Conceptual modelling, ontology design, and semantic interoperability

Professional master on technologies for e-government
Lecture 7

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Formal ontology

Brings together two senses of *formal*.

- ▷ **Rigorous**: formal framework, e.g., logic.
- ▷ **Foundational**: search for invariants across domains (Husserl).

Rigorous

Starting from the informal conceptual analysis it is necessary to choose:

- ▷ the kinds of entities we accept in the domain;
- ▷ the formal language (logical framework + primitives);
- ▷ the properties of the primitives (axioms).

Foundational

Basic tools for ontology development:

▷ import results coming from *logic*, *philosophy*, *linguistics*, *cognitive science*, *mathematics*, *computer science*, etc. in order to identify a set of well founded conceptual primitives that are general enough to be applied to several domains constituting the basis for the development of more domain oriented models.

$\Rightarrow$ Intrinsically multidisciplinary.

Formal ontology again

Facilitates:

- ▷ a precise representation (and communication) based on cognitively transparent and theoretically well founded primitives;
- ▷ the **reuse** of theories in different contexts and domains;
- ▷ a modular approach to ontology building;
- ▷ the **comparison** between different theories that correspond to alternative ontological positions.

Basic notions

But what are these basic notions coming from different disciplines?

- ▷ Essentiality and identity.
- ▷ Properties and qualities.
- ▷ Dependence.
- ▷ Parthood, connection, and unity.
- ▷ Time, space, and space-time.
- ▷ Objects, constitution, and composition.
- ▷ Events, participation, and causality.

Outline

- ▷ Orders
- ▷ Mereologies
- ▷ Theories of time

Orders (1/5)

Orders are quite simple and abstract structures that are useful for general and heterogeneous comparisons between entities:

- ▷ "is more/less ... than": "is bigger than", "is smaller than", "is later than" ...
- ▷ "is to the left of", "is an ancestor of", "is a divisor of", "is part of"...

Note: in the following we will consider as formal framework the FOL with identity.

Orders

- A **partial order** is a binary relation $\leq$ such that:

A1	$x \leq x$	(*reflexivity*)
A2	$x \leq y \wedge y \leq z \rightarrow x \leq z$	(*transitivity*)
A3	$x \leq y \wedge y \leq x \rightarrow x = y$	(*antisymmetry*)

- A **strict partial order** is a binary relation $<$ such that:

A4	$x < y \wedge y < z \rightarrow x < z$	(*transitivity*)
A5	$x < y \rightarrow \neg y < x$	(*asymmetry*)
T1	$\neg x < x$	(*irreflexivity*)

Orders

- ▷ In a partial order, defining $<$ as:

  D1 $x < y \triangleq x \leq y \wedge \neg y \leq x$

  we obtain a strict partial order.

- ▷ In a strict partial order, defining $\leq$ as

  D2 $x \leq y \triangleq x < y \vee x = y$

  we obtain a partial order.

- ▷ Therefore it is the same to start from the $\leq$ or the $<$ primitive.

Orders **(4/5)**

▷ Classical additional properties for orders

A6	$x \leq y \vee y \leq x$	(*total/linear order*)
A7	$x < y \rightarrow \exists z(x < z \wedge z < y)$	(*dense order*)
A8	$x < y \rightarrow \exists z(x < z \leq y \wedge \neg\exists v(x < v < z))$	
	$x < y \rightarrow \exists z(x \leq z < y \wedge \neg\exists v(z < v < y))$	(*discrete order*)
A9	$\exists x \forall y(x \leq y)$	(*left bounded*)
A10	$\exists x \forall y(y \leq x)$	(*right bounded*)
A11	$\forall x \exists y(y < x)$	(*left unbounded*)
A12	$\forall x \exists y(x < y)$	(*right unbounded*)

▷ On finite domains, all orders are discrete and bounded.

▷ **Examples**: $\langle \mathbb{N}, \leq \rangle$, $\langle \mathbb{Z}, \leq \rangle$, $\langle \mathbb{Q}, \leq \rangle$, $\langle \mathbb{R}, \leq \rangle$, $\langle 2^A, \subseteq \rangle$.

Orders (5/5)

- ▷ Many orders on many domains!
  – weights, heights, numbers, instants (precedence), preferences...
  these are not parthood!

- ▷ Specificities of Parthood?
  – Surely not a linear order
  – Dense, discrete, bounded, unbounded: all possible options

  So?

A bit of history of mereologies

Mereology: from the greek meros, 'the theory of parthood'

- ▷ Lesniewski 1927-1931, On the Foundations of Mathematics. Alternative to Set Theory for escaping Russells paradox
  - No null individual (no empty set)
  - No distinction between *urelements* ($\in$) and *sets* ($\subseteq$): a single relation of parthood
- ▷ Tarski 1935. Link with algebra.
- ▷ Leonard and Goodman 1940. The calculus of individuals, nominalism.
- ▷ Contemporary studies: Peter Simons (1986), Achille Varzi (1996).

Why is the Parthood relation important?

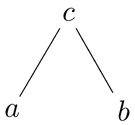
Today, the majority of ontologies use at least a parthood relation.

- ▷ Philosophical, cognitive and linguistic relevance.
- ▷ Spatial and temporal reasoning based on vague information: impossibility to use exact coordinates, trajectories in terms of mathematical functions, and calculus.
- ▷ Reference to extended entities (e.g., temporal periods, spatial regions), possibly composed of parts of the same nature.
- ▷ No calculus, yet still a rigorous formal approach: logical theories.

Note: similarly to the case of orders, no one single mereology, but a plurality of different mereologies.

Hasse diagrams

- Graph on finite domains
- **Convention**: all vertical or oblique arcs are implicitly oriented from bottom to top and indicate a strict order.
- **Example**: the following diagram graphically represents the formula $a < c \wedge b < c$



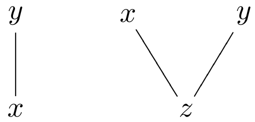
Basic mereology (M)

- We note the parthood relation with P.
- A *basic mereology* M is a reflexive, transitive, and antisymmetric relation, i.e. it is a partial order.
- Useful definitions (*proper part* corresponds to strict order):

  D3 $\mathsf{PP}(x, y) \triangleq \mathsf{P}(x, y) \wedge \neg\mathsf{P}(y, x)$ (*proper part*)

  D4 $\mathsf{O}(x, y) \triangleq \exists z(\mathsf{P}(z, x) \wedge \mathsf{P}(z, y))$ (*overlap*)

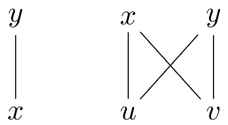
  D5 $\mathsf{PO}(x, y) \triangleq \mathsf{O}(x, y) \wedge \neg\mathsf{P}(x, y) \wedge \neg\mathsf{P}(y, x)$ (*proper overlap*)



Minimal and extensional mereologies (1/2)

▷ Supplementation

$$\begin{array}{lll} \textsf{ws} & \mathsf{PP}(x,y) \rightarrow \exists z(\mathsf{P}(z,y) \wedge \neg \mathsf{O}(z,x)) & (\textit{weak suppl.}) \\ \textsf{ss} & \neg \mathsf{P}(x,y) \rightarrow \exists z(\mathsf{P}(z,y) \wedge \neg \mathsf{O}(z,x)) & (\textit{strong suppl.}) \end{array}$$



▷ Extensionality:

$$\begin{array}{lll} \textsf{we} & \exists z(\mathsf{PP}(z,x)) \wedge \forall z(\mathsf{PP}(z,x) \leftrightarrow \mathsf{PP}(z,y))) \rightarrow x = y & (\textit{weak}) \\ \textsf{se} & \forall z(\mathsf{O}(z,x) \leftrightarrow \mathsf{O}(z,y)) \rightarrow x = y & (\textit{strong}) \end{array}$$

note that we does not follows from ws, see theorem (T5).

Minimal and extensional mereologies (2/2)

$\mathsf{MM} = \mathsf{M} \cup \{\mathsf{ws}\}$ *minimal mereology*

$\mathsf{EM} = \mathsf{M} \cup \{\mathsf{ss}\}$ *extensional mereology*

▷ Theorems:

T2 $\mathsf{EM} \vdash \mathsf{ws}$

T3 $\mathsf{EM} \vdash \mathsf{we}$

T4 $\mathsf{EM} \vdash \mathsf{se}$

T5 $\mathsf{MM} \nvdash \mathsf{we}$

T6 $\mathsf{M} \cup \{\mathsf{we}\} \nvdash \mathsf{se}$

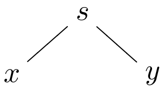
T7 $\mathsf{M} \cup \{\mathsf{se}\} \vdash \mathsf{ws}$

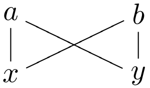
Closure mereology (1/3)

▷ Sum:

D6 $\mathsf{Sum}(s, x, y) \triangleq \forall z(\mathsf{O}(z, s) \leftrightarrow (\mathsf{O}(z, x) \vee \mathsf{O}(z, y)))$

sum $\forall x, y\, \exists s(\mathsf{Sum}(s, x, y))$

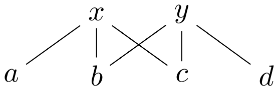




▷ Product:

D7 $\mathsf{Prod}(p, x, y) \triangleq \forall z(\mathsf{P}(z, p) \leftrightarrow (\mathsf{P}(z, x) \wedge \mathsf{P}(z, y)))$

prd $\mathsf{O}(x, y) \rightarrow \exists p(\mathsf{Prod}(p, x, y))$



Closure mereology (2/3)

▷ Difference:

$$\textbf{D8}\quad \mathsf{Diff}(d,x,y) \triangleq \forall z(\mathsf{P}(z,d) \leftrightarrow (\mathsf{P}(z,x) \wedge \neg \mathsf{O}(z,y)))$$

$$\mathsf{dif}\quad \exists z(\mathsf{P}(z,x) \wedge \neg \mathsf{O}(z,y)) \rightarrow \exists d(\mathsf{Diff}(d,x,y))$$

▷ Complement:

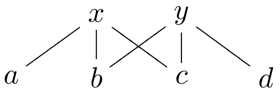
$$\textbf{D9}\quad \mathsf{Compl}(c,x) \triangleq \forall z(\mathsf{P}(z,c) \leftrightarrow \neg \mathsf{O}(z,x))$$

$$\mathsf{cmp}\quad \exists z(\neg \mathsf{O}(z,x)) \rightarrow \exists c(\mathsf{Compl}(c,x))$$

▷ Universe (noted with Un):

$$\mathsf{uni}\quad \exists x \forall y(\mathsf{P}(y,x))$$

$$\textbf{T8}\quad \exists z(z = \mathsf{Un}) \wedge x \neq \mathsf{Un} \rightarrow (\mathsf{Compl}(c,x) \leftrightarrow \mathsf{Diff}(c,\mathsf{Un},x))$$



Closure mereology **(3/3)**

$\mathsf{CM} = \mathsf{M} \cup \{\mathsf{sum}, \mathsf{prd}\}$ *closure mereology*
$\mathsf{CMM} = \mathsf{CM} \cup \{\mathsf{ws}\}$ *closure minimal mereology*
$\mathsf{CEM} = \mathsf{CM} \cup \{\mathsf{ss}\}$ *closure extensional mereology*

▷ Theorems:

T9 $\mathsf{CMM} \vdash \mathsf{ss}$ therefore CMM and CEM are equivalent.

T10 $\mathsf{CEM} \vdash \mathsf{Sum}(s, x, y) \wedge \mathsf{Sum}(s', x, y) \rightarrow s = s'$

T11 $\mathsf{CEM} \vdash \mathsf{Prod}(p, x, y) \wedge \mathsf{Prod}(p', x, y) \rightarrow p = p'$

T12 $\mathsf{CEM} \vdash \mathsf{dif}$

T13 $\mathsf{CEM} \vdash \mathsf{Diff}(d, x, y) \wedge \mathsf{Diff}(d', x, y) \rightarrow d = d'$

T14 $\mathsf{CEM} \vdash \mathsf{Compl}(c, x) \wedge \mathsf{Compl}(c', x) \rightarrow c = c'$

Atomic mereology

- ▷ Atoms, atomicity, and atomic essentialism:

D10 $\mathsf{At}(x) \triangleq \forall y(\mathsf{P}(y,x) \rightarrow y = x)$ (*atom*)

at $\forall x \exists y(\mathsf{At}(y) \wedge \mathsf{P}(y,x))$ (*atomicity*)

ate $(\mathsf{At}(z) \rightarrow (\mathsf{P}(z,x) \rightarrow \mathsf{P}(z,y))) \rightarrow \mathsf{P}(x,y)$ (*at. essent.*)

- ▷ Atomic mereologies

$\mathsf{AEM} = \mathsf{EM} \cup \{\mathsf{at}\}$

the same for all other mereologies

- ▷ Theorems:

T15 $\mathsf{AEM} \vdash \mathsf{ate}$

T16 $\mathsf{EM} \cup \{\mathsf{ate}\} \vdash \mathsf{at}$

T17 $\mathsf{M} \cup \{\mathsf{ate}\} \vdash \mathsf{ss}$

T18 $\mathsf{AM} \cup \{\mathsf{we}\} \vdash \mathsf{se}$

Back to mereology requirements

▷ No distinction between *urelements* ($\in$) and *sets* ($\subseteq$)

- in the Sum "operator", the *sum* and the *addenda* are at the same level

▷ No null individual (no empty set)

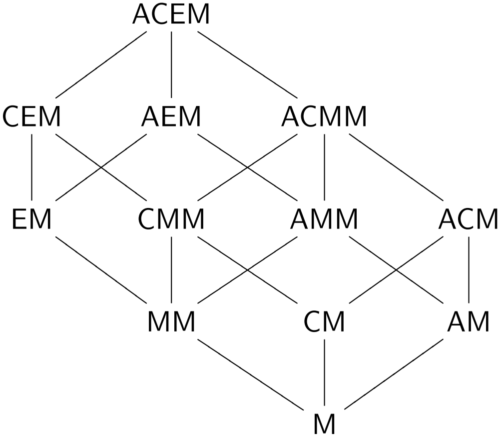
- the mereologies we considered are not based on an axiom like

  A13 $\exists x(\forall y(\mathsf{P}(x, y)))$

Which mereology?

- $\triangleright$ **Extensionality**
  - – Identity between my body and the collection of my organs
  - • parthood vs. spatial inclusion + theory of constitution and levels of reality
- $\triangleright$ **Closure**
  - – sum of my nose and Caesars toe
  - • mereotopology (part+connection) to identify *wholes*
- $\triangleright$ **Transitivity**
  - – My hand is part of me, and
    I'm part of the University of Trento, but
    My hand is not part of the Committee

The space of mereologies



Libraries of theories

- ▷ The *libraries* of theories as, for example, the library/space of mereologies are very important *conceptual* tools because they 'encapsulate' a deep analysis of different notions that are intuitively linked to the a general one (parthood for mereologies).
- ▷ Different theories can be *adequate* to specific modeling requirements:the user selects the theory that better matches his needs.
- ▷ No monolithic/standardized approach: the links between the theories in the library make explicit their (in)compatibilities.

- In this lecture, we will see how different mereologies can be *reused* for modeling some perspectives on time.
- In the following of the course you will see their application to: space, physical objects, qualities, organizations, etc.