Founding properties on measurement

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Abstract. Taking for granted an ontological standpoint independent of any empirical or epistemological perspective, philosophical theories of properties are actually quite rarely adopted in the knowledge representation community. The theory of *qualities* introduced in the DOLCE-CORE ontology [4] allows for representing different viewpoints on the world in a unified framework but it does not refer to any process used in empirical or epistemic investigations. In this paper, I will found the theory of qualities on the measurement theory proposed in [8]. In this new perspective, the stability of properties is not assumed a priori but it is founded on the stability of specific (physical) objects: the measurement systems and standards.

Keywords. ontology, theory of properties, theory of qualities, measurement, empirical and epistemic foundations

Introduction

The nature of properties, how objects have properties, and how objects change through time, are fundamental issues not only in logic and philosophy, but, more generally in science. Most philosophical theories take for granted the *objectivity* (independence from cognition) and the *stability* (independence from time) of properties. These assumptions often reflect a *realistic* viewpoint on properties that some philosophers of science (see, for example, [32]) claim to be very important for clarifying the metaphysics of *measurement*. This realistic view is often opposed to *operationalism* or *conventionalism* (see [33] for a short survey) that consider properties as created by the application of measurement operations or conventions.

In this paper, I propose a framework that accounts for the empirical and epistemic foundation of properties. Being neutral with respect to the realistic vs. operationalistic controversy, my framework, differently from philosophical theories of properties, has no *explanatory power*. It does not explain what exists in reality to make possible (what are the *truth-makers*) for an object to have a property. It just analyzes how properties can be *ascribed* to objects on the basis of an *epistemic theory of measurement*, in particular, the one introduced in [17,8]. There exist different positions relative to the role of epistemology in measurement. Following [17], measures can be seen as: (1) “inherent properties of the measured things”, a quite pure ontological perspective; (2) “the results of operations that preserve the relations observed among measured things”, here observations introduce an epistemic component; (3) “the results of operations recognized as adequate for their goal of obtaining information of measured things”, where the epistemic dimension becomes central. In all these views, properties can be *associated* to measures by assuming a general schema exemplified by: an object has the property of ‘being 1m long’ if and only if the result of its length measurement is *1m*. Objective facts "that trascend particular methods of measuring it" [32] are not denied at priori but in my approach, as in cases (2) and (3), they do not play any role in establishing the objectivity and the stability of measures (and therefore properties). How objects can be measured in a communicable and inter-subjective way, and how they can be diachronically compared, will be explained by relying on the objectivity and stability of *measurement systems* and *measurement standards* without any reference to the truth-makers necessary for objects to have specific measures. While disappointing from a purely ontological perspective, the ontological 'neutrality' of the proposed approach allows its application to a variety of domains without excluding possible refinements to account for specific applicative or ontological needs. I will support this generality by providing an explicit interpretation of both the *theory of qualities* introduced in the DOLCE ontology [19] (later modified in [18] and [4] to account for different epistemic point of views) and the *theory of social concepts* proposed in [20] in terms of this general framework.

Philosophical, empirical, and cognitive approaches to properties

Universalism (see [1] for an overview) claims that properties are *universals*, i.e. immutable entities wholly present in each one of their *instances*. *Trope Theory* (see [6] for an overview) introduces *tropes*, individualized properties that *inhere* in single objects.[1] Properties reduce to equivalent classes of (exactly) *resembling* tropes. Even though, for example, two apples have the same color property, their color tropes, the redness of the first apple and the redness of the second one, are numerically distinct entities. *Resemblance Nominalism* (see [26]) rejects both universals and tropes. By means of a *resemblance* relation directly defined on objects, it is not necessary to refer to properties because "what makes it true that a particular has a property is that it resembles other particulars" ([26], p.57). Properties are then devoid of any ontological relevance, however they can be at least *associated* to classes of resembling objects.

In all these theories, properties are independent from time and they form an objective and stable *framework*.[2] However, to diachronically compare objects, to take into account possible changes in objects, or, more generally, to analyze the expression 'the object a has the property P at time t' one has to clarify not only how *properties* persist through time but also whether and how *objects* persist. *Endurantism* (see [2] for a recent defense) claims that the *whole* a can exist at different times, therefore the 'has' relation needs to be temporally qualified specifying at what time a has the property P: 'a has-at-t P'. This position has been criticized because it excludes the possibility for objects to have properties *simpliciter*, independently of any temporal specification [16]. *Perdurantism* (see [15] and [29]) commits to the existence of temporal slices of objects, i.e. at every time t at which it exists, a has a different temporal slice, a-at-t. Therefore, 'a has P at t' because 'a-at-t has P'. *Stage Theory* (see [11]) denies the persistence of objects: *stages*, i.e. objects with an instantaneous existence, are the only true ontological entities. Common-sense persisting objects are the result of a conceptual construction that collects together similar stages. While, from an ontological point of view, Perdurantism and Stage

[1] I use the term *object* as synonymous of (the more technical) *particular* or *individual*.

[2] This is evident in the case of Universalism, while in Trope Theory and Resemblance Nominalism it follows from the purely ontological nature of the *resemblance* relations.

Theory have different commitments, their analyses of 'a has P at t' both predicate the property P on an instantaneous part of a (the temporal slice or the stage).

Some theories of properties commit to specific theories of persistence. Trope Theory relies on a generalization of the Stage Theory where tropes are not necessarily instantaneous but cannot change. For example, the change in color of a is explained by a trope *substitution*: the old color trope is substituted with a (non exactly resembling) new one. Therefore 'a has P at t' if and only if, at t, there exists a trope member of P (a set of resembling tropes) that inheres in a. In Resemblance Nominalism, the resemblance relation is a (non temporally qualified) transtemporal binary relation, i.e. it may apply both synchronically and diachronically. Differently from tropes, objects can change through time, therefore Resemblance Nominalism defines resemblance only on instantaneous slices of objects, i.e. it commits to Perdurantism.[3]

On the one hand, Universalism is not committed to a specific theory of persistence but it is founded on the quite abstract notion of universal that seems very difficult to relate to empirical theories. On the other hand, Trope Theory and Resemblance Nominalism are based on resemblance relations that, by means of their intuitive connection with *similarity* relations, offer a 'handle' to cognitive science and measurement theories, but they commit to stages, tropes, or temporal slices.

In Universalism, Trope theory, and Resemblance nominalism, properties are independent of any conceptualization of the world, thus necessarily objective. Vice versa, empirical and epistemic perspectives do not rely on this aprioristic objectivity. Following [17], *evaluation*, e.g. personal judgment or estimation, can be distinguished from *measurement* on the basis of the objectivity and the communicability of the results of measurement. To ground the objectivity, it is necessary to specify how a community of subjects can *share* a point of view on reality, i.e. how this community is able to measure (or at least classify) objects in a uniform and communicable way. In this context, I prefer to use the term *inter-subjectivity* instead of *objectivity*. In the following I will ground the inter-subjectivity on empirical and epistemic approaches. This grounding allows for a non absolutist and scientific interpretation of objectivity.

In cognitive science, inter-subjectivity is often implicitly assumed. Humans share the same cognitive system, therefore they (communities of them) also share a (basic) system of categories and a categorization mechanism. *Prototype Theory* (see [23] for a brief introduction) reduces *categorization* to judgments of *similarity* between the objects under analysis and the *prototypes*, i.e. shared archetypal examples of objects belonging to a category. It focuses on the reduction of similarity to more basic notions and not on what grounds inter-subjectivity. For example, in *featural* (*set-theoretical*) *models* [34], an object is represented by a set of features, and the similarity between two entities is expressed as a (linear) combination of the measure of the common and distinctive features. Or, in *geometrical* (*spatial*) *models*, an object is represented by a point in a multidimensional space, and the similarity between two objects is represented by the distance between the points that represent them. More specifically, the approach called *Multidimensional Scaling* (MDS) (see [14] for a review) derives a spatial representation typically starting from one or more matrices of *proximity* (subjective) judgments and "MDS techniques work backward to discover both the number and the nature of the stimulus attributes or dimensions that were used to make those judgments, and to estimate the loca-

[3] Resemblance Nominalism faces other difficult problems, like, for example the co-extensionality of properties. [26] discusses in details these problems and proposes some solutions to them.

tions of the stimuli along those dimensions" [14]. The MDS approach aims at finding the dimensions, the scales, and the distance that are able to 'explain' the collected empirical data but points (and features) are implicitly assumed as inter-subjective. In addition, as in the case of Resemblance Nominalism, similarity judgments are not relative to a specific aspect of the objects under analysis. In [10], Goodman claims that generic similarity judgments do not make sense, an entity is similar to another one only with respect to a specific dimension of comparison. This position undermines the MDS approach because it presupposes that the dimension of comparison must be known *before* any similarity judgment and not vice versa. By accepting Goodman's view, one needs to make explicit what a dimension is and how the dimension, as well as the similarity judgments, can be shared. Cognitive theories become close to scientific ones where the dimensions and their structures "are tightly connected to the *measurement methods* employed to determine the values on the dimensions in experimental situations" ([9], p.21).

In the following I will propose an analysis of the expression 'a has P at t' based on an epistemic measurement theory that identifies specific dimensions of comparisons without presupposing any aprioristic inter-subjectivity. In addition, it does not require stages, slices, tropes or individual qualities and, in fact, neither properties that, however, can be *associated* to the internal states of (calibrated) measurement systems. My analysis assumes the form 'a has P-at-t' already discussed in [35].[4] Later, it will be clear why I cannot assume the stability of properties a priori, however, as stated in [35], to seriously take into account this analysis, the *similarity* between P-at-t and P-at-t', two different properties, needs to be explained.[5] I will found this *similarity* on the stability of a framework of *objects*: the measurement systems and the measurement standards.

(Epistemic) theories of measurement

Representational Measurement Theory (RMT) (see [31]) is one of the best known measurement theories. According to RMT, measurement consists in building a mapping from an *empirical relational structure* to a *numerical relational structure* such that the relations among numbers *represent* the empirical relations among objects. More formally, measurement consists in building a *homomorphism*[6] (also called *scale*) ϕ from an *empirical* structure $\mathcal{E} = \langle D, R_1, \ldots, R_n \rangle$ (where D is a set of empirical objects and R_i are empirical relations on D) to a *numerical* structure $\mathcal{N} = \langle N, S_1, \ldots, S_n \rangle$ (where N is a set of numbers and S_i are relations on N) coupled with (1) a *representation theorem* that states that the axioms in $\mathcal{E}$ are preserved by ϕ in $\mathcal{N}$; and (2) a *uniqueness theorem* that establishes the permissible transformations $\phi \rightarrow \phi'$ that yield homomorphisms into the *same* numerical structure [31]. Given an empirical structure $\mathcal{E}$ regulated by specific (empirical) laws encoded by axioms, measurement aims at finding a unique (i.e. up to a class of transformations) scale into a numerical structure $\mathcal{N}$ that represents $\mathcal{E}$.

[4] In [35] such form is considered as a notational variant of 'a has-at-t P'. In the following I will argue against this equivalence.

[5] Notice that perdurantism faces a similar difficulty regarding the link between a-at-t and a-at-t', two different objects. This difficulty is usually solved adopting some notion of (spatio-temporal) continuity.

[6] ϕ is an homomorphism from $\langle D, R_1, \ldots, R_n \rangle$ to $\langle M, S_1, \ldots, S_n \rangle$ iff (i) $\phi : D \rightarrow M$ and (ii) ϕ maps R_i to S_i, $i = 1, \ldots, n$, i.e. $R_i(x_1, \ldots, x_m)$ iff $S_i(\phi(x_1), \ldots, \phi(x_m))$. Note that ϕ can map different empirical objects to the same number, i.e. it individuates sets of empirical objects that are ϕ equivalent.

RMT defines a notion of measurement in precise mathematical terms, but, as stated in [8], this notion is so abstract that in practice it is not used in empirical contexts (and, in particular, in physical metrology). One problem of RTM regards the nature of the empirical structure. The relations and the axioms (corresponding to physical laws) in this structure are the result of an abstractive process that is not considered in RMT. Taking the empirical structure as given, RTM moves to a lower level of abstraction the problem of founding measurement on empirical procedures or methods without solving it.

[8] proposes an alternative general model of measurement that addresses the empirical foundation of measurement by giving a central role to *measurement systems* (MSs). Roughly speaking, an MS is a system – a (physical) object describable by a set of states discernible in empirical terms – able to interact with the *system under measurement* (SUM) and "provided with instructions specifying how such interaction must be performed and interpreted" [8]. The output of the interaction between an MS and an SUM is a piece of (symbolic) information.

Following [8], an MS is formally represented by the tuple $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda\rangle$ where:[7]

- m is a (physical) object (the *support* of the MS).
- $\mathcal{E} = \langle U, R_1, \ldots, R_n\rangle$, the *empirical structure*, describes m in terms of the potential interactions with objects. U is the set of the *empirically discernible* states of all possible complex systems resulting from the interaction of any object $o \in O$ (the domain of objects) with m (noted by $m \bullet o$). R_i are relations among states in U, i.e. they are observable relations among the internal states of the complex systems. U and the R_i empirically anchor, respectively, the symbols and the relations in the symbolic (numerical) structure. Notice that $\mathcal{E}$ refers to potential interactions with objects, i.e., by abstracting from specific objects, it depends only on the (physical) structure of m.
- $\kappa\colon O \to U$ is the *interaction* function that associates to an object $o \in O$ the internal state of the complex system $m \bullet o$, i.e. it specifies, in terms of internal states in U, what the interaction of an object with m yields. The elements of U induce a partition on objects: $o \approx o'$ iff $\kappa(o) = \kappa(o')$, i.e. U gives the *resolution* of m. Similarly, each R_i induces a relation on objects: $R_i^O(o_1, \ldots, o_n)$ iff $R_i(\kappa(o_1), \ldots, \kappa(o_n))$. Therefore, differently from RMT, the empirical structure is determined by the (physical) structure of the MS that *induces* a structure on objects (by interacting with them) i.e. an MS (and the measurement procedure) provides a specific point of view on the reality.
- $\mathcal{S} = \langle V, S_1, \ldots, S_n\rangle$, the *symbolic structure*, is necessary to abstract from the internal states of the complex system. It translates (i) the internal states in U into symbols in V that can be used to communicate inter-subjectively; and (ii) each relation R_i between states into a relation S_i between symbols. Notice that, differently from RMT, $\mathcal{S}$ is not necessarily a *numerical* structure.
- $\lambda\colon U \to V$, the *symbolization* function, is a *one-to-one* function between U and V, such that $R_i(u_1, \ldots, u_n)$ iff $S_i(\lambda(u_1), \ldots, \lambda(u_n))$. Note that λ is an isomor-

[7] I omit here the set-up and reset procedures, i.e. I assume that the MS is always ready to correctly interact with an SUM. In addition, I do not address other important properties of MSs like (i) *selectivity*, i.e. measures must be related only to the objects under measurement and not to the environment in which they are situated (measurement must be independent from environment conditions as much as possible); or (ii) *invasivity*, i.e. in the measurement process, the MSs must interact with objects without changing them (at least with respect to the dimension under measurement).

phism, i.e. $\mathcal{S}$ contains the whole information in $\mathcal{E}$ but in a *communicable* and *manipulable* form. $\lambda \circ \kappa$ is an homomorphism that associates symbols (relations among symbols) to objects (relations among objects induced by an MS).

Summing up, in this model of measurement, the relations on objects are not assumed but they are induced by the structure of the MS and by the way its support interacts with the objects. Therefore, in addition to determining a symbolic structure, an MS selects a dimension along which the objects are measured and how objects compare with respect to this dimension. I.e., an MS sets a perspective on the world, the way one looks at the objects and what one can catch about them (the collectible *data*). This view places the MSs at the core of an epistemological foundation of measurement and allows for a simple position: "measurements are evaluations performed by means of measuring system" ([17], p.28). In general, an MS rests on the basis of a *theoretical framework* that needs to be validated from an empirical standpoint. However, an MS can also be linked to different theories, it can survive the change of the background theoretical framework, and the same theory can be associated to different measurement systems. In addition, according to the previous model, an MS has a unique support during its whole life. A different conception that allows the MSs to change their supports through time is possible[8] but I will not consider it in this paper.

To have *inter-subjective* measures that "convey the same information to different observers" ([17], p.28), MSs need to be *calibrated*. Calibration is based on *measurement standards* (mST), i.e. a set of reference objects, with a *conventionally* associated symbolic structure. [8] does not formalize mSTs. In this paper, I represent an mST by a tuple $\langle R, \mathcal{R}, \alpha \rangle$, where:

- R is the set of reference objects;
- $\mathcal{R} = \langle M, S_1^M, \ldots, S_n^M \rangle$ is a symbolic structure;
- α: $R \to M$ is a one-to-one function that conventionally assigns to each object in R a symbol in M. In this case too, each S_i^M *induces* a relation on R.[9]

The MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda \rangle$ is calibrated with respect to the mST $\langle R, \mathcal{R}, \alpha \rangle$, iff (i) $\mathcal{S} = \mathcal{R}$, i.e. the MS has the same symbolic structure as the mST, and (ii) for each $r, r_1, \ldots, r_n \in R$, $\lambda(\kappa(r)) = \alpha(r)$ and $S_i(\lambda(\kappa(r_1)), \ldots, \lambda(\kappa(r_n)))$ iff $S_i^M(\alpha(r_1), \ldots, \alpha(r_n))$, i.e. measuring the reference objects in R with the MS under calibration, one obtains the same results established in the mST. Because both α and λ are one-to-one functions, $|R| = |U|$, i.e. the resolution of the MS is reducible to the reference objects.[10] Notice that, for a structurally integral MS, the calibration process reduces to the *tuning* of the λ function (that at this point can be seen as the composition of a physical and a tuning function). Two MS, m_1 and m_2, are *aligned* iff there exists an mST s, such that both m_1 and m_2 are calibrated with respect to s. Measurement standards together with the notion of alignment allow to abstract from single MSs, i.e. given an mST s, all the systems calibrated with respect to s are equivalent from the point of view of measurement, even if, physically, they can differ.

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[8] One can recognize the classical problems linked to the *constitution* of objects (see [25] for an overview).

[9] Here I assume the S_i^M relations as given, but they could also be linked to a reference MS by convention.

[10] There are some calibration procedures that do not need to test the MS with all the objects it is able to resolve. For example, in the case of temperature, one can set the 0°C and the 100°C and check the linearity of the MS. This is very important from the practical point of view, but it does not conceptually change the scenario and it introduces the problem of checking, in the example, the linearity of the MS.

Sketching an empirical theory of properties

Given an mST s with symbolic structure $\langle M, S_1^M, \ldots, S_n^M \rangle$, it is possible to associate to each $s_p \in M$ a property P: 'a has P' (or 'a is an *instance* of P') if and only if there exists an MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda \rangle$ calibrated with respect to s such that $\lambda(\kappa(a)) = s_p$. First of all, note that according to this definition, 'a has P' only if it has been measured producing the result s_p. The definition can be generalized by introducing a potential aspect, i.e. 'a has P' if it *could* interact with the MS producing the result s_p. I don't consider this option because it moves away from a pure empirical approach and it seems to involve the difficult notion of *disposition* (see [22]). Second, different properties (e.g. properties associated to non-aligned MSs) can in principle have the same instances, i.e. properties are not *extensional*. The *intensions* of properties are linked to the MSs, the mSTs, and the procedures of measurement and calibration. Third, by means of the S_i^M relations, the mST s and the calibrated MSs individuate not only a set of properties but also how they are structured. Fourth, the properties that correspond to symbols cannot be further refined, they represent the *resolution* of the MSs calibrated with respect to s. In philosophy, these properties are called *determinate* properties (see [13,12]) and they are distinct from more general properties called *determinable* properties. Let us assume that 'being scarlet' and 'being crimson' are determinate properties. 'Being red' is a determinable property because it is the generalization of 'being scarlet', 'being crimson', etc., i.e. to be red an object needs to be scarlet, or crimson, etc. Coming back to the mSTs, *determinable* properties correspond to *sets* of symbols, i.e. 'a has the (determinable) property Q' iff there exists an MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda \rangle$ calibrated with respect to s such that there exists an $s \in S_q \subseteq M$ (S_q is the set of symbols of s associated to Q) such that $\lambda(\kappa(a)) = s$.[11]

Moving now to the analysis of the form 'a has P at t', I already observed that both theories of properties and theories of persistence need to be considered. Unsurprisingly, measurement theories rely on positions quite close to philosophical ones. [8] assumes that a is a *state* of an object, i.e. what an object is at a specific time. The notion of state is not further analyzed, thus it is not clear if states are similar to temporal slices or to more abstract entities like *states of affairs*. RTM has an even more obscure position that, in any case, seems very similar to the previous one. Cognitive sciences often refer to *stimuli* (like sounds, visual inputs, etc.) that abstract from objects by considering just the input of the human sensory system. However, stimuli too can have different properties at different times, therefore, as in the case of objects, one has to refer to how stimuli are at specific times. [7] defines a *wide sense* of measurement: "a process of empirical, objective assignment of symbols to attributes of objects and events of the real world, in a such way to represent them, or to describe them" ([7], p.41). According to this definition, measurement concerns the assignment of symbols to *attributes* of objects that seem quite close to *tropes* or to a generalization of them (see the *qualities* in the DOLCE approach described below).

I will show that giving a central and *referential* role to mSTs and MSs the existential commitment to temporal slices, states, tropes, or stimuli can be avoided by assuming objects as *direct subjects* of measurement. In this view, what objects under measurement are and how they can be recognized, selected, observed, measured, and re-identified through time are central questions. Admittedly, I have no definite answers, only few ob-

[11] To have a uniform representation, determinate properties can be associated to *singletons*.

servations. First of all, note that committing to temporal slices, states, or tropes, one still faces the problem of recognizing, selecting, and measuring them. Second, to account for change, one has to refer to persistent objects even though they are just conceptual constructions instead of real entitites. My approach requires the capability of re-identifying objects through time without committing to their ontological nature and to the way they persist. Third, in classical mechanics, observations and measures are attributed to *particles* that can be seen as a kind of objects. Particles constitute more complex *systems* the states of which are represented in multi-dimensional spaces. Classical mechanics does not directly refer to 'mesoscopic or macroscopic objects'. However, these objects can be mathematically reconstructed (and re-identified through time) from the representations of the systems without committing to their ontological nature [21].[12] Four, in cognitive science, the concept of object individuation, identification, and persistence across time, space, occlusion, property change, and loss of cohesion have been deeply studied starting from experience-dependent constructivist approaches to approaches that assume innate representation mechanisms for objects (see [27] for a recent review). In addition, some psychological theories of persistence have been linked to philosophical ones (in particular endurantism and perdurantism) [28]. Five, an MS selects the kind of entities it is able to interact with, therefore the nature of objects the MS can measure is (at least partially) determined by it.

Let us now consider how the form 'a has P at t' can be analyzed in terms of measurement just committing to objects. Assuming instantaneous processes of measurement, each measure (that can be seen as a *datum* or *observation* about an object) is collected by using an MS at a precise time. Given an MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda \rangle$, $[t](\lambda(\kappa(a)) = s_p)$ represents the fact that m and a interacted at t with the result s_p, i.e., $[t]$ is a sort of temporal operator that indicates *when* a has been measured.[13] From these premises, 'a has P at t' can be reduced to the following statement: given an mST s with symbolic structure $\langle M, S_1^M, \ldots, S_n^M \rangle$ such that P is associated to the symbol $s_p \in M$, there exists an MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda \rangle$ calibrated, *at* t, with respect to s, such that $[t](\lambda(\kappa(a)) = s_p)$. Objects can be diachronically compared by examining the measures collected at different times by MSs calibrated with respect to the same mST. For example, if m and m' are two MSs calibrated with respect to s, respectively, at t and t', then, from $[t](\lambda(\kappa(a)) = s_p)$ and $[t'](\lambda'(\kappa'(b)) = s_p)$, one can conclude that a and b share the property P.[14] Despite its apparent triviality, this conclusion relies on important additional hypotheses about the stability of mSTs and MSs. First of all, in the previous analysis, the property P is associated to a symbol of an mST that, by definition of mST, identifies a (physical) reference object. The diachronic alignment of the MSs relies on the calibration, at different times, with respect to *the same* mST. Therefore, the change (across time) of the reference objects that constitute an mST can invalidate this alignment. Second, the interaction and symbolization functions of an MS depend on the (physical) structure of the support m

[12] Things are more complex in the case of quantum mechanics.

[13] In an MS, a symbol corresponds to an internal state of the complex system resulting from the interaction between the support of the MS and the measurand. However, from an empirical point of view, one does not need to define what states or configurations are. It is enough to be able to observe (or measure), *at a specific time*, some characteristics of the complex system, i.e. states can be reduced to observations (or measures).

[14] I assume here the stability (through time) of symbols and natural laws.

that can change across time.[15] Therefore MSs too can change. If an mST s is stable, then by (diachronically) calibrating an MS with respect to s one makes sure that the MS is stable (at least for what concerns its measurement capacities). However, even though the measurement is instantaneous, from an empirical perspective it is very hard to assume that (i) all the MSs are re-calibrated every time they are used, and (ii) that the calibration and the measurement can be synchronically performed. These observations make clear that to diachronically compare objects one must rely on the stability of mSTs and MSs.[16] Once the mSTs are considered as *reference systems*, i.e., systems that are stable by *assumption*, and the MSs are assumed to be stable (at least from the calibration to the measurement), then the comparison between the state of $m \bullet a$ and the state of $m' \bullet b$ (where m and m' are the supports of two diachronically aligned MSs) depends exclusively on a and b. One can then conclude that two objects share a property but this conclusion relies on a framework of objects assumed to be stable.[17]

[3] discusses two alternative methods to diachronically tune systems. To align measures taken at different times, one method maximizes the stability of the objects, i.e. it assumes that the world is quite stable, the other one maximizes the stability of the structure of the measures, i.e. it assumes that the MSs are quite stable. On one hand, similarly to the previous analysis, these methods too assume the stability of objects or MSs. However, they are less precise because different tunings can maximize the stability of objects or MSs. On the other hand, by relying on the availability, at every time t, of the measures of every object a that exists at t with respect to any dimension that makes sense for a, they are unusable when, at each time, one has a partial knowledge of the world.

From an ontological perspective, in absence of the stability of the mSTs and the MSs, the fact that two objects measure the same does not imply any similarity between them. The identity of measures could be caused by a misalignment of the MSs used to collect these measures. If properties are associated to symbols of mSTs that identify reference objects, then properties do not depend only on symbols (that are assumed to be stable here) but on reference objects too because these objects are essential for the calibration process that makes possible the diachronic comparison of objects. In this perspective, a property can change accordingly to the referent object to which it is associated. Therefore one has to assume that, at t, a property depends on how the referent object identified by the symbol associated to P is at t. In principle, properties can change at every time and the *similarity* between 'P-at-t' and 'P-at-t'' is founded on the similarity (stability) of the reference object identified by the symbol associated to P. Instead of re-identifying objects on the basis of a stable framework of properties, here we are 're-identifying' properties on the basis of a stable framework of objects. But, on what empirical evidences can the stability of mSTs and MSs be grounded? To empirically justify the stability of an mST, one needs to diachronically compare its reference objects (at least along some dimensions). To do that one needs other mSTs and MSs, the stability of which, in turn, needs to be founded. It is evident that this approach suffers a circularity or infinite regression problem, very frequent at the foundational level. Instead of considering one

[15] To explicitly represent this dependence one has to analyze the temporal operator considered in the form $[t](\lambda(\kappa(a)) = s)$. For example, it is possible to add a temporal parameter to the λ and κ functions or to consider λ and κ as functions that individuate a function at every time. I don't consider this interesting aspect here.

[16] Statistical analysis on relative changes of calibrated MSs helps in discovering needs for system re-calibration, but does not solve the theoretical problem.

[17] Again, the approaches that commit to states, temporal slices, or tropes suffer the same problems.

single mST (or MS), one can consider the global framework that includes all the mSTs and the MSs. The stability of this global framework can be founded only on the stability of the mutual relationships between its components, and the mutual relationships are measured by the global framework itself. Changes are always relative to a framework, but once the global one is considered, then they represent the maximal resolution possible for an observer. Still, the global framework could be changing in absolute terms, i.e. the observer could ascribe the same property to objects that, in absolute ontological terms, do not share anything. However, even though an observer disposes of the whole data collected by the global framework, (s)he cannot detect this dissimilarity.

In the following, I will assume that all the mSTs and the MSs are stable. In this way the calibration too is stable and diachronic comparisons are always possible.

Qualities and measurement

In this section I will link one of the theories of qualities discussed in [18] (that avoids individual qualities) with the measurement theory above introduced.

DOLCE [19] adopts a theory of qualities that relies on the notions of (*individual*) *quality*, *quality kind*, and *space* (*of qualities*). Individual qualities, e.g. 'the color of my car', 'the weight of John', *inhere* in their unique hosts, but, differently from tropes, they can change. While tropes are individualizations (with respect to the host) of a *determinate* property (e.g. 'being 1m long', 'being 1g heavy'), individual qualities are individualizations of *determinable* properties that correspond to specific dimensions of comparison (e.g. 'being colored', 'being shaped') represented by quality kinds, i.e. properties that collect the individual qualities relative to the same dimension. Each space is associated to a unique quality kind. The determinate properties relative to the dimensions individuated by quality kinds are represented by the *atomic regions* of the spaces associated to these kinds.[18] These regions can be organized in taxonomies, or in more complex geometrical ways. DOLCE-CORE [4] extends DOLCE by allowing several spaces, in general with different structures motivated by epistemic or empirical considerations, to be associated to the same quality kind. An object can have only one individual quality of a specific kind, that, at a given time, can be *located* in different spaces (associated to such kind). In [18], the theory called ES avoids individual qualities by adopting a *location* relation directly defined on objects.[19] $\mathsf{L}(r, o, t)$ stands for 'the region r is the *location* of the object o at time t'. Regions correspond to properties therefore 'o has P at t' can be analyzed as: at t, the object o is located in the region r_p (where r_p corresponds to the property P). Here again several spaces can be associated to the same dimension of comparison (e.g. color, weight, etc.). While in DOLCE and DOLCE-CORE the quality kind allows to individuate all the spaces relative to the same dimension, in ES, to cluster all the regions that represent properties relative to the same dimension (*generalized spaces*), an additional primitive is required. I think that generalized spaces can be 'simulated', in measurement terms, by clustering mSTs (and MSs), however here I don't consider this aspect.

ES assumes a finite number of mutually disjoint spaces noted with SP_i. In each space a *classical extensional mereology* (see [5,30]) is assumed. Here I have no space for

[18] Determinable properties are represented by non-atomic regions.

[19] ES considers *endurants* (persons, cars, etc.) but excludes *perdurants* (events, processes, etc.). I follow this simplification, even though I think that the following results can be adapted to the case of perdurants.

taking into account this aspect therefore I consider all the regions as 'atomic'.[20] Finally, the primitive $\mathsf{EX}(o, t)$ stands for 'the object o *exists* at time t'. As already said, I do not address here the problem of persistence of objects, therefore I do not provide any epistemic interpretation of the primitive EX, I just assume to (empirically) know at what times an object exists. In addition, differently from ES, I consider only temporal instants. These moves simplify the formal framework without reducing its conceptual relevance.

Summing up, I consider a theory with the following extra-logical vocabulary: $\textsc{ob}$ (object), $\textsc{tm}$ (time), $\textsc{sp}_1, \ldots, \textsc{sp}_n$ (spaces), EX (existence), L (location). This is a general theory that represents properties abstracting from empirical, epistemic, or conceptual considerations. I will provide an interpretation of this theory in terms of *measurement structures*. A measurement structure, is a structure $\langle O, T, S, F, EX\rangle$ where O is a set of 'objects', T is a set of 'times', S is a set of 'symbols', F is a set of (*calibrated*) *measurement frameworks* (MFs), and $EX \subseteq O \times T$. A (calibrated) measurement framework $\mathcal{M}_i$ is a couple $\langle \mathsf{s}_i, M_i^*\rangle$ where s_i is an mST, and M_i^* is a set of MSs calibrated[21] with respect to s_i such that the supports of the MSs in M_i^* and the reference objects in s_i belong to O, and $M_i \subseteq S$, where M_i is the set of symbols of s_i.[22] A measurement structure of dimension n is a measurement structure such that $|F| = n$.

At this point the interpretation function $\mathcal{I}$ becomes trivial (a theory that assumes n spaces $\textsc{sp}_1, \ldots, \textsc{sp}_n$ is interpreted in a measurement structure of dimension n or greater):

(*a*) $\textsc{ob}^{\mathcal{I}} \subseteq O$;

(*b*) $\textsc{tm}^{\mathcal{I}} \subseteq T$;

(*c*) $\textsc{sp}_i^{\mathcal{I}} \subseteq M_i$ (the set of symbols of the mST s_i in an MF of F);

(*d*) $\mathsf{EX}^{\mathcal{I}} \subseteq EX$;

(*e*) $\mathsf{L}^{\mathcal{I}} \subseteq S \times O \times T$ and
$\langle r, o, t\rangle \in \mathsf{L}^{\mathcal{I}}$ iff there exists an MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda\rangle$ belonging to some M_i^* (in one measurement framework) such that $[t](\lambda(\kappa(o)) = r)$.[23]

According to (*c*), the regions of the space $\textsc{sp}_i$ correspond to the symbols of the mST s_i. All the MSs calibrated with respect to s_i have the same symbols, therefore the regions of the space $\textsc{sp}_i$ correspond to the symbols of the MF $\mathcal{M}_i$. This introduces a correspondence between spaces and MFs. Structural relations among regions belonging to a space are then mapped to structural relations among symbols in the corresponding MF (that all the MSs in it share). According to (*e*), at t, an object is located in a region of a space $\textsc{sp}_i$ if and only if there is an MS belonging to the M_i^* in the MF associated with $\textsc{sp}_i$ that, at t, interacted with the object giving as result the symbol associated to the given region. This makes clear in which sense spaces represent an empirical point of view. However, note that the correspondence between regions and symbols is valid only by rejecting non atomic regions. As said, symbols correspond only to determinate

[20] ES is compatible also with non-atomic spaces, i.e. it accepts the existence of determinable properties that are not partitioned by determinate properties. From both the conceptual and empirical point of views, I find this option quite debatable.

[21] Remember that the calibration is assumed to be stable.

[22] Every MS calibrated with respect to an mST s has the same symbols of s, therefore all the MSs belonging to M_i^* have the same symbols of s_i (a subset of S).

[23] Note that one needs to ensure that $[t](\lambda(\kappa(o)) = r)$ implies that both m and o exist at t. This constraint does not really regard the interpretation function but the notion of MS.

properties while non atomic regions correspond to determinable properties because they have atomic parts (in the case of an atomic space) that, in turn, correspond to determinate properties. This means that the parthood relation cannot be associated to a structural relation between symbols. If needed, as already observed, the interpretation function can be modified by associating *sets of* symbols to regions and the (set-theoretical) *inclusion* relation to the parthood relation. Alternatively, one can add some conventions in the measurement frameworks: an mST determines both the conventional symbols assigned to the reference objects and the conventional symbols assigned to the sets of reference objects. In this case, the parthood relation can be associated to the inclusion relation between sets of reference objects conventionally associated to symbols.

Let us now discuss, in the light of the given interpretation, the main axioms proposed in [18] and [4].

a1 $\mathsf{L}(r, o, t) \wedge \mathsf{L}(r', o, t) \wedge \textsc{sp}_i(r) \wedge \textsc{sp}_i(r') \rightarrow r = r'$

a2 $\mathsf{EX}(o, t') \wedge \exists r(\mathsf{L}(r, o, t) \wedge \textsc{sp}_i(r)) \rightarrow \exists r'(\mathsf{L}(r', o, t') \wedge \textsc{sp}_i(r'))$

a3 $\mathsf{EX}(o, t) \rightarrow \bigvee_i (\exists r(\mathsf{L}(r, o, t) \wedge \textsc{sp}_i(r)))$

(a1) states that, in a given space and at a given time, the location of an object is unique. In the interpretation, the premises in (a1) imply that, in the MF associated to $\textsc{sp}_i$, there exist $\mathsf{m} = \langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda\rangle$ and $\mathsf{m}' = \langle m', \mathcal{E}', \kappa', \mathcal{S}', \lambda'\rangle$ such that $[t](\lambda(\kappa(o)) = r)$ and $[t](\lambda'(\kappa'(o)) = r')$. But m and m' are in the same MF, thus they are aligned and, by definition of alignment, $r = r'$. Nevertheless, note that, from an empirical standpoint, this means that two MSs can be used at the same time to measure the same object, i.e. objects can simultaneously interact with different MSs (of the same kind). If needed it is easy to exclude this situation. (a2) states that if, at a given time, an object is located in a specific space, then it is located in this space at every time at which it exists. This seems ontologically but not empirically plausible. The fact that o has been measured at t does not imply that o has been measured (relatively to the same dimension) at every time at which it exists. Similarly in the case of (a3): it is possible that at a specific t, o has not been measured in any way. Axioms (a2) and (a3) make evident that the empirical theory does not presuppose a complete information about the world.[24]

[24] takes into account the cognitive processes underlying classification and measurement by extending the theory of qualities in DOLCE with the notions of *absolute magnitude* (human independent and objective value), *perceived magnitude* (subjective mental entity evoked by the interaction with objects) and *communicable symbol* (entity that allows the communication of a perceived magnitude). The theory proposed in [24] aims at understanding the *cognitive* foundations of measurement. This important aspect is left for future work.

Perspectives: concepts and measurement

The proposed measurement theory can also provide an interpretation of the theory of *social concepts* introduced in [20] and simplified in [4]. In this theory, concepts depend on communities of agents, who, by some sort of *social conventions*, create, make use of, communicate about, *define*, and accept them. These social conventions are represented

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[24] This is the kind of information on which the methods proposed in [3] rely.

by *descriptions* that (i) are created by agents, (ii) are encoded in some 'public' (formal or informal) language, (iii) need to have at least a (but, in general, several) physical support, and (iv) are accepted by agents. Both concepts and descriptions are in time. $\mathsf{DF}(c, d)$ stands for 'the concept c is defined by the description d'. Objects can be classified by concepts, $\mathsf{CF}(c, o, t)$ stands for 'the objects o, as it is at t, is classified by the concept c'. I propose the following interpretation function $\mathcal{I}$ that maps the extra-logical vocabulary $\textsc{ob}$ (object), $\textsc{tm}$ (time), $\textsc{cn}$ (concept), $\textsc{ds}$ (description), DF (definition), CF (classification) into the measurement structures previously introduced:

(a) $\textsc{ob}^{\mathcal{I}} \subseteq O$;
(b) $\textsc{tm}^{\mathcal{I}} \subseteq T$;
(c) $\textsc{cn}^{\mathcal{I}} \subseteq S$;
(d) $\textsc{ds}^{\mathcal{I}} \subseteq F$;
(e) $\mathsf{DF}^{\mathcal{I}} \subseteq S \times F$ and $\langle c, \mathcal{M}_i \rangle \in \mathsf{DF}^{\mathcal{I}}$ iff $c \in M_i$ (the set of symbols of $\mathcal{M}_i$);
(f) $\mathsf{CF}^{\mathcal{I}} \subseteq S \times O \times T$ and
$\langle c, o, t \rangle \in \mathsf{CF}^{\mathcal{I}}$ iff there exists an MS $\langle m, \mathcal{E}, \kappa, \mathcal{N}, \lambda \rangle$ such that $[t](\lambda(\kappa(o)) = c)$.

According to this interpretation, concepts are associated to symbols of MFs (condition (c)), and descriptions are associated to MFs (condition (d)). In general, an MF determines a (complex) symbolic structure, therefore, by (e), a description can define several (structured) concepts. This holds also for (mathematical, scientific, philosophical) *theories* in which the primitive notions are usually characterized by the way they are interrelated. Second, an MF $\mathcal{M}_i$ can be seen as the maximal collection of the physical supports of a description. One may think it would be more appropriate to map a description to the theory behind the $\mathcal{M}_i$. As already observed, this theory does not always exist. In addition, different theories can be associated to the same MF (different theories at different times, or different theories associated to different, but aligned, MSs in the same MF). Theories are then not essential neither to define MFs nor the 'meaning' of the symbols, i.e., for measurement, the *operative* side seems more important than the theoretical one. This allows also to clearly establish the temporal extensions of descriptions and concepts, even though a deeper analysis of the persistence conditions of MSs, mSTs, and MFs is required.[25] The interpretation of CF (condition (f)) is very similar to the one of L. Spaces can thus be seen as networks of social concepts the acceptance and sharing of which can be grounded on calibration and alignment. The question whether this interpretation is compatible with the axioms in [20] and general enough to account for all kinds of social concepts, e.g. abstract concepts or concepts defined in terms of temporal or modal relations, is left for future work.

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