{O}] \circ [\mathtt{N}, \mathtt{P}] = [\mathtt{NI} \circ \mathtt{N}, \mathtt{O} \circ \mathtt{P}] = [(\mathtt{N}, \mathtt{NI}), (\mathtt{O}, \mathtt{P})] =$

$[(\mathtt{N}, \mathtt{O}), (\mathtt{N}, \mathtt{P}), (\mathtt{NI}, \mathtt{O}), (\mathtt{NI}, \mathtt{P})] =$

$(\hat{O}, \hat{F}, \hat{O}\hat{I}, \hat{H}\hat{I})$

The Region Occlusion Calculus

Randell, Witkowski, Shanahan, 2001

Called *ROC-20*, it extends the Lines of Sight calculus by allowing concave shaped objects.

Restrictions:

(1) The position of the observer is a point in space

(2) The objects are non-rigid

(3) Shape is irrelevant

The Basis of *ROC*

(1) The universe of discourse comprises 3 distinct types of entities: bodies (3D), regions (which are divided in two subtypes: 3D and 2D regions), and points (0D)

(2) Two primitive functions:
- $reg(x)$ to indicate the 3D region occupied by body x
- $image(x, v)$ to indicate the 2D region which is the image of body x wrt the 0D point v (informally, the observer's viewpoint)

(3) The mereo-topological level of *ROC-20* is given by *RCC-8*

(4) Basic axioms (maps to *RCC*)

$\forall xy[\Phi(reg(x), reg(y)) \rightarrow \forall v[\Phi(image(x, v), image(y, v))]]$
where $\Phi \in \{C, O, P, PP, NTPP, EQ\}$

Q.: something wrong with this axiom?

The Occlusion Primitive

$TO(x, y, v)$ stands for

"body x totally occludes body y wrt viewpoint v"

Axioms

(1) Irreflexive and transitive for any fixed v

(2) $\forall xyzv[[TO(x, y, v) \wedge P(reg(z), reg(y))] \rightarrow TO(x, z, v)]$

(3) $\forall xyv[TO(x, y, v) \rightarrow \forall z[P(reg(z), reg(y)) \rightarrow \neg TO(z, x, v)]]$

(4) $\forall xyv[TO(x, y, v) \rightarrow$
$\forall zu[[P(reg(z), reg(x)) \wedge P(reg(u), reg(y))] \rightarrow \neg TO(u, z, v)]]$

(5) $\forall xv\exists yz[P(reg(y), reg(x)) \wedge P(reg(z), reg(x)) \wedge TO(y, z, v)]$

(6) $\forall xyv[TO(x, y, v) \rightarrow P(image(y, v), image(x, v))]$

Initial ROC Definitions

From TO and parthood we can define:

(1) $Occludes(x, y, v) \equiv$
$\exists z, u[P(reg(z), reg(x)) \wedge P(reg(u), reg(y)) \wedge TO(z, u, v)]$

(2) $PartiallyOccludes(x, y, v) \equiv$
$Occludes(x, y, v) \wedge \neg TO(x, y, v) \wedge \neg Occludes(y, x, v)]$

(3) $MutuallyOccludes(x, y, v) \equiv$
$Occludes(x, y, v) \wedge Occludes(y, x, v)]$

(4) $NonOccludes(x, y, v) \equiv \neg Occludes(x, y, v) \wedge \neg Occludes(y, x, v)]$

<u>Maps to RCC for a fixed veiwpoint:</u>

$NonOccludes \rightarrow DR$

$PartiallyOccludes \rightarrow \{PO, PP\}$

$MutuallyOccludes \rightarrow \{PO, P, PI\}$

Refining the definitions

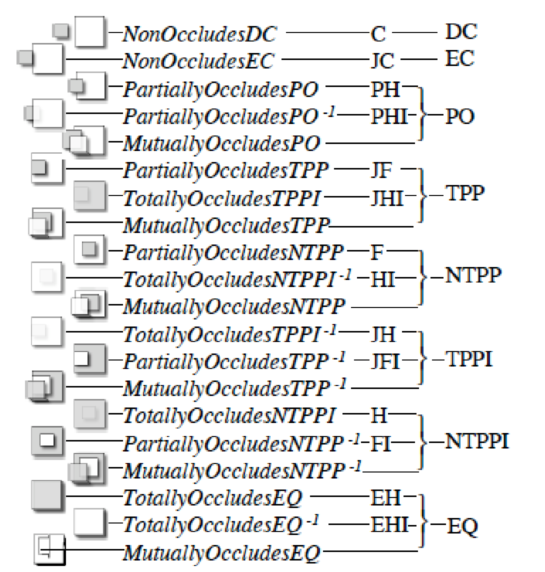
ROC-20 is obtained by refining the above definitions using *RCC*

(1) $TO\&EQ(x, y, v)$
(2) $TO\&TPPI(x, y, v)$
(3) $TO\&NTPPI(x, y, v)$
(4) $NonOccludes\&DC(x, y, v)$
(5) $NonOccludes\&EC(x, y, v)$
(6) $PartiallyOccludes\&PO(x, y, v)$
(7) $PartiallyOccludes\&TPP(x, y, v)$
(8) $PartiallyOccludes\&NTPP(x, y, v)$
(9) etc.

E.g. $TO\&EQ(x, y, v) \equiv TO(x, y, v) \wedge EQ(image(x, v), image(y, v))$

ROC-20, LOS-14, RCC comparison

In the left column region A is always gray, B is white.



The Convex Hull Calculus
Cohn, 1995

Goal:
to provide a qualitative description of the shape of objects.

Primitives:

(1) Connection, C (to get *RCC*)

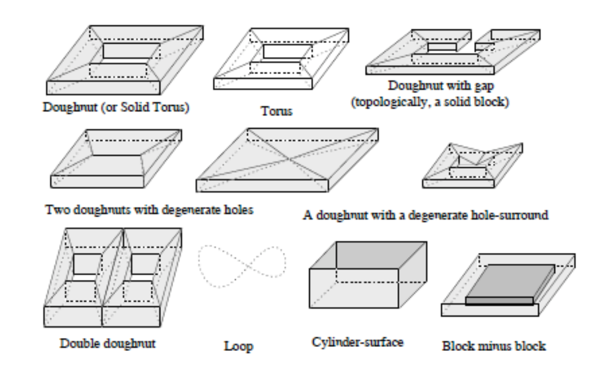
(2) Convex Hull, $Conv$

Notions and Shapes using Connection Alone

(1) Self-connected: $CON(x) \equiv \neg\exists yz[x = y + z \wedge DC(y, z)]$

(2) Manifold: $Manifold(x) \equiv \forall yz[x = y + z \rightarrow \exists w[O(w, y) \wedge O(w, z) \wedge DC(w, compl(x)) \wedge CON(w)]]$

(3) Topological Component: $MAX\_P(x, y) \equiv Manifold(x) \wedge P(x, y) \wedge \neg\exists z[PP(x, z) \wedge P(z, y) \wedge Manifold(z)]$



Axioms for $Conv$

Let $CONV(x) =_{\text{def}}\ x = conv(x)$.

(1) $\forall x\,[conv(conv(x)) = conv(x)]$

(2) $\forall x\,[\neg x = conv(x) \rightarrow TPP(x, conv(x))]$ thus $P(x, conv(x))$

(3) $\forall xy\,[P(x, y) \rightarrow P(conv(x), conv(y))]$

(4) $\forall xy\,[P(conv(x) + conv(y), conv(x + y))]$

(5) $\forall xy\,[conv(x) = conv(y) \rightarrow C(x, y)]$ any comment?

(6) $\forall xy\,[conv(x) * conv(y) = conv(conv(x) * conv(y))]$

(7) $\forall xy\,[DC(x, y) \rightarrow \neg CONV(x + y)]$

(8) $\forall xy\,[NTPP(x, y) \rightarrow \neg CONV(y - x)]$

(9) $\forall xy\,[[CONV(x) \wedge CONV(y)] \rightarrow CONV(x * y)]$

do you feel you have we seen this one already?

(10) $\forall xyz\,[[EC(x, y) \wedge CONV(x+y) \wedge EC(y, z) \wedge CONV(y+z) \wedge DC(x, z)] \rightarrow CONV(y)]$

Shapes using Connection and Convex Hull

- $Concavity(x, y) \equiv MAX\_P(x, inside(y))$
- $Adjacent(i1, i2, x) \equiv Concavity(i1, x) \wedge Concavity(i2, x) \wedge \exists z\,[EC(z, i1) \wedge EC(z, i2) \wedge PP(z, x) \wedge CON(z) \wedge DC(z, inside(x) - i1 - i2) \wedge \forall i3, i4[[Concavity(i3, x) \wedge Concavity(i3, x) \wedge DC(i1 + i2, i3 + i4)] \rightarrow \exists w\,[CON(w) \wedge PP(w, x) \wedge EC(w, i3) \wedge EC(w, i4) \wedge DR(w, z)]]]$
- $SameSide(i1, i2, x) \equiv Concavity(i1, x) \wedge Concavity(i2, x) \wedge \exists z\,[P(z, x) \wedge Manifold(i1 + i2 + z) \wedge Manifold(x - z) \wedge O(i1, conv(x - z)) \wedge O(i2, conv(x - z))]$

(Note that this last definition does not work... can you fix it?)

In *RCC+conv*, one can distinguish the following shapes:

Defining Stripe from Convex Hull

Mereology augmented with the operator "$conv$" (convex hull) can define "$Strp$" (being a stripe)

- A Cut_C is a pair of non-overlapping convex regions whose sum is the universe
  $Cut_C(x, y) =_{\text{def}} CONV(x) \wedge CONV(y) \wedge \neg Oxy \wedge \forall z.P(z, x + y)$
  (x and y are complementary half-planes)
- A *half-plane* is any region forming a cut
  $HP(x) =_{\text{def}} \exists y\, Cut_C(x, y)$ (x is half-plane)
- A *stripe* x is a convex non-HP region that can be added to a non-overlapping HP to form a HP
  $Strp(y) =_{\text{def}} CONV(y) \wedge \neg HP(y) \wedge \exists x.HP(x) \wedge \neg O(y, x) \wedge HP(x + y)$ (x is a stripe)

Defining Convex Hull from Stripe - 1

Mereology augmented with the predicate "$Strp$" (being a stripe) can define operator "$conv$" (convex hull)

- A region is *finite* if it is part of two overlapping stripes that do not have a stripe in common
  $FReg(x) =_{\text{def}} \exists yz\,[Strp(y) \wedge Strp(z) \wedge O(y,z) \wedge \neg Strp(y*z) \wedge P(x, y*z)]$
  (x is a finite region)

- A *half-plane* is any region that contains only some stripes, for each finite part there is a stripe in the region that contains such a part, any two stripes not in it are contained in a stripe not in it, and any two overlapping stripes in it have a stripe in common.
  $$\begin{aligned} HP(x) =_{\text{def}} & \exists yz\,[Strp(y) \wedge \neg O(y,x) \wedge Strp(z) \wedge P(z,x) \wedge \\ & \forall y((P(y,x) \wedge FReg(y)) \rightarrow \exists z(P(z,x) \wedge Strp(z) \wedge P(y,z))) \wedge \\ & \forall u,v((Strp(u) \wedge Strp(v) \wedge \neg O(u,x) \wedge \neg O(v,x)) \rightarrow \\ & \qquad \exists w(Strp(w) \wedge \neg O(w,x) \wedge P(u+v,x))) \wedge \\ & \forall u,v((Strp(u) \wedge Strp(v) \wedge P(u+v,x) \wedge O(u,v)) \rightarrow \\ & \qquad \exists w(Strp(w) \wedge P(w,u) \wedge P(w,v)))] \end{aligned}$$
  (x is half-plane)

Defining Convex Hull from Stripe - 2

- A Cut_S is a pair of non-overlapping half-planes whose sum is the universe
  $Cut_S(x,y) =_{\text{def}} HP(x) \wedge HP(y) \wedge \neg O(x,y) \wedge \forall z.P(z, x+y)$
  (x and y are complementary half-planes)
- A convex region x is a region such that no larger region can be contained in exactly the same half-planes containing x.
  $CONV(x) =_{\text{def}} \forall u\, PP(x,u) \rightarrow \exists y.(HP(y) \wedge P(x,y) \wedge \neg P(u,y))$
  (x is convex)
- $x = conv(y)$ iff $P(y,x) \wedge CONV(x) \wedge \forall z\,(P(y,z) \wedge CONV(z)) \rightarrow P(x,z)$

...but why do we care about “Stripe”?

Some Results

(Grzegorczyk, 1951): both the first-order theory of *RCC* and the first-order theory containing P (parthood) and $conv$ are undecidable.

(Davis, Gotts, Cohn 1999) on bounded regular regions:

THEOREM If region s is an affine transformation of region r , then r and s cannot be distinguished by any first-order formula over RCC_8+$conv$.

A *constraint* is a predicate applied to constant symbols.
A *constraint network* is a finite conjunction of constraints.
A *constraint language* is a language in which sentences are constraint networks.

THEOREM The constraint language over RCC_8+$conv$ is decidable.

THEOREM A constraint network $\mathcal{N}$ over RCC_8+$conv$ is consistent if and only if the system of algebraic constraints *corresponding* to $\mathcal{N}$ is consistent over the reals.