Laboratory for Applied Ontology

Institute of Cognitive Science and Technology
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Ontological Analysis

4 - The tools of Formal Ontology

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The problem of primitives

The formal tools of ontological analysis

- **Theory of Essence and Identity**
- **Theory of Parts (Mereology)**
- **Theory of Unity and Plurality**
- **Theory of Dependence**
- **Theory of Composition and Constitution**
- **Theory of Properties and Qualities**

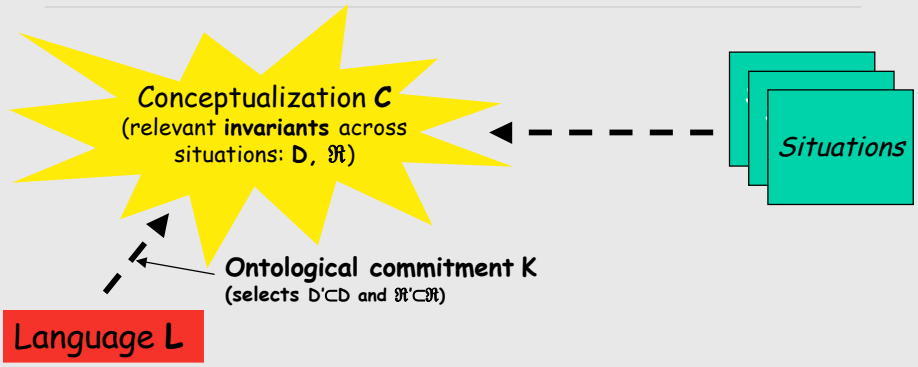
The basis for a common ontology vocabulary

Idea of Chris Welty, IBM Watson Research Centre, while visiting our lab in 2000

Formal Ontology

- Theory of ***formal distinctions and connections*** within:
  - entities of the world, as we perceive it (***particulars***)
  - categories we use to talk about such entities (***universals***)
- Why ***formal***?
  - Two meanings: ***rigorous*** and ***general***
  - Formal logic: connections between truths - neutral wrt ***truth***
  - Formal ontology: connections between things - neutral wrt ***reality***
- ***NOTE:*** **"represented in a formal language" is not enough for being formal in the above sense!**
- **(*Analytic ontology* may be a better term to avoid this confusion)**

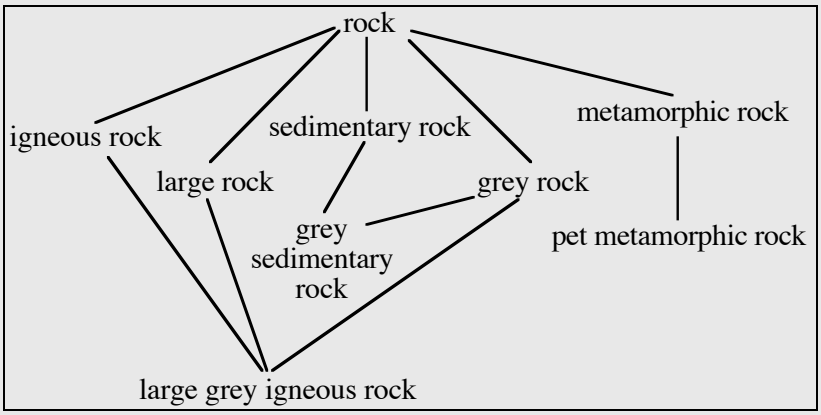
The first steps of ontological analysis



- Be clear about the ***domain of discourse*** (existence...)
- Choose the relevant ***concepts and conceptual relations***
- Choose the ***primive relations***
- Choose meaningful ***names*** for these

Terminological competence - kinds

How many rock kinds are there?



[From Brachman, R ., R. F ikes, et al. 1983. "Krypton: A Functional Approach to Knowledge Representation", *IEEE Computer*]

Terminological competence - kinds of relations

- Woods' "What's in a link?" (1975):

  JOHN
  HEIGHT: 6 FEET
  KISSED: MARY

- "no longer do the link names stand for attributes of a node, but rather arbitrary relations between the node and other nodes"
- different notations should be used

Structured concepts: a broader picture

JOHN
HEIGHT: 6 FEET
RIGHT-LEG: BROKEN
MOTHER: JANE
KISSED: MARY
JOB: RESEARCHER

quality
part
role
external relation
meta-level assertion

We need different primitives to express ***different structuring relationships*** among concepts

We need to represent ***non-structuring relationships*** separately

Current description logics collapse **EVERYTHING**!

Mereology as an example of formal ontological analysis

- Primitive: ***proper part-of*** relation (PP)
  - asymmetric
  - transitive
  - **Pxy =$_{def}$ PPxy ∨ x=y**
  - **Oxy =$_{def}$ ∃ z(Pzx ∧ Pzy)**
- Axioms:

supplementation: **PPxy → ∃z (PPzy ∧ ¬ Ozx)**

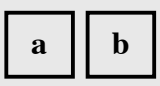
principle of sum: **∃z ∀w (Owz ↔ (Owx ∨ Owy))**

extensionality: **x = y ↔ ∀w(Pwx ↔ Pwy)**

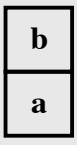
Excluded models:

Part, Constitution, and Identity

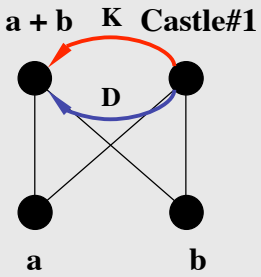
- ***Parts not enough*** *to make the whole:* structure changes identity
- ***Mereological extensionality*** is lost
- ***Constitution*** links the two entities
- **Constitution is** asymmetric (implies ***dependence***)



Two blocks



A castle



Sets and mereological sums

- Sets of concrete things are abstract
- Sums of concrete things are concrete!
- No analogous of membership relation and empty set for mereological sums

Essential properties

- For an individual
  - John must have a brain
  - John must be a human
  - John must be alive
- For a type
  - All human beings must have a brain
  - All human beings must be “a whole” (all of a piece)

Essential properties and rigidity

- Certain entities must have some properties in order to *exist*
  - John must have a brain
  - John must be a person.
- Certain properties are essential to ***all*** their instances (***being a person*** vs. ***being hard***).
- These properties are ***rigid*** - Their extension is the same in all possible worlds. If an entity is ever an instance of a rigid property, it must necessarily be such.

- By the way, what's the meaning of *exist*?
  - Being an element of the domain of discourse
  - Being present *at a certain time* (or in a certain world...)

Formal Rigidity

- ϕ is rigid (+R): $\forall x\ (\Diamond\phi(x) \rightarrow \Box\phi(x))$
  - e.g. Person, Apple
- ϕ is non-rigid (-R): $\exists x\ (\Diamond\phi(x) \wedge \neg \Box\phi(x))$
  - e.g. Red, Male
- ϕ is anti-rigid (~R): $\forall x\ (\Diamond\phi(x) \rightarrow \neg \Box\phi(x))$ e.g. Student, Agent

Meta-properties

Formal rigidity - variations

- **Takint actual existence into account:**

$$\Box\forall x(\phi(x) \rightarrow \Box(E(x) \rightarrow \phi(x)))$$

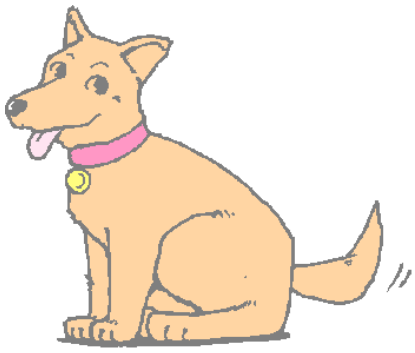
- **Taking time and actual existence into account:**

$$\Box\forall xt((E(x,t) \wedge \phi(x,t)) \rightarrow \Box\forall t'(E(x,t') \rightarrow \phi(x)))$$

- *Welty, C. and Andersen, W. Towards OntoClean 2.0: A framework for rigidity (Applied Ontology 1(1), 2006)*

Unity, Identity, and Essence

- **Unity**: is the collar part of my dog?
  - ***Being a whole*** (of a certain kind) is also a (relevant) essential property
  - It is a (weak) ***identity condition*** for dogs.



Defining unity

- A tentative formulation: *x* ***is a whole*** under a unifying relation U iff U is an *equivalence relation* that binds together all the parts of *x*, such that, ***necessarily***

$$P(y,x) \rightarrow (P(z,x) \leftrightarrow U(y,z))$$

but not

$$U(y,z) \leftrightarrow \exists x(P(y,x) \wedge P(z,x))$$

- P is the ***part-of*** relation
- U can be seen as a ***generalized indirect connection***

Unity Refined

Problem: the unity relation may not link together all the parts (think of a family as a whole)

$\delta_U(x) =_{df} U(x, x)$ ***(x belongs to the domain of U)***

$\mathbf{U}_U(x) =_{df} \Sigma_{\delta_U}(x) \wedge \forall y,z((\delta_U(y) \wedge \delta_U(z) \wedge P(y, x) \wedge P(z, x)) \rightarrow U(y, z))$

(x is unified by U)

$\mathbf{W}_U(x) =_{df} Max_{\mathbf{U}_U}(x)$ ***(x is a whole under U)***

$\Sigma_\phi(x) =_{df} \forall y(P(y, x) \rightarrow \exists z(\phi(z) \wedge P(z, x) \wedge O(z, y))$ ***(sum of ϕs)***

Kinds of Whole

- Depending on the **nature of the *unifying relation***, we can distinguish:
  - ***Topological wholes*** (a piece of coal, a heap of coal)
  - ***Morphological wholes*** (a constellation)
  - ***Functional wholes*** (a hammer, a bikini)
  - ***Social wholes*** (a population)

* a whole can have ***parts that are themselves wholes*** (with a different unifying relation)

Unity and Plurality

- *Ordinary objects:* ***wholes or sums of wholes***
  - *Singular*: ***no wholes as proper parts***
  - *Plural*: ***sums of wholes***
    - *Plural wholes* (the sum is ***also a whole***)
    - *Collections* (the sum is not a whole)