Time and Space

ESSLLI 2005 introductory course on
Formal Ontology for Semanticists - Lesson 4

Edinburgh – 11 August 2005

Outline

- Basic ontological choices
- Time structures
  - Instants
  - Intervals
  - From instants to intervals and vice-versa
  - Events
- Space structures
  - Mereotopology
- Further notes

Time and space: domains?

Absolute or relative time / space?

- Absolute: Separate (existentially independent) domain of purely temporal / spatial entities, a *substrate*
  Concrete entities related to them by a *location* relation
  - Standard physics
  - DOLCE's Time and Space quality spaces
- Relative: Time / space is an implicit structure induced by temporal / spatial relations
  - Leibniz - Newton controversy
  - commonsense and psychological evidence
- Linguistic reference to “times” (*today*, *August 11 2005*) or “places” (*here, room EM-2.44*) not a clear evidence for absolutism

Which primitive entities?

Absolute case

1. “instants”, “moments” / “points”: non-extended entities
2. “intervals”, “periods” / “regions”: extended entities, mereological structure

Relative case

- Concrete entities: generally extended
- Do all entities participate in the temporal (spatial) structure?

The 3D / 4D debate

[Simons, 1987, Sider, 2003]

- Three- vs. four-dimensionalism
  - do all entities have temporal parts?
  - objects / events, endurants / perdurants, continuants / occurrents
- Co-localization, multiplicationism and identity criteria
  - mereology: things that have the same parts are identical
  - does a given spatio-temporal “worm” identify a single entity? (strong four-dimentionalism)
- Identity across time
  - Is Tibbles the cat identical to Tib?

Absolute case: which location relation?

- Time and space as quality spaces: two relations (cf. lesson 3)
  - qt, relation between a concrete entity and its quality
  - ql, relation between the quality and its quale (temporalized or not)
- Spatial location: e.g., L(x,s) (cf. lesson 3), or rather L(x,s,t)
- Temporal location: e.g., Occurs(e,t)
  - Not to be confused with meta-predicates of reified temporal logics (e.g., Holds(p,t))
    Propositions correspond to eventuality types, not to event tokens

Which structure?

- Which domain?
- Which relations?
- Which axioms?

Structures of time

1- Instant structures

Instant theories: orders

Primitive relations and basic axioms: some basic maths...

- identity $=$: first-order logic with identity assumed
- precedence $<$: strict order
  - transitive: $\forall x, y, z\ ((x < y \wedge y < z) \rightarrow x < z)$
  - asymmetric: $\forall x, y\ (x < y \rightarrow \neg y < x)$
  - irreflexive (theorem): $\forall x\ \neg(x < x)$
  - non-strict order: $x \leq y =_d x < y \vee x = y$
- equivalent variants with non-strict order $\leq$ as a primitive

Which order?

Total or partial

- Total (linear)
  - $\forall x, y\ (x \leq y \vee y \leq x)$
- Partial (branching)
  - parallel times: alternative worlds
  - linear to the left: (possibly) branching future
    $\forall x, y, z\ ((x < z \wedge y < z) \rightarrow (x \leq y \vee y \leq x))$

Which order?

Bounded or unbounded

- Bounded: $\exists x, y\, \forall z\, (x \leq z \wedge z \leq y)$
- Bounded to the left: $\exists x\, \forall y\, (x \leq y)$
- Unbounded: $\forall x\, \exists y, z\, (y \leq x \wedge x \leq z)$
- Unbounded to the right: $\forall x\, \exists y\, (x \leq y)$

Dense or discrete

- Dense: $\forall x, y\, (x < y \rightarrow \exists z\, (x < z \wedge z < y))$
- Discrete: $\forall x, y\, (x < y \rightarrow \exists z, t\, ((z < y \wedge \forall u\, \neg(z < u \wedge u < y))$
  $\wedge (x < t \wedge \forall u\, \neg(x < u \wedge u < t))))$

Completeness

When should we stop adding axioms?

- Syntactic completeness
  - $\mathcal{T}$, axiomatic theory in first-order language $\mathcal{L}$
  - $\mathcal{T}$ is syntactically complete iff
    for any $\phi \in \mathcal{L}$, $\mathcal{T} \vdash \phi$ or $\mathcal{T} \vdash \neg\phi$
- Isomorphims of models with respect to $\mathcal{L}$ (infinite models: modulo cardinality)

Classical examples of complete linear order structures

- $\langle \mathbb{N}, < \rangle$: total, left-bounded, right-unbounded, discrete
- $\langle \mathbb{Z}, < \rangle$: total, unbounded, discrete
- $\langle \mathbb{Q}, < \rangle$; $\langle \mathbb{R}, < \rangle$: total, unbounded, dense

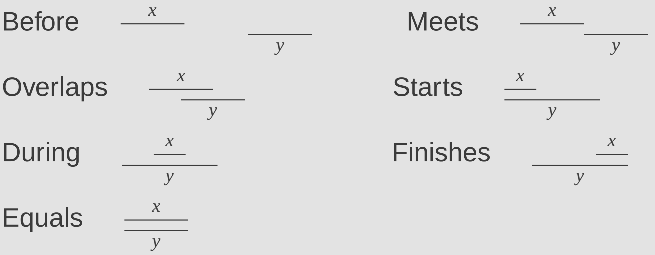
Structures of time

2- Interval structures

Allen's interval theory

[Allen, 1983, Allen, 1984, Allen and Hayes, 1985]

- *Convex* "intervals"
- Allen's relations
- 13 possible relations between any ordered pair of intervals



- Inverse relations: After, Met-by, Overlapped-by, Started-by, Contains, Finished-by
  $After(x,y) \leftrightarrow Before(y,x)$

Allen's interval theory - 2

- Allen & Hayes's theory [Allen and Hayes, 1985], [Ladkin, 1987]
  - based on a unique primitive: Meets, noted $\|$
  - $Before(x, y) =_d \exists z(x\|z \wedge z\|y)$
  - $Equals(x, y) =_d \exists z, t(z\|x \wedge x\|t \wedge z\|y \wedge y\|t)$
  - unbounded, "continuous" and linear time
- Axiomatization [Hajnicz]

UNI1	$\forall x, y, z, v\ ((x\|z \wedge x\|v \wedge y\|z) \rightarrow y\|v)$
UNI2	$\forall x, y, z, v\ ((x\|z\|y \wedge x\|v\|y) \rightarrow z = v)$
LIN	$\forall x, y, z, v\ ((x\|y \wedge z\|v) \rightarrow (x\|v\ \nabla\ \exists s(x\|s\|v)\ \nabla\ \exists s(z\|s\|y)))$
UNB	$\forall x\ \exists y, z\ (y\|x\|z)$
SUM	$\forall x, y\ (x\|y \rightarrow \exists z, v, s(z\|x\|y\|v \wedge z\|s\|v))$
DENSM	$\forall x, y, z, u\ (\mathrm{P}_<(x, y, z, u) \rightarrow$ $\exists v, w\ (\mathrm{P}_<(x, y, v, w) \wedge \mathrm{P}_<(v, w, z, u)))$

with $\mathrm{P}_<(x, y, z, u) =_d x\|y \wedge z\|u \wedge \exists w(x\|w\|u)$

From instants to intervals and vice-versa

- Let $\langle \mathbb{Q}, < \rangle$ be a linear, unbounded, dense strict order structure, then
  - $\langle I, \| \rangle$ s.t.
    - $I = \{(q_1, q_2) \mid q_1, q_2 \in \mathbb{Q}$ and $q_1 < q_2\}$
      where $(q_1, q_2) = \{q \in \mathbb{Q} \mid q_1 < q < q_2\}$
    - $(q_1, q_2) \| (q_3, q_4)$ iff $q_2 = q_3$

    is a model of interval theory (an interval structure)
  - Interval theory is countably categorical, i.e., all models of cardinality ω_0 are isomorphic to $\langle I, \| \rangle$
- Let $\langle I, \| \rangle$ be an interval structure, then
  - $\langle P, < \rangle$ s.t.
    - $P = \{[x, y] \mid x, y \in I$ and $x \| y\}$
      where $[x, y] = \{(z, v) \mid z, v \in I$ and $x \| v$ and $z \| y\}$
    - $[x, y] < [z, v]$ iff $\exists w\, (x \| w \| v)$.

    is a linear, unbounded, dense strict order structure

van Benthem's "period" theory

[van Benthem, 1983]

- Two order relations: a precedence $\prec$ and a mereology $\sqsubseteq$
- $\prec$ is an unbounded strict order, "discrete" (continuous):
  - $\forall x,y\,(x \prec y \rightarrow (\exists z_1\,(x \prec z_1 \wedge \neg\exists u\,(x \prec u \prec z_1))\wedge$
    $\exists z_2\,(z_2 \prec y \wedge \neg\exists u\,(z_2 \prec u \prec y))))$
- $\sqsubseteq$ is reflexive, transitive, antisymmetric, *and*

  Supplementation: $\forall x,y\,(\forall z\,(z \sqsubseteq x \rightarrow z\mathsf{O}y) \rightarrow x \sqsubseteq y)$

  where $x\mathsf{O}y =_d \exists z\,(z \sqsubseteq x \wedge z \sqsubseteq y)$

  Existence of the product:

  $\forall x,y\,(x\mathsf{O}y \rightarrow \exists z(z \sqsubseteq x \wedge z \sqsubseteq y \wedge \forall u((u \sqsubseteq x \wedge u \sqsubseteq y) \rightarrow u \sqsubseteq z)))$

  z is noted $x \sqcap y$

  "Underlap": $\forall x,y\; x\mathsf{U}y$

  where $x\mathsf{U}y =_d \exists z\,(x \sqsubseteq z \wedge y \sqsubseteq z)$

  Existence of the "convex sum":

  $\forall x,y\,(x\mathsf{U}y \rightarrow \exists z\,(x \sqsubseteq z \wedge y \sqsubseteq z \wedge \forall u((x \sqsubseteq u \wedge y \sqsubseteq u) \rightarrow z \sqsubseteq u)))$

  z is noted $x \sqcup y$

Period theory – Linking $\prec$ and $\sqsubseteq$

Density: $\forall x \exists y, z (y \prec z \wedge x = y + z)$

where $x = y + z$ iff $x = y \sqcup z \wedge \forall w\ (w \sqsubseteq x \rightarrow (w\mathrm{O}y \vee w\mathrm{O}z))$

Mon-1: $\forall x, y\ (x \prec y \rightarrow \forall z((z \sqsubseteq x \rightarrow z \prec y) \wedge (z \sqsubseteq y \rightarrow x \prec z)))$

Mon-2: $\forall x, y\ (x \prec y \rightarrow (\forall z\ (z \prec y \rightarrow (x \sqcup z) \prec y)$
$\wedge \forall z\ (y \prec z \rightarrow y \prec (x \sqcup z))))$

Conv: $\forall x, y, z\ ((x \prec y \wedge y \prec z) \rightarrow \forall u\ ((x \sqsubseteq u \wedge z \sqsubseteq u) \rightarrow y \sqsubseteq u))$

Lin-1: $\forall x, y(x \prec y \vee y \prec x \vee \exists z(z \sqsubseteq x \wedge z \sqsubseteq y))$

Lin-2: $\forall x, y\ (x\mathrm{O}y \rightarrow (x = y \vee$
$(x \sqsubseteq y \wedge (\exists z\ (x \prec z \wedge y = x + z) \vee \exists z\ (z \prec x \wedge y = x + z) \vee$
$\exists z_1, z_2\ (z_1 \prec x \prec z_2 \wedge y = (z_1 + x) + z_2)))$
$(y \sqsubseteq x \wedge (\exists z\ (y \prec z \wedge x = y + z) \vee \exists z\ (z \prec y \wedge x = y + z) \vee$
$\exists z_1, z_2\ (z_1 \prec y \prec z_2 \wedge x = (z_1 + y) + z_2))) \vee$
$\exists z_1, z_2\ (z_1 \prec z_2 \wedge x = z_1 + z_2 \wedge z_1 \prec y \wedge z_2 \sqsubseteq y \wedge$
$\exists z_3 (y = z_2 + z_3 \wedge x \prec z_3))) \vee$
$\exists z_1, z_2\ (z_1 \prec z_2 \wedge y = z_1 + z_2 \wedge z_1 \prec y \wedge z_2 \sqsubseteq x \wedge$
$\exists z_3\ (x = z_2 + z_3 \wedge y \prec z_3)))))$

From points to periods and vice-versa

- Let $\langle \mathbb{Q}, < \rangle$ be a linear, unbounded, dense strict order structure, then
  - $\langle I, \prec, \sqsubseteq \rangle$ s.t.
    - $I = \{(q_1, q_2) \mid q_1, q_2 \in \mathbb{Q}$ and $q_1 < q_2\}$ where $(q_1, q_2) = \{q \in \mathbb{Q} \mid q_1 < q < q_2\}$
    - $(q_1, q_2) \prec (q_3, q_4)$ iff $q_2 \leq q_3$
    - $(q_1, q_2) \sqsubseteq (q_3, q_4)$ iff $q_3 \leq q_1 < q_2 \leq q_4$

    is a model of the period theory (a period structure)
  - Period theory is countably categorical, i.e., all models of cardinality ω_0 are isomorphic to $\langle I, \prec, \sqsubseteq \rangle$
- Let $\langle I, \prec, \sqsubseteq \rangle$ be a period structure, then
  - $\langle P, < \rangle$ s.t. (filter construction)
    - $P = \{F \subseteq I | (\forall x \in F)(\forall s \in I)(x \sqsubseteq s \rightarrow s \in F)$ and $(\forall x, y \in F)(x \sqcap y \in F)\}$;
    - $F_1 < F_2$ iff $(\exists x \in F_1)(\exists y \in F_2)(x \prec y)$.

    is a linear, unbounded, dense strict order structure

Structures of time

3- Events

Temporal theory of events [Kamp, 1979]

- Simultaneity possible
  No existential assumptions
- Precedence $\prec$: a strict partial order
- Overlap o: a reflexive and symmetric relation
- Mixed axioms
  - $\forall x, y\ (x \prec y \rightarrow \neg xoy)$
  - $\forall x, y, z, t\ ((x \prec y \wedge yoz \wedge z \prec t) \rightarrow x \prec t)$
  - $\forall x, y\ (x \prec y \vee xoy \vee y \prec x)$
- Construction of an instant structure from an event structure (Russell-Wiener)
  - $\langle E, \prec, o \rangle$: event structure
  - instants are maximal sets of two by two overlapping events $I \subseteq 2^E$ s.t. for any $i \in I$ and $x, y \in i$, xoy and $\forall x \in E\ (x \notin i \rightarrow \exists y\ (y \in i \wedge \neg xoy))$
  - for all $i, j \in I$, $i \leq j$ iff $\exists x, y\ (x \in i \wedge y \in j \wedge x \prec y)$
  - $\langle I, \leq \rangle$: instant structure

Events and intervals

- An interval structure can be built on top of the instant structure
- Not an isomorphism between the original event structure and this interval structure
  - Simultaneous different events: “more” events than intervals
  - No sum and no product existence imposed on events: “more” intervals than events
  - Atomic events generate instants: degenerate intervals are needed
- Hypothesis that only “real events” (accomplishments and achievements) contribute to time structure
  - every state and activity is started and ended by a (change of state) event

Beyond time: what are events?

[Casati and Varzi, 1996]

What events are not

- Events vs. objects
  - Endurant / perdurant discussion
  - Strong four-dimentionalism: stages of objects
  - Objects and events colocate differently:
    the ball / the piece of metal
    the spinning of the ball / the warming up of the ball
    the music going on / the smoke filling up the room
  - Objects can move, events cannot
  - What relationship? existential dependence, participation

What events are not - 2

- Events vs. facts, propositions and states of affairs
  - Caesar's death / that Caesar died, my standing here / that I am standing here
  - Events are concrete (= situated in space-time), facts and soa are abstract
  - Events occur once, propositions and soas can repeatedly be the case / obtain
  - Caesar's death = Caesar's violent death, that Caesar died $\neq$ that Caesar died violently

How many events?

- The spinning of the ball
  The warming up of the ball
- John's answering my question
  John's shouting
- Brutus's stabbing Caesar
  Brutus's killing Caesar
  Caesar's death
- My alerting the burglar
  My illuminating the room
  My turning on the light
  My pushing on the button
  My moving my finger . . .

Event identity

- "No entity without identity"
- Identity criteria
  - Co-localization, *but* strong four-dimentionalism
  - Causal equivalence, *but* temporal shifts
  - Logical equivalence, *but* slingshot argument
  - Many different properties: exemplification of properties at a time
- A general semantic problem? (cf. definite descriptions)
- Multiplicationism, again...

Structures of space

Spaces without points

- Space of regions, i.e., extended primitive entities
- [Nicod, 1962, de Laguna, 1922, Whitehead, 1929, Tarski, 1956]
- Modern accounts based on mereology
- First step: adding topological concepts, “mereotopology”
- [Clarke, 1981, Randell et al., 1992, Asher and Vieu, 1995, Borgo et al., 1996, Varzi, 1996, Masolo and Vieu, 1999]
- Primitive relation of “connection” (Whitehead)
  intended semantics: at least a *point* in common
  what happens at the boundaries is taken into account

Basic Mereotopology

- Mereology
  - P1 $P(x,x)$
  - P2 $(P(x,y) \wedge P(y,x)) \rightarrow x = y$
  - P3 $(P(x,y) \wedge P(y,z)) \rightarrow P(x,z)$
- Connection
  - C1 $C(x,x)$
  - C2 $C(x,y) \rightarrow C(y,x)$
  - C3 $P(x,y) \rightarrow \forall z\ (C(z,x) \rightarrow C(z,y))$
- Strong basic mereotopology
  - C4 $\forall z\ (C(z,x) \rightarrow C(z,y)) \rightarrow P(x,y)$
  - $P(x,y) =_d \forall z\ (C(z,x) \rightarrow C(z,y))$

Eight possible relations

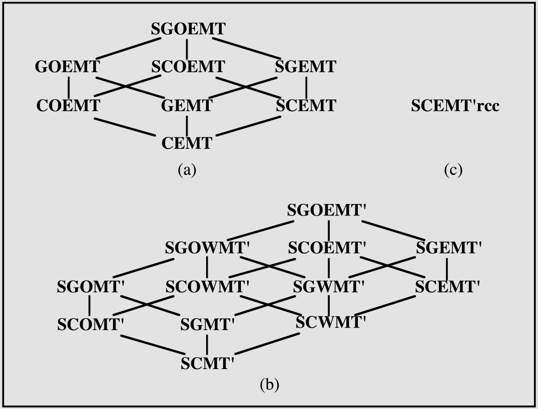
- Mutually exhaustive, pairwise disjoint
  - O $\exists z\, P(z,x) \wedge P(z,y)$
  - =
  - EC $EC(x,y) =_d C(x,y) \wedge \neg O(x,y)$
  - TPP $TPP(x,y) =_d PP(x,y) \wedge \exists z\, (C(z,x) \wedge C(z,y))$
  - NTPP $NTPP(x,y) =_d PP(x,y) \wedge \neg\exists z\, (C(z,x) \wedge C(z,y))$
  - DC $DC(x,y) =_d \neg C(x,y)$

O = EC TPP NTPP DC

Closed / General Mereotopology: operators

- Which extensionality?
  - important identity criteria
  - basis for definition of operators of sum, difference and fusion
- Mereology
  - O: strong supplementation
  - $\forall z\ (O(z,x) \leftrightarrow O(z,y)) \rightarrow x = y$
- Mereotopology
  - Choice of O (strong supplementation) or
  - C: strong mereotopology (C4)
  - $\forall z\ (C(z,x) \leftrightarrow C(z,y)) \rightarrow x = y$
- Topological operators
  - Interior: fusion of all the NTPP
  - Closure: complement of the interior of the complement

Families of mereotopologies



Atomicity / divisibility critical

Model-theoretical results for some of these theories only

- Two trends: topological spaces, connection algebras

Mereogeometries

Beyond mereotopology: geometric concepts
Several approaches [Borgo & Masolo, to appear]

- ternary can-connect [de Laguna, 1922] [Donnelly, 2001]
- unary sphere [Tarski, 1956, Bennett, 2001]
- ternary or quaternary distance [Nicod, 1962]
- binary congruence [Borgo et al., 1996]

What about relative space?

- Like for event and interval temporal theories, spatial relative theories are very similar to those we have just seen
- Possible co-localization requires:
  - Part-of relation replaced by spatial inclusion
  - Identity replaced by “spatial equivalence”
- Connection replaced? Yes, if interpretation more than spatial, e.g., other unity criteria
- More serious problem: unrestricted sums, products, differences? fusion? existence of a universe?

What about space-time?

[Muller, 1998]

- A single domain of primitive entities: space-time “worms”
- Primitive relations: spatio-temporal ones and purely temporal ones
  - P and C with spatio-temporal interpretation
  - precedence and temporal connection
- Definition of temporal inclusion, temporal equivalence, temporal part, “temporal slice” operator
- Characterization of spatio-temporal “continuity”
- Characterization of motion

A methodological observation

Model-theoretical tools

- very powerful
- very useful to understand in depth the nature of the entities and relations described in the axiomatic theories

... When applicable!

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