Parthood Simpliciter vs. Temporary Parthood

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Abstract

Starting from the formal characterization of four-dimensionalism (perdurantism) provided by Theodore Sider, I study the interconnections between the theories of *parthood simpliciter* (classical mereologies) and the theories of *temporary parthood* (parthood at a time). On the basis of this formal analysis, I propose a definition of temporary parthood in terms of parthood simpliciter that does not commit to the existence of temporal parts. In this way, I hope this definition can be accepted by endurantists.

Introduction

According to Sally Haslanger (Haslanger 2003, pp. 316–317), most of the puzzles about change through time rely on general conditions that, when integrally accepted, generate a contradiction. She individuates five general conditions:

1. Objects persist through change.
2. The properties involved in a change are incompatible.
3. Nothing can have incompatible properties.
4. The object before the change is one and the same object after the change.
5. The object undergoing the change is itself the proper subject of the properties involved in the change.

Let us consider, for example, a rose r that persists through the change from 'red' (R) to 'brown' (B), two incompatible properties, i.e., $\neg\exists x(R(x) \wedge B(x))$. Accepting the previous conditions, $R(r) \wedge B(r)$ holds, leading to a contradiction, that, to be solved, requires the rejection of (at least) one of the conditions (1)–(5).

In this paper I will focus on two positions on persistence through time, *perdurantism* and *endurantism*[1], that avoid the previous contradiction by rejecting, respectively, condition (5) and (2).

Perdurantism assumes that all the objects persist by *perduring*, i.e., similarly to the extension through space, objects are extended in time by having different (temporal) parts at different times. At each time, only a part of a persisting object is present, i.e. at one time persisting objects are only *partially present*. The subjects of temporal properties are temporal parts. In the previous example, 'r is P at t' must be read as 'r-at-t is P' where 'r-at-t' is the temporal part of r at t. Because r-at-t and r-at-t' are two different temporal parts of r (if $t \neq t'$), the contradiction disappears.

Endurantism assumes that some objects undergoing the change *endure*[2], i.e. they are *wholly present* at any time at which they exist, they maintain their identity through change and they are the subjects of properties, but these properties need to be *temporally qualified*. Red and brown are incompatible only if stated at the same time (about the same object), the fact that r is red-at-t and brown-at-t' does not lead to any contradiction. Different readings of 'P-at-t' are accepted by endurantists (e.g. modal or relational readings are considered, see (Varzi 2003)) that however refuse the applicability of the perdurantist view to all kinds of objects.

While the notion of *being partially present* has been quite precisely stated (Sider 1997; 2001), the notion of *being wholly present* is still quite obscure, even though some attempts to characterize it exist (Crisp and Smith 2005). This complicates the formal comparison between perdurantism and endurantism that often reduces to different positions on *parthood*: endurantists claim that, for enduring objects, a temporally qualified parthood (called here *temporary parthood*) is required, while perdurantists often refer to an atemporal parthood (called here *parthood simpliciter* or simply parthood) that is enough (together with a predicate of existence in time) to define temporal parts (see next section for the details).

To overcome this 'deadlock', Theodore Sider introduced a formal characterization of perdurantism based on temporary parthood (Sider 1997; 2001). On one side, perdurantists are able to accept his formulation simply analyzing 'x is part of y at t' as 'the temporal part of x at t is part of the temporal part of y at t'. On the other side, he hopes that the formalization of perdurantism in terms of temporary parthood can be 'intelligible' by endurantists.

In this paper, I don't provide a characterization of endurantism, I will just show that endurantists do not neces-

[1] I prefer to use the terms 'endurantim' and 'perdurantism' instead of *three-* and *four-dimensionalism*, because I will concentrate on persistence through time ignoring the spatial dimension. All the results are valid in any n-dimensional space-time (with $n \geq 2$).

[2] Usually endurantists also accept perduring objects, e.g. *processes* or *events*, as opposed to endurants, e.g. persons or cars.

sarily need to consider *temporary parthood* as primitive. I will prove that the axioms for temporary parthood can be 'recovered' in a theory based on parthood simpliciter *without* assuming the existence of temporal parts. This requires a new definition of temporary parthood (see (d9)) in terms of parthood simpliciter (and existence in time) that does not rely on temporal parts. Endurantists could accept this formulation analyzing 'x is part (simpliciter) of y' just as *constant parthood*, i.e. 'at every time at which x exists, x is part of y'. I think that this analysis prevents an a priori refutation of having parthood simpliciter as primitive and it offers an alternative to the usual tensed interpretation that reduces 'part-of' to 'part-of, now'. In addition to that, I formally analyze the interconnections between theories of parthood and theories of temporary parthood and how these interconnections depend on existential conditions (about the entities in the domain), a particularly important aspect to uncover the ontological commitment of perdurantism and endurantism.

One may wonder if a deeper understanding of perdurantism and endurantism is relevant for representing commonsense knowledge. I do not have a definite answer, but only few considerations. First, perdurantism is not incompatible with commonsense. Commonsense and natural language are deeply related and perdurantism offers an alternative ontological foundation to the semantics of natural language that can handle a number of well-known semantic phenomena (Muller 2007). Second, Patrick Hayes, in his seminal work (Hayes 1985), already encountered the problem of understanding temporal parts: the ontological status of the couples $\langle objects, time \rangle$ he uses has not been clarified. Third, my analysis is quite general and can be helpful in formalizing different domains. For example some qualitative theories of space-time and movement are based on four-dimensional entities (Muller 1998). Fourth, perdurantism has recently been adopted in some applications not only to overcome some technical difficulties (as in the case of the representation of n-ary relations in *description logics* (Welty and Fikes 2006)) but also advocating its adequateness, conceptual simplicity and practical advantages for representing dynamic environments (Stell and West 2004; West 2004).

Formal characterization of perdurantism

Following Sider, temporal existence is represented by the primitive $\mathsf{EX}xt$ whose informal reading is "at time t, x exists". I'm concerned here with persistence through time, therefore I focus only on objects that *are* in time, objects that *exist* at some times:

a1 $\exists t(\mathsf{EX}xt)$

EX has to be intended just as a *representational surrogate* that does not necessarily commit neither on the existence/nature of times nor on the fact that existence is an extrinsic relation between objects and times. Times could be constructed from events like in (Kamp 1979) or just be the reification of the worlds of a (modal) temporal logic. Existence in time can be reduced to 'being simultaneous with' others entities (Simons 1991) or, assuming a Newtonian view in which time is an independent container, to a *location* relation. Times can be punctual or extended and different structures (discrete vs. continuous, linear vs. branching, etc.) can be imposed on them. For the purpose of this paper, time can just be considered as a set of indexes, i.e. a set of atomic entities that are related only by *identity*.

The notion of *parthood simpliciter* is represented by the predicate $\mathsf{P}xy$ that can be read as "x is part of y".

On the basis of parthood simpliter and existence in time, the (perdurantist) notion of *temporal part* (also called *temporal slice*) can be defined. x is a temporal part of y at t, formally $\mathsf{TP}xyt$, if x is a maximal part of y that exists *only* at t. Formally (using the relation $\mathsf{O}xy$ defined in (d1) that stands for "x overlaps y"):

d1 $\mathsf{O}xy \triangleq \exists z(\mathsf{P}zx \wedge \mathsf{P}zy)$

d2 $\mathsf{TP}xyt \triangleq \mathsf{EX}xt \wedge \mathsf{EX}yt \wedge \neg\exists t'(\mathsf{EX}xt' \wedge t' \neq t) \wedge \mathsf{P}xy \wedge \forall z(\mathsf{P}zy \wedge \mathsf{EX}zt \rightarrow \mathsf{O}zx)$

Following the schema adopted by perdurantists for all the temporary properties and relations, *temporary parthood* ($\mathsf{tP}xyt$ stands for "x is part of y at t") can be defined as:

d3 $\mathsf{tP}xyt \triangleq \exists zw(\mathsf{TP}zxt \wedge \mathsf{TP}wyt \wedge \mathsf{P}zw)$

Because endurantists accept objects that do not necessarily have temporal parts at every time at which they exist, they refuse (d3) as a *general* definition of temporary parthood. For *enduring* objects temporary parthood has to be taken as primitive, or an alternative to (d3) that does not rely on temporal parts needs to be provided.

Sider's formulation

In (Sider 1997; 2001), Sider proposes a formulation of perdurantism based on the primitive of *temporary parthood* instead of parthood simpliciter. He hopes that this move can lead to a theory 'intelligible' both to perdurantists and endurantists, allowing for a formal comparison of the two positions.

The axioms and definitions considered by Sider are reported below (see (Sider 2001, pp. 58–59)) where $\mathsf{tO}xyt$ stands for "x overlaps y at t", and $\mathsf{tTP}xyt$ stands for "x is a temporal part of y at t"[3]:

d4 $\mathsf{tO}xyt \triangleq \exists z(\mathsf{tP}zxt \wedge \mathsf{tP}zyt)$

d5 $\mathsf{tTP}xyt \triangleq \neg\exists t'(\mathsf{EX}xt' \wedge t' \neq t) \wedge \mathsf{tP}xyt \wedge \forall z(\mathsf{tP}zyt \rightarrow \mathsf{tO}zxt)$

a2 $\mathsf{tP}xyt \rightarrow \mathsf{EX}xt \wedge \mathsf{EX}yt$

a3 $\mathsf{EX}xt \rightarrow \mathsf{tP}xxt$

a4 $\mathsf{tP}xyt \wedge \mathsf{tP}yzt \rightarrow \mathsf{tP}xzt$

a5 $\mathsf{EX}xt \wedge \mathsf{EX}yt \wedge \neg\mathsf{tP}xyt \rightarrow \exists z(\mathsf{tP}zxt \wedge \neg\mathsf{tO}zyt)$

Sider characterizes perdurantism (four-dimensionalism in his vocabulary) as:

[3] I use different symbols to represent the temporal part relation defined in terms of temporary parthood (d5) from the one defined in terms of parthood simpliciter (d2).

"[N]ecessarily, each spatiotemporal object has a temporal part at every moment at which it exists." (Sider 2001, p. 59)[4]

This claim seems a restriction of the one given in (Sider 1997, p. 206) where Sider refers to objects in *time* instead of in *space-time*. In the original work of Lesniewski (Lesniewski 1991) mereology is not intended as a theory necessarily related to space or space-time but as a pure formal theory (that applies to all kinds of entities) aimed at avoiding some (ontological) assumptions of set-theory, namely, the existence of the *empty set* and the distinction between *urelements* and *sets*. In this sense mereology does not commit to existence in space or time. Even though a theory of persistence must consider entities in time, I do not see any reason to exclude entities that (according to some researchers) do not have a clear spatial location, e.g. mental attitudes, concepts, mathematical theories, societies. I thus prefer the following characterization:

"Each object that exists in time has a temporal part at every time at which it exists.", i.e. formally:

pd $\mathsf{EX}xt \rightarrow \exists y(\mathsf{tTP}yxt)$

$\mathcal{T}_{\mathsf{tP}} = \{$(a1)–(a5), (pd)$\}$ denotes Sider's theory where tO and tTP are respectively defined by (d4) and (d5).

Theorem (t1) shows that at a given time, the temporal parts are not necessarily unique. A counterexample is provided by a model with two *different* elements a and b, both existing only at t, such that $\langle a,a,t\rangle$, $\langle b,b,t\rangle$, $\langle a,b,t\rangle$, $\langle b,a,t\rangle \in \mathsf{tP}^{\mathcal{I}}$. In this case, it is easy to verify that $\langle a,a,t\rangle$, $\langle b,a,t\rangle \in \mathsf{tTP}^{\mathcal{I}}$.

(t2) shows that different entities can coincide (they are part one of the other) during their whole life. The previous model is a counterexample because both a and b exist only at t and $\langle a,b,t\rangle, \langle b,a,t\rangle \in \mathsf{tP}^{\mathcal{I}}$, but $a \neq b$ by hypothesis.

t1 $\mathcal{T}_{\mathsf{tP}} \nvdash \mathsf{tTP}xyt \wedge \mathsf{tTP}zyt \rightarrow x = z$

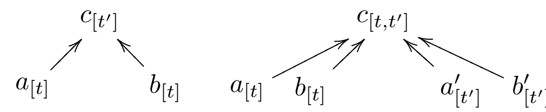
t2 $\mathcal{T}_{\mathsf{tP}} \nvdash \forall t(\mathsf{EX}xt \rightarrow \mathsf{tP}xyt) \wedge \forall t(\mathsf{EX}yt \rightarrow \mathsf{tP}yxt) \rightarrow x = y$

Formulation based on parthood simpliciter

Sider shows that P and EX allow to define the notions of *temporal part* and *temporary parthood* (respectively by (d2) and (d3)) and to characterize perdurantism by an axiom similar to (pd). However, Sider does not clarify what axioms on P and EX are necessary to have a theory *equivalent* to $\mathcal{T}_{\mathsf{tP}}$. I intend equivalence in the following way: (i) all the axioms in $\mathcal{T}_{\mathsf{tP}}$ can be proved in this new theory by *assuming* the 'same' EX and the definition (d3) for tP; (ii) the new theory does not add new properties on tP and EX.

According to (Simons 1987) and (Casati and Varzi 1999), parthood is minimally characterized as a *partial order*, i.e., a reflexive, antisymmetric, and transitive binary relation (axioms (a6), (a7), and (a8)). The inclusion of the *extensionality* (axiom (a9)) guarantees the identity of objects that have the same parts (t3) or that overlap the same objects (t4). The theory $\mathcal{M}_{\mathcal{E}} = \{$(a6)–(a9)$\}$ is called *extensional mereology*.

[4]Sider does not explicitly introduce a modal operator in his formulation. However note that in a classical first order logic all the formula can be considered as 'necessary'.



(a) **(b)**

Figure 1: (pdn) is independent from (a10).

a6 $\mathsf{P}xx$

a7 $\mathsf{P}xy \wedge \mathsf{P}yx \rightarrow x = y$

a8 $\mathsf{P}xy \wedge \mathsf{P}yz \rightarrow \mathsf{P}xz$

a9 $\neg\mathsf{P}xy \rightarrow \exists z(\mathsf{P}zx \wedge \neg\mathsf{O}zy)$

t3 $\mathcal{M}_{\mathcal{E}} \vdash \forall z(\mathsf{P}zx \leftrightarrow \mathsf{P}zy) \rightarrow x = y$

t4 $\mathcal{M}_{\mathcal{E}} \vdash \forall z(\mathsf{O}zx \leftrightarrow \mathsf{O}zy) \rightarrow x = y$

It is possible to characterize perdurantism by introducing an axiom analogous to (pd):

pdn $\mathsf{EX}xt \rightarrow \exists y(\mathsf{TP}yxt)$

Because, as already observed, P can in general apply to all kinds of objects, standard mereologies do not analyze how P and EX are related. (pdn) is a weak link between P and EX that does not rule out models like the one in figure 1.a where some of the parts of c (namely, a and b) have temporal extensions disjoint from the one of c.[5]

(a10) rules out these models by ensuring that the temporal extension of the part is included in the one of the whole. First of all note that (a10) and (pdn) are independent: the model in figure 1.a satisfies (pdn) but not (a10) and vice versa for the model in figure 1.b. Secondly, and more importantly, by defining parthood simpliciter as *constant parthood* (d6), (t5) shows that, in $\mathcal{T}_{\mathsf{tP}}$, (a10) holds. Therefore, assuming (d6), the lack of (a10) prevents any equivalence between $\mathcal{T}_{\mathsf{tP}}$ and the theory based on parthood simpliciter we are building.

a10 $\mathsf{P}xy \wedge \mathsf{EX}xt \rightarrow \mathsf{EX}yt$

d6 $\mathsf{P}xy \triangleq \forall t(\mathsf{EX}xt \rightarrow \mathsf{tP}xyt)$

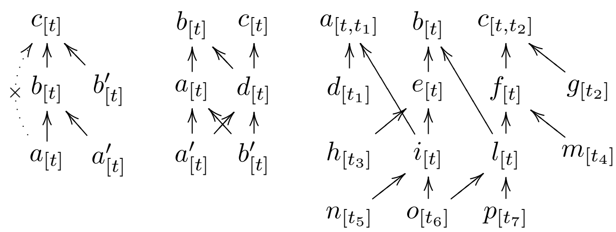
t5 $\mathcal{T}_{\mathsf{tP}} \vdash_{(\mathrm{d6})} \{(\mathrm{a10})\}$

Let $\mathcal{T}_{\mathsf{P}} = \{$(a1),(a6)–(a10),(pdn)$\}$, where O, TP, and tP are defined by (d1)–(d3).

(t6) shows that $\mathcal{T}_{\mathsf{P}}$ is at least as strong as $\mathcal{T}_{\mathsf{tP}}$, i.e., all the axioms in $\mathcal{T}_{\mathsf{tP}}$ can be proved in $\mathcal{T}_{\mathsf{P}}$ by assuming the same EX and the definition (d3) for tP.

(t7) and (t8) show that $\mathcal{T}_{\mathsf{P}}$ is strictly stronger than $\mathcal{T}_{\mathsf{tP}}$, because in $\mathcal{T}_{\mathsf{tP}}$ temporal parts (at a specific time) are not unique and different entities can coincide (see (t1) and (t2)).

[5]The graphical notation adopted follows four conventions: (i) the times at which an entity exists are subscribed between square brackets; (ii) an arc from a to b without labels stands for parthood, i.e., $\langle a,b\rangle \in \mathsf{P}^{\mathcal{I}}$; (iii) an arc from a to b with label t stands for temporary parthood at t, i.e., $\langle a,b,t\rangle \in \mathsf{tP}^{\mathcal{I}}$; (iv) all the arcs due to reflexivity and transitivity closure of parthood are omitted. For example the graph in figure 1.a depicts the following model: $D = \{a,b,c,t,t'\}$, $\mathsf{EX}^{\mathcal{I}} = \{\langle a,t\rangle, \langle b,t\rangle, \langle c,t'\rangle\}$, and $\mathsf{P}^{\mathcal{I}} = \{\langle a,a\rangle, \langle b,b\rangle, \langle c,c\rangle, \langle a,c\rangle, \langle b,c\rangle\}$.



(a) **(b)** **(c)**

Figure 2: Counterexamples to the transitivity of the temporary parthood.

t6 $\mathcal{T}_{\mathsf{P}} \vdash_{(\text{d3})} \mathcal{T}_{\mathsf{tP}}$

t7 $\mathcal{T}_{\mathsf{P}} \vdash_{(\text{d3})} \mathsf{tTP}yxt \wedge \mathsf{tTP}yzt \rightarrow y = z$

t8 $\mathcal{T}_{\mathsf{P}} \vdash_{(\text{d3})} \forall t(\mathsf{EX}xt \rightarrow \mathsf{tP}xyt) \wedge \forall t(\mathsf{EX}yt \rightarrow \mathsf{tP}yxt) \rightarrow x = y$

To find a theory based on P equivalent to $\mathcal{T}_{\mathsf{tP}}$ it is then necessary to weaken $\mathcal{T}_{\mathsf{P}}$.

t9 $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a8)}\} \nvdash_{(\text{d3})}$ (a4)

t10 $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a9)}\} \nvdash_{(\text{d3})}$ (a4)

t11 $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a10)}\} \nvdash_{(\text{d3})}$ (a4)

t12 $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a7)}\} \vdash_{(\text{d3})} \mathcal{T}_{\mathsf{tP}}$

t13 $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a7)}\} \nvdash \mathsf{TP}yxt \wedge \mathsf{TP}zxt \rightarrow y = z$

t14 $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a7)}\} \nvdash_{(\text{d3})} \mathsf{tTP}yxt \wedge \mathsf{tTP}zxt \rightarrow y = z$

t15 $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a7)}\} \nvdash_{(\text{d3})} \forall t(\mathsf{EX}xt \rightarrow \mathsf{tP}xyt) \wedge \forall t(\mathsf{EX}yt \rightarrow \mathsf{tP}yxt) \rightarrow x = y$

t16 $\mathcal{T}_{\mathsf{tP}} \vdash_{(\text{d6})} \mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a7)}\}$

(t9), (t10), and (t11) show that weakening $\mathcal{T}_{\mathsf{P}}$ by respectively dropping the transitivity, the extensionality or the 'temporal monotonicity' of P lead to a too weak theory in which the transitivity of the temporary parthood (defined via (d3)) does not hold: figures 2.a, 2.b, and 2.c respectively illustrate a model of $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a8)}\}$, $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a9)}\}$, and $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a10)}\}$ in which $\langle a, b, t\rangle, \langle b, c, t\rangle \in \mathsf{tP}^{\mathcal{I}}$ but $\langle a, c, t\rangle \notin \mathsf{tP}^{\mathcal{I}}$ (in figure 2.a, the curved arrow on the left makes explicit that in this case the transitivity closure is not valid, i.e. we have $\langle a, c\rangle \notin \mathsf{P}^{\mathcal{I}}$).

(t12)–(t15) show that, dropping the antisymmetry of parthood, the embedding is maintained but the uniqueness of TP and tTP does not holds and it is possible to have different coincident objects. As a counterexample, let us consider: $\mathsf{EX}^{\mathcal{I}} = \{\langle a, t\rangle, \langle b, t\rangle\}$, $\mathsf{P}^{\mathcal{I}} = \{\langle a, a\rangle, \langle b, b\rangle, \langle a, b\rangle, \langle b, a\rangle\}$.

(t16) shows that $\mathcal{T}_{\mathsf{tP}}$ can be embedded in $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a7)}\}$ via (d6). In addition, it is possible to prove that expanding the definition of P in terms of the vocabulary of $\mathcal{T}_{\mathsf{tP}}$, and, successively expanding the formula obtained using the definition of tP given in $\mathcal{T}_{\mathsf{P}}$, we re-obtain P; similarly starting from the expansion of the definition of tP in terms of the vocabulary of $\mathcal{T}_{\mathsf{P}}$. Therefore $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a7)}\}$ and $\mathcal{T}_{\mathsf{tP}}$ are *equivalent*.

It is also possible to strengthen $\mathcal{T}_{\mathsf{tP}}$ via (a11) (an axiom that directly corresponds to the antisymmetry of P) to prove the equivalence between $\mathcal{T}_{\mathsf{P}}$ and $\mathcal{T}_{\mathsf{tP}} \cup \{\text{(a11)}\}$.

a11 $\forall t(\mathsf{EX}xt \rightarrow \mathsf{tP}xyt) \wedge \forall t(\mathsf{EX}yt \rightarrow \mathsf{tP}yxt) \rightarrow x = y$

The two equivalences and the theorems (t1) and (t2) show that the main difference between $\mathcal{T}_{\mathsf{tP}}$ and $\mathcal{T}_{\mathsf{P}}$ concerns the uniqueness of the temporal parts and the acceptance of the *coincidence* of different objects (different objects that are one part the other during their whole life).

These topics have been deeply discussed in the literature on (material) *constitution* (see (Rea 1997) for a good review). According to (a11), if, for example, the clay that constitutes a statue and the statue itself are different, they cannot coincide during their whole life (even though the distinction is based on a difference in modal behavior).[6] From my point of view, this represents a genuine difference between perdurantism and endurantism. While perdurantists, identifying coincidence with identity, tend to reduce differences among objects to mereological ones (in particular spatio-temporal ones), endurantists tend to accept coincidence between different objects motivating this distinction by, not necessarily mereological, different temporary property. While perdurantists have a multiplicative approach towards parts, endurantists have a multiplicative approach towards coincident objects.

Avoiding temporal parts

In this section, I introduce an alternative definition of temporary parthood in terms of parthood and I show which existential conditions are necessary to embed the theory based on parthood in the one based on temporary parthood.

Let us start observing that, as showed by (t17) and (t18), the equivalence between $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(a7)}\}$ and $\mathcal{T}_{\mathsf{tP}}$ and the one between $\mathcal{T}_{\mathsf{P}}$ and $\mathcal{T}_{\mathsf{tP}} \cup \{\text{(a11)}\}$ both rely on the existence of temporal parts.

t17 $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(pdn)}\} \nvdash_{(\text{d3})}$ (a3)

t18 $\mathcal{T}_{\mathsf{tP}} \cup \{\text{(a11)}\} \smallsetminus \{\text{(pd)}\} \nvdash_{(\text{d6})}$ (a9)

The situation in figure 1.b is a model of $\mathcal{T}_{\mathsf{P}} \smallsetminus \{\text{(pdn)}\}$ in which $\langle c, t\rangle \in \mathsf{EX}^{\mathcal{I}}$ and $\langle c, c, t\rangle \notin \mathsf{tP}^{\mathcal{I}}$ (because c has no temporal parts).

$\mathsf{EX}^{\mathcal{I}} = \{\langle a, t\rangle, \langle b, t\rangle, \langle b, t'\rangle\}$, $\mathsf{tP}^{\mathcal{I}} = \{\langle a, a, t\rangle, \langle b, b, t\rangle, \langle a, b, t\rangle, \langle b, a, t\rangle\}$ is a model of $\mathcal{T}_{\mathsf{tP}} \cup \{\text{(a11)}\} \smallsetminus \{\text{(pd)}\}$ in which $\langle b, a\rangle \notin \mathsf{P}^{\mathcal{I}}$ but, because a is part of itself and it is also part of b, both a and b overlap a, i.e. $\langle a, a\rangle, \langle a, b\rangle \in \mathsf{O}^{\mathcal{I}}$. This situation fails to satisfy (a9) because the only part (simpliciter) of b different from b (a *proper* part of b) is a that does not exists at t'. Therefore (a5) does not introduce any new object because it applies neither at t (a and b coincide at t) nor at t' (only b exists at t'). (pd) allows to prove (a9) by introducing the temporal part of b at t'.

According to (t17), by refusing (pdn), endurantists cannot accept (d3) as a general definition of tP in terms of P. In the following, I propose an alternative definition that commits to existential conditions weaker than (pdn). More specifically, I consider an *extensional closure mereology* (Casati and Varzi 1999) extended just with (a10), i.e. the theory

$\mathcal{T}^{c}_{\mathsf{P}} = \{\text{(a1), (a6)–(a10), (a12), (a13)}\}$,

[6] Interpreting parthood as spatio-temporal inclusion, (a11) excludes the possibility of having spatio-temporally co-located entities.

where SUM (SUMsxy stands for "s is a sum of x and y") and DIF (DIFdxy stands for "d is a difference between x and y") are defined by (d7)–(d8). Note that to avoid 'empty objects', according to (a13), the difference between x and y exists only in case x is not part of y.

d7 $\mathsf{SUM}sxy \triangleq \forall z(\mathsf{O}zs \leftrightarrow \mathsf{O}zx \vee \mathsf{O}zy)$

d8 $\mathsf{DIF}dxy \triangleq \forall z(\mathsf{P}zd \leftrightarrow \mathsf{P}zx \wedge \neg \mathsf{O}zy)$

a12 $\exists s(\mathsf{SUM}sxy)$

a13 $\neg \mathsf{P}xy \rightarrow \exists d(\mathsf{DIF}dxy)$

(d9) is my alternative to (d3). Informally, (d9) may be explained in the following way: let us suppose that both x and y exist at t, then x is part of y at t if and only if (i) x is part of y at every time at which it exists (and therefore, in particular, at t); or (ii) if x is part of y only during a part of its life (the life of x), then this part of life includes t. The condition (ii) can be restated: if x is not part of y at t (and x exists at t), then the difference between x and y exists at t because some parts of x that exist at t are not part of y.

(t19) allows for interpreting parthood as constant part.

d9 $\mathsf{tP}xyt \triangleq \mathsf{EX}xt \wedge \mathsf{EX}yt \wedge (\mathsf{P}xy \vee \exists d(\mathsf{DIF}dxy \wedge \neg \mathsf{EX}dt))$[7]

t19 $\mathcal{T}^c_{\mathsf{P}} \vdash_{(\mathrm{d9})} \mathsf{P}xy \leftrightarrow \forall t(\mathsf{EX}xt \rightarrow \mathsf{tP}xyt)$

t20 $\mathcal{T}^c_{\mathsf{P}} \nvdash_{(\mathrm{d9})}$ (a4)

(t20) shows that $\mathcal{T}^c_{\mathsf{P}}$ is too weak. Figure 3 depicts[8] a situation in which $\langle A, B\rangle, \langle B, C\rangle \in \mathsf{tP}^{\mathcal{I}}$ but $\langle A, C\rangle \notin \mathsf{tP}^{\mathcal{I}}$. To understand why, let us note that $\langle a, A, B\rangle$, $\langle b, B, C\rangle$, $\langle A, A, C\rangle \in \mathsf{DIF}^{\mathcal{I}}$, but only A (that is the only difference between A and C) exists at t and this fact prevents the possibility of having $\langle A, C, t\rangle \in \mathsf{tP}^{\mathcal{I}}$. Notice that A exists at t even though all its proper parts (a and b) exist only at t'.

Therefore to embed $\mathcal{T}^c_{\mathsf{P}}$ in $\mathcal{T}_{\mathsf{tP}} \cup \{(\mathrm{a11})\} \smallsetminus \{(\mathrm{pd})\}$ we need to strengthen $\mathcal{T}^c_{\mathsf{P}}$. (t22), (t23) and (t24) show that (a14) (from which (t21) follows directly) does the job without committing to the existence of temporal parts. A situation with only one object that exists at two different times is a simple counterexample to both (pdn) and (pd), but it is possible also to build complex counterexamples following the situations in figure 4.

a14 $\mathsf{DIF}dxy \wedge \mathsf{EX}xt \wedge \neg \mathsf{EX}yt \rightarrow \mathsf{EX}dt$

t21 $\mathsf{SUM}sxy \wedge \mathsf{EX}st \rightarrow (\mathsf{EX}xt \vee \mathsf{EX}yt)$

t22 $\mathcal{T}^c_{\mathsf{P}} \cup \{(\mathrm{a14})\} \vdash_{(\mathrm{d9})} \mathcal{T}_{\mathsf{tP}} \cup \{(\mathrm{a11})\} \smallsetminus \{(\mathrm{pd})\}$

t23 $\mathcal{T}^c_{\mathsf{P}} \cup \{(\mathrm{a14})\} \nvdash$ (pdn)

t24 $\mathcal{T}^c_{\mathsf{P}} \cup \{(\mathrm{a14})\} \nvdash_{(\mathrm{d9})}$ (pd)

Without committing to temporal parts, $\mathcal{T}^c_{\mathsf{P}} \cup \{(\mathrm{a14})\}$ and the definition (d9) offers endurantists the possibility to choose parthood simpliciter as primitive, informally reading this relation as 'constant parthood' which, in my understanding, does not violate any endurantist principle.

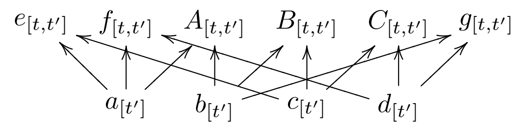


Figure 3: Counterexample to the transitivity of tP.

(a) **(b)**

Figure 4: Counterexamples to (pd).

It is clear that the equivalence between $\mathcal{T}^c_{\mathsf{P}} \cup \{(\mathrm{a14})\}$ and $\mathcal{T}_{\mathsf{tP}} \cup \{(\mathrm{a11})\} \smallsetminus \{(\mathrm{pd})\}$ cannot be proved. Strongly, (t18) shows that $\mathcal{T}_{\mathsf{tP}} \cup \{(\mathrm{a11})\} \smallsetminus \{(\mathrm{pd})\}$ is too weak to prove the extensionality of P via (d6).

This last problem can be solved by substituting {(pd), (a5)} with (a15): $\mathcal{T}^n_{\mathsf{tP}} = \{(\mathrm{a1})\text{–}(\mathrm{a4}), (\mathrm{a15})\}$. (t25), (t26), and (t27) show $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\mathrm{a11})\}$ does not commit to temporal parts but it is strong enough to 'recover' the extensionality of P. In addition, (t28) shows that $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\mathrm{a11})\}$ is not too strong with respect to $\mathcal{T}^c_{\mathsf{P}} \cup \{(\mathrm{a14})\}$.

a15 $\mathsf{EX}xt \wedge \neg \mathsf{tP}xyt \rightarrow \exists z(\mathsf{tP}zxt \wedge \forall t'(\neg \mathsf{tO}zyt'))$

t25 $\mathcal{T}^n_{\mathsf{tP}} \vdash$ (a5)

t26 $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\mathrm{a11})\} \nvdash$ (pd)

t27 $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\mathrm{a11})\} \vdash_{(\mathrm{d6})} \mathcal{T}_{\mathsf{P}} \smallsetminus \{(\mathrm{pdn})\}$

t28 $\mathcal{T}^c_{\mathsf{P}} \cup \{(\mathrm{a14})\} \vdash_{(\mathrm{d9})} \mathcal{T}^n_{\mathsf{tP}} \cup \{(\mathrm{a11})\}$

Figure 4 depicts two counterexamples to (pd) in $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\mathrm{a11})\}$. The example in figure 4.a does not satisfy the 'maximality' imposed to temporal parts, while in the example in figure 4.b the fact that objects need to have a temporal part at any time at which they exist does not hold. However, the existential commitment imposed by (a15) is inevitably stronger than the one imposed by (a5).

However, the embedding of $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\mathrm{a11})\}$ in $\mathcal{T}^c_{\mathsf{P}} \cup \{(\mathrm{a14})\}$ does not hold, i.e. $\mathcal{T}^c_{\mathsf{P}} \cup \{(\mathrm{a14})\}$ is strictly stronger than $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\mathrm{a11})\}$. In particular, the existential commitment provided by (a15) is too weak to guarantee the existence of the difference (a13). Informally, (a13) requires that (i) the difference between x and y is a part of x and (ii) that all the objects that are part of x and do not overlap y are part of the difference. These conditions are not imposed by (a15). The example in figure 5 does not satisfy the above condition (ii) but it satisfies (a15) (and all the other axioms of $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\mathrm{a11})\}$): at t, a is not part of b, but c satisfies the constraint in (a15). At t, a is not part of c, but b satisfies the constraint in (a15). At t', a is neither part of b nor c, but d satisfies the constraint in (a15) in both cases. Because a is present at t but, at this time, a is not part of b, then by (d6), a is not part simpliciter of b, then the hypothesis of (a13) is

[7] In an extensional closure mereology, if x is not part of y then the difference exists and it is *unique*, therefore (d9) is equivalent to $\mathsf{tP}xyt \triangleq \mathsf{EX}xt \wedge \mathsf{EX}yt \wedge (\neg \mathsf{P}xy \rightarrow \forall d(\mathsf{DIF}dxy \rightarrow \neg \mathsf{EX}dt))$.

[8] For the sake of conciseness, in the figure are reported only the sums of couples of atomic objects. The graph needs to be completed with the sums of three and four atomic objects that however are not relevant for the proof of (t20).

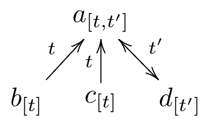


Figure 5: Counterexample to (a13).

satisfied and the existence of the difference between a and b must be proved. The only candidates for the difference are c and d. c cannot be the difference because d is part of a at t', d does not overlap b at any time, but d is not part of c at any time. d cannot be the difference because c is part of a at t, c does not overlap b at any time, but c is not part of d at any time.

In addition to that, in $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\text{a11})\}$ nothing guarantees the existence of sums, for example models with two objects, one existing only at t, the other one existing only at t' are not ruled out.

Conclusions and further work

In this work I studied some interconnections between theories based on parthood simpliter and theories based on temporary parthood. I showed that, to build a theory based on parthood simpliciter equivalent to the theory of Sider, the antisymmetry of parthood cannot be included. I analyzed how this result can explain some divergences between endurantism and perdurantism. In addition to that, theorem (t19) and theorems (t22)–(t24), together with (d9), make explicit the possibility to have a characterization of temporary parthood in terms of parthood simpliciter that, without committing to the existence of temporal parts, may be accepted by endurantists (at least from my point of view).

Some formal results are still lacking. In particular I do not know how the theory $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\text{a11})\}$ can be extended in order to prove the equivalence with $\mathcal{T}^c_{\mathsf{P}} \cup \{(\text{a14})\}$. A straightforward possibility consists in adding to $\mathcal{T}^n_{\mathsf{tP}} \cup \{(\text{a11})\}$ the analogue of axioms (a12) and (a13), but the proof of equivalence with $\mathcal{T}^c_{\mathsf{P}} \cup \{(\text{a14})\}$ is not trivial. Another open problem concerns the independence of the existence of differences from (a15) plus the existence of sums: in presence of (a15), is the existence of sums enough to guarantee the existence of differences?

Finally, I think that, at least in the case of applications, one of the main motivations to follow a perdurantist approach concerns the possibility to reduce the predication of a property an object has at t, to the predication on the temporal part of the object at t. By avoiding temporal parts, my analysis does not provide any alternative to this reduction. I think that a possible alternative compatible with the endurantist view is offered by *trope theory* (see (Daly 1997) for a good survey) that conceives change as trope substitution. But this theory, that in any case commits to a new kind of objects called tropes, requires other basic primitives notions as *inherence* and *resemblance* that cannot be grasped only in terms of parthood.

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