# Conceptual modelling, ontology design, and semantic interoperability

Professional master on technologies for e-government
**Lecture 8, part I**

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Trento, 24.10.2007

# The importance of time

- An important philosophical domain of inquiry.
- A foundational ontology.
- An essential domain in physics.
- An essential domain in knowledge representation and reasoning.
- A basic ingredient of most linguistic statements.

# Instant theories of time

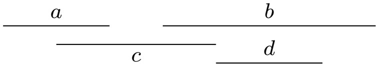
- ▷ Time is *absolute*, i.e. it is a container in which entities exist, are located, take place, or happen.
- ▷ **Domain**: punctual and extensionless temporal entities, i.e. *instants*, *moments*.
- ▷ **Primitive relation**: *precedence* ($<$), i.e. a strict order relation between instants.

# Structure of instants

- ▷ Total (linear) or partial (branching)
  - parallel times: alternative *worlds*
  - linear to the left, branching future: *planning*
  - linear to the right, branching past: *diagnostic*
- ▷ Bounded or unbounded
- ▷ Dense or discrete
  - commonsense?
  - Achille and the turtle paradox
  - computers are discrete but calculus assume real time

# Period theories of time

- ▷ Time is *absolute*.
- ▷ **Domain**: extended temporal entities (*convex* stretches of time), i.e. *interval*, *periods*.
- ▷ **Primitive relations**: to take into account the extension of the periods, we need two order relations: *precedence* ($\prec$), and *parthood/inclusion* ($\sqsubseteq$).




- ▷ Similarly to instants there is a huge space of possibilities.
- ▷ We present here the period theory axiomatizing the structure consisting of intervals in $\mathbb{Q}$ (Van Benthem 1980).

# Van Benthem theory of periods (1/3)

▷ Let us start to characterize $\prec$ and $\sqsubseteq$ by *reusing* some of the axioms already discussed:

- $\prec$ is unbounded and discrete strict order;
- $\sqsubseteq$ is an $\mathsf{EM}$ plus the existence of the product ($\mathsf{prd}$);

▷ A domain with extended convex entities complicates the theory:

- not all the sums of convex periods are convex; therefore we need to modify the standard sum ($\mathsf{sum}$):

**D1** $\mathsf{cSum}(s,x,y) \triangleq x \sqsubseteq s \land y \sqsubseteq s \land \forall u((x \sqsubseteq u \land y \sqsubseteq u) \rightarrow s \sqsubseteq u))$

$\mathsf{csum}$ $\exists z(x \sqsubseteq z \land y \sqsubseteq z) \rightarrow \exists s(\mathsf{cSum}(s,x,y))$

# Van Benthem theory of periods (2/3)

- To assure the existence of convex sums:

  **A1** $\forall x, y \exists z (x \sqsubseteq z \wedge y \sqsubseteq z)$

- A *convexity* axiom is needed:

  **A2** $x \prec y \wedge y \prec z \rightarrow \forall u ((x \sqsubseteq u \wedge z \sqsubseteq u) \rightarrow y \sqsubseteq u)$

- The *density* axiom becomes($\mathsf{fSum}$: convex sum without gaps):

  **D2** $\mathsf{fSum}(s, x, y) \triangleq \mathsf{cSum}(s, x, y) \wedge \forall z (z \sqsubseteq s \rightarrow (\mathsf{O}(z, x) \vee \mathsf{O}(z, y))$

  **A3** $\forall x \exists y, z (y \prec z \wedge \mathsf{fSum}(x, y, z))$

- The *linearity* axiom becomes:

  **A4** $x \prec y \vee y \prec x \vee \exists z (z \sqsubseteq x \wedge z \sqsubseteq y)$

# Van Benthem theory of periods (3/3)

New axioms linking the two primitives are required.

- Order monotonicity:

  **A5** $x \prec y \to \forall z((z \sqsubseteq x \to z \prec y) \land (z \sqsubseteq y \to x \prec z))$

- Sum monotonicity:

  **A6** $x \prec y \to (\forall z(z \prec y \land \mathsf{cSum}(s, x, z) \to s \prec y) \land \forall z(x \prec z \land \mathsf{cSum}(s, y, z) \to x \prec s))$

- Van Benthem considered other axioms can be added, but here we have not the time to enter into the details.

# Allen's theory of periods (1983-89)

- **Domain**: convex intervals.
- Time is absolute, unbounded and linear.
- One primitive: *meets* that combines order and adjacency.
- 13 possible relations between any ordered pair of intervals (the six below + identity + the inverse relations):

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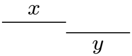
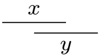
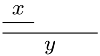
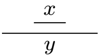
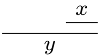
Meets 


Overlaps 


Starts 


During 


Finishes 

# Event theories of time

- **Relative approach**: time is an implicit structure induced by temporal relations over events (Leibniz - Newton controversy).
- Commonsense and psychological evidence.
- Simultaneity is not identity.

# The need for events

- ▷ Causal reasoning and planning are based on events, esp. actions
- ▷ Linguistic evidence: event names, event anaphora, verb modification...
- ▷ But: identity criteria for events are not obvious
  - co-localization
    - spatio-temporal, not temporal: simultaneity does not entail identity
    - distinction object / event: myself and my life
    - distinction between events: the spinning of the ball and the warming up of the ball
  - causal equivalence, logical equivalence...

# Kamp's theory of events (1979)

- **Domain**: events.
- **Primitives**: *precedence* ($\lhd$) and (temporal) *overlap* ($\mathsf{O}$) between events.
- The precedence is a strict partial order and the overlap is reflexive and symmetric relation. In addition mixed axioms:

  **A7** $\forall x, y(x \lhd y \rightarrow \neg \mathsf{O}(x, y))$

  **A8** $\forall x, y, z, t((x \lhd y \wedge \mathsf{O}(y, z) \wedge z \lhd t) \rightarrow x \lhd t)$

  **A9** $\forall x, y(x \lhd y \vee \mathsf{O}(x, y) \vee y \lhd x)$

- Defining (temporal) parthood/inclusion as:

  **D3** $\mathsf{P}(x, y) \triangleq \forall z(\mathsf{O}(z, x) \rightarrow \mathsf{O}(z, y))$

## Summing up (1/2)

We have seen that:

- ▷ there exists a *space* of orders and mereologies, i.e. that the multitude of orders and mereologies that can be organized according to some formal and practical dimensions
  $\Rightarrow$ *library of theories*; and
- ▷ orders and mereologies have been *reused* in the theories of time
  $\Rightarrow$ library of *theories* $\approx$ library of *routines*
  $\Rightarrow$ *modularization* and incremental development of theories.

**but...**

# Summing up (2/2)

Theories of time disagree on:

- **domain**: instants vs. periods vs. events (there are also theories that consider both instants and periods)
- **primitives**: even theories that agree on the domain can disagree on primitives (precedence+parthood vs. meets)
- **axioms**: there are a huge space of possible characterizations of the primitives that identify different structures of time

How is it possible to integrate systems based on different theories or at least to allow for their interoperability?

# Extending the comparison

- ▷ The library of theories offers a way of comparing/integrating theories, or at least to understand (in)compatibilities.
- ▷ Axioms+FOL links are particularly useful for comparing theories that agree on the domain but disagree on primitives/axioms.
- ▷ But, they are less useful for comparing theories that disagree on the domain (instant vs. period vs. event theories of time) because often the links between the domains require more expressive power.

⇒ We can consider set-theoretical mappings between structures that are models of the theories.

# Comparing theories via structures

- ▷ We need to define how formula talking about temporal relations among instants translate in formula about temporal relations among periods or events (and viceversa).
- ▷ We consider the following structures:
  - $\langle I, < \rangle$: $<$ is a linear, unbounded, dense strict order on $I$. (*instant structure*)
  - $\langle P, \prec, \sqsubseteq \rangle$ (*VB period structure*)
  - $\langle P, \parallel \rangle$ (*AH period structure*)
  - $\langle E, \lhd, \mathsf{O} \rangle$ (*KA event structure*)

# From instant to VB period structures

Let $\langle I, < \rangle$ be a *instant structure*, then $\langle P, \prec, \sqsubseteq \rangle$ such that

- $P = \{(i_1, i_2) \mid i_1, i_2 \in I \text{ and } i_1 < i_2\}$, where $(i_1, i_2) = \{i \in I \mid i_1 < i < i_2\}$
- $(i_1, i_2) \prec (i_3, i_4)$ iff $i_2 \leqslant i_3$
- $(i_1, i_2) \sqsubseteq (i_3, i_4)$ iff $i_3 \leqslant i_1 < i_2 \leqslant i_4$

is a *VB period structure*.

**Note 1**: we can define periods as couples of instants instead of (convex) sets of instants.

**Note 2**: we can define this link in FOL assuming two kinds of entities (instants and periods) and one *incident* relation between instants and periods.

# From VB period to instant structures

Let $\langle P, \prec, \sqsubseteq \rangle$ a *VB period structure* then $\langle I, < \rangle$ such that

- $I = \{F \subseteq P \,|\, \forall x \in F, s \in P(x \sqsubseteq s \rightarrow s \in F)$ and
$\forall x, y \in F, p \in P(\mathsf{Prod}(p, x, y) \rightarrow p \in F)\}$
- $F_1 < F_2$ iff $\exists x \in F_1, y \in F_2(x \prec y)$

is an *instant structure*.

**Note 1**: the sets $F$ are called *filters*.

**Note 2**: the construction of instants by means of filters is quite set-theoretically oriented and, with respect the previous one, much more difficult to simulate in FOL.

# From instant to AH period structures

Let $\langle I, < \rangle$ be a *instant structure*, then $\langle P, \| \rangle$ such that

- $P = \{(i_1, i_2) \mid i_1, i_2 \in I \text{ and } i_1 < i_2\}$, where $(i_1, i_2) = \{i \in I \mid i_1 < i < i_2\}$
- $(i_1, i_2) \| (i_3, i_4) \text{ iff } i_2 = i_3$

is a *AH period structure*.

**Note**: the construction of instants is the identical to the one for VB period structures.

# From AH period to instant structures

Let $\langle P, \| \rangle$ a *AH period structure* then $\langle I, < \rangle$ such that

- $I = \{[x, y] \,|\, x, y \in P \,\text{and}\, x \parallel y\}$, where
  $[x, y] = \{(z, v) \,|\, z, v \in P \text{ and } x \parallel v \text{ and } z \parallel y\}$
- $[x, y] < [z, v]$ iff $\exists w \; (x \parallel w \parallel v)$

is an *instant structure.*

# From KA event to instant structures

Let $\langle E, \lhd, \mathsf{O} \rangle$ a *KA even structure* then $\langle I, < \rangle$ such that

- $I = \{F \subseteq E \mid \forall x, y \in F(\mathsf{O}(x,y))$ and
  $\forall x \in E(x \not\in F \rightarrow \exists y(y \in F \wedge \neg \mathsf{O}(x,y)))\}$
- $F_1 < F_2$ iff $\exists x \in F_1, y \in F_2(x \lhd y)$

is an *instant structure*.

**Note 1**: the construction of instants if quite similar to the one in VB period structures.

**Note 2**: from the instant structure is then possible to construct the interval structure following the previous mapping, therefore it is possible to map KA event structures to all the other ones.

**Note 3**: the construction of event structures from period or instants structure is not interesting because events are more general.