# Qualities in Formal Ontology

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**Abstract.** We characterize and compare four different ways of representing qualities in formal ontology. Our goal is to discuss their ontological commitments and their adequacy in applications. We also show how the frameworks here presented relate to other approaches in ontology (trope theory), in cognitive science (conceptual spaces), and in physics (International System of Units). The work we present focuses on ontological construction; we do not discuss issues specifically related to measurements, metrics and the like.

## 1 Introduction

It is hard to conceive ontology and, more generally knowledge representation, without thinking about conceptualization and representation of endurants (objects like chairs and cars) and perdurants (events like driving and sneezing). People differentiate these entities because of a variety of aspects and characteristics, hereafter called *qualities*, that can be recognized and classified like color, weight or duration. Nonetheless, when asked to list the qualities of an endurant, we cannot do more than listing a few of them (like shape, color, size, weight, temperature, duration, smell and so on). This observation is surprising if we consider the crucial relevance qualities have in our life.

However, the lack of a set of qualities on which people agree upon is not the only deficiency in this area and, perhaps, not even the most important. Indeed, the research community has not yet isolated a systematic and ontologically sound framework to compare and analyze qualities. There are a few approaches (essentially based on the notions of *individual* quality, *trope* and *property*), but their systematic comparison from the formal and ontological perspective has not been carried out.

This paper gives a contribution in this area by formally presenting a set of frameworks and by discussing their relationships and ontological commitments. These frameworks build on a variety of entities like

- *Individual qualities*, e.g. "the weight of John". Individuals qualities *inhere in* specific individuals, that is, "the color of John" is different from "the color of Mary", and they can change through time since "the color of John" can match color *red* today and color *rose* tomorrow.
- *Qualia*, e.g. a specific color. These entities are obtained by abstracting individual qualities from time and from their hosts. If the color of John and the color of Mary match the same shade of red, then they have the same (color)

quale. In this sense qualia represent perfect and "objective" similarity between (aspects of) objects.
– *Regions* and *spaces*. These entities corresponds to different ways of "organizing" qualia. They are motivated by "subjective" (context dependent, qualitative, etc.) similarity between (aspects of) objects. By means of spaces, a structure can be imposed on qualia (for example a geometry or a topology) and this makes it possible to differentiate several quantitative and qualitative degrees of similarity.

We refer to [1] for a deeper discussion of the notions of quality and quale. Although the presentation there develops within the framework of the DOLCE ontology (Descriptive Ontology for Linguistic and Cognitive Engineering), our paper does not commit (nor is limited to) that specific ontology.

Sections 3 to 6 present and discuss four different (yet related) approaches to quality representation. In section 7 we analyze the ontological nature of qualia and we make explicit the link with approaches based on *tropes* and *universals*. We will see that all these approaches are comparable in expressive power but differ in their *ontological commitment*. It is then important to understand in which cases it is better to choose one approach rather than another one. In sections 8 and 9 we analyze the *adequacy* of these approaches with respect to the theory of conceptual spaces [2] and the International System of Units[1].

## 2 Focus and basic notions

In this work we concentrate only on qualities of *endurants*, i.e. qualities of entities that are *wholly* present at any time they are present, e.g. a car, Einstein, the K2, a law, some gold, etc.[2]

The approaches we present are founded on the following (formal) basic notions and distinctions:

– *Parthood.* $\mathsf{P}(x, y)$ stands for "$x$ is part of $y$". We assume a classical extensional mereology ($\mathsf{CEM}$) (see [3, 4] for the axioms) defined only in restricted domains that we will make explicit in the following. In addition, the classical definition of $\mathsf{PP}$ is considered.
– *Endurants.* $ED(x)$ stands for "$x$ is an endurant".
– *Time intervals or instants.* $T(x)$ stands for "$x$ is a time interval or instant (briefly a time)". Our presentation does not commit to a specific notion of time even if we introduce the parthood relation in $T$.

[1] See http://www.physics.nist.gov/cuu/Units/introduction.html
[2] We think that, with small changes, the frameworks introduced below can be applied also to *perdurants* (entities that are only *partially* present at any time they are present like a process evolving in time). We do not consider these changes in this paper.

– *Being present.* $\mathsf{PRE}(x,t)$ stands for "$x$ is present (exists) during the time $t$". In the case of an endurant, we require that there exists a time during which the endurant is present.

(A1) $\mathsf{PRE}(x,t) \to T(t)$
(A2) $ED(x) \to \exists t(\mathsf{PRE}(x,t))$

– We assume that all temporal relations we introduce enjoy the dissectivity property, that is, given relations $A(x,t)$ and $B(x,y,t)$, where $t$ is a time, we assume

$$A(x,t) \to \forall t'(\mathsf{P}(t',t) \to A(x,t'))$$
$$B(x,y,t) \to \forall t'(\mathsf{P}(t',t) \to B(x,y,t'))$$

in particular,

(A3) $\mathsf{PRE}(x,t) \to \forall t'(\mathsf{P}(t',t) \to \mathsf{PRE}(x,t'))$

– Finally, we assume that the categories/types/domains introduced (for example $ED$ and $T$) are all disjoint.

## 3 Endurants, Qualities, Qualia, and Spaces (EQQS)

We begin our analysis with the more sophisticated system we are going to consider. New classes of entities are introduced among which *qualities* and *qualia*.

– $QT(x)$ stands for "$x$ is a quality". Qualities are partitioned into $n$ non-empty subtypes: $QT_1, \ldots, QT_n$. Thus, given a quality $x$ there is an index $i$ such that $QT_i(x)$.
– $QL(x)$ stands for "$x$ is a quale". Qualia are also partitioned into $n$ non-empty subtypes: $QL_1, \ldots, QL_n$.[3] Analogously to the case of qualities, from $QL(x)$ it follows that there is an index $i$ such that $QL_i(x)$. We assume the $\mathsf{CEM}$ in each $QL_i$.
– For $1 \leq i \leq n$, fix $m_i\,(m_i \geq 1)$ non-empty spaces that we indicate by $S_i^1, \ldots, S_i^{m_i}$. As working thypothesis, we assume $\mathsf{CEM}$ holds in each $S_i^j$, but a richer structure can be added (e.g. topological or geometrical properties). Indeed, some spaces may be atomic and others atomless; some may satisfy complex mathematical properties and others just basic mereology.
Entities in a space $S_i^j$ are called *regions*. We write $RG(x)$ to mean "there are $i, j$ such that $x$ is in space $S_i^j$", in addition, spaces can be regrouped in *generalized spaces*, we write $GS_i(x)$ to mean "there is a $j$ such that $x$ is in space $S_i^j$". Formally

(D1) $RG(x) \triangleq \bigvee_{i,j} S_i^j(x)$

[3] Axiom (A7), see below, assures a correspondence between the $QT_i$ and the $QL_i$.

(D2) $GS_i(x) \triangleq \bigvee_j S_i^j(x)$

The mereological sum is defined within each space $S_i^j$ but not across spaces. In particular, a region belongs to exactly one space $S_i^j$ (for some $i, j$).

Now we look at the relationships among the entities so far introduced.

We say that a quality $x$ *inheres in* an endurant $y$, formally $\mathsf{inh}(x, y)$, if $y$ is the host of quality $x$. The relationship between qualities and qualia is dubbed *abstract* (abs). Expression $\mathsf{abs}(x, y, t)$ stands for "$x$ is the quale of quality $y$ at time $t$". Abstraction is a ternary relation since the quality of endurants may vary over time (e.g. a color might fade) and so the matching between quality and qualia is time-dependent. Informally, abs captures a form of relative identity among qualities in the sense that if both $\mathsf{abs}(x, y, t)$ and $\mathsf{abs}(x, y', t)$ hold, then qualities $y$ and $y'$ (if different) can have only one distinguishing characteristic at time $t$: they inhere in different hosts. Finally, regions are interpreted as qualia *positions* ($\mathsf{pos_{QL}}$) in a space. We write $\mathsf{pos_{QL}}(x, y)$ to mean "$x$ is a position of the quale $y$."

*Domain restrictions*
(A4) $\mathsf{inh}(x, y) \rightarrow QT(x) \wedge ED(y)$
(A5) $\mathsf{abs}(x, y, t) \rightarrow QL(x) \wedge QT(y) \wedge T(t)$
(A6) $\mathsf{pos_{QL}}(x, y) \rightarrow RG(x) \wedge QL(y)$
*Correspondences*
(A7) $\mathsf{abs}(x, y, t) \rightarrow \bigwedge_i (QL_i(x) \leftrightarrow QT_i(y))$
(A8) $\mathsf{pos_{QL}}(x, y) \rightarrow \bigwedge_i (\bigvee_j S_i^j(x) \leftrightarrow QL_i(y))$

**Notation.** Given a relation $A$ and a predicate $B$, we will write $A(x|B, y)$ for $A(x, y) \wedge B(x)$. If $A$ is ternary, then $A(x|B, y, z)$ stands for $A(x, y, z) \wedge B(x)$.

### 3.1 (Direct) Inherence

Each quality inheres in (has) a unique host that is an endurant (A12)+(A9), and it is present as long as its host is present (A11)[4]. In addition an endurant cannot have more than one quality of type $i$ (A10).

(A9) $\mathsf{inh}(x, y) \wedge \mathsf{inh}(x, y') \rightarrow y = y'$
(A10) $\mathsf{inh}(x|QT_i, y) \wedge \mathsf{inh}(x'|QT_i, y) \rightarrow x = x'$
(A11) $\mathsf{inh}(x, y) \rightarrow \forall t(\mathsf{PRE}(x, t) \leftrightarrow \mathsf{PRE}(y, t))$
(A12) $QT(x) \rightarrow \exists y(\mathsf{inh}(x, y))$

### 3.2 Abstraction

Qualities are mapped to qualia only when they are present (A13), actually, they are necessarily mapped to qualia when present (A15)[5]. Given a time $t$, a quality is

[4] Here it is assumed that PRE is defined on qualities.
[5] The existential condition on $t'$ is introduced in order to avoid a commitment on temporal atoms. This axiom (together with CEM and (A3)) assures that the temporal extension of a quality can be

mapped to only one quale (A14). Also, recall that $\mathsf{abs}$, being a temporal relation, is dissective. So far we have seen that relation $\mathsf{PRE}$ is defined over endurants and qualities. We do not define it over qualia since qualia are atemporal.

(A13) $\mathsf{abs}(x, y, t) \to \mathsf{PRE}(y, t)$
(A14) $\mathsf{abs}(x, y, t) \wedge \mathsf{abs}(x', y, t) \to x = x'$
(A15) $QT(x) \wedge \mathsf{PRE}(x, t) \to \exists y, t'(\mathsf{P}(t', t) \wedge \mathsf{abs}(y, x, t'))$

### 3.3 (Exact) Position

A quale is associated to at most one position (region) in one space (A16) and it has a position in every space associate to its quale type (A17). These spaces provide a structure to compare and evaluate qualia and, indirectly, the corresponding qualities.

(A16) $\mathsf{pos_{QL}}(x|S_i^j, y) \wedge \mathsf{pos_{QL}}(x'|S_i^j, y) \to x = x'$
(A17) $QL_i(x) \to \bigwedge_j(\exists y(\mathsf{pos_{QL}}(y|S_i^j, x)))$
(T1) $\mathsf{pos_{QL}}(x, y) \wedge (\mathsf{PP}(x', x) \vee \mathsf{PP}(x, x')) \to \neg\mathsf{pos_{QL}}(x', y)$ (from (A16))

**Refinement.** On the basis of the relation $\mathsf{pos_{QL}}$, one can define a *refinement* relation ($\mathsf{refin}$) between regions in different spaces (provided these refer to the same quality type) and extend it to spaces themselves ($\mathsf{refin_S}$).[6]

(D3) $\mathsf{refin}(i, x, j, y, k) \triangleq S_i^j(x) \wedge S_i^k(y) \wedge \forall z(\mathsf{pos_{QL}}(x, z) \to \mathsf{pos_{QL}}(y, z)) \wedge \forall z(\mathsf{pos_{QL}}(y, z) \to \exists w(\mathsf{pos_{QL}}(x, w)))$
(region $x$ is a refinement of region $y$)
(D4) $\mathsf{refin_S}(i, j, k) \triangleq \forall y(S_i^k(y) \to \exists x(\mathsf{refin}(i, x, j, y, k)))$
(space $S_i^j$ is a refinement of space $S_i^k$)
(T2) $\mathsf{refin_S}(i, j, k) \to \neg\exists x, y, y', a, b(\mathsf{pos_{QL}}(x|S_i^j, a) \wedge \mathsf{pos_{QL}}(x|S_i^j, b) \wedge \mathsf{pos_{QL}}(y|S_i^k, a) \wedge \mathsf{pos_{QL}}(y'|S_i^k, b) \wedge y \neq y')$
(from (A16), (D3), and (D4))

## 4 Endurants, Qualities, and Spaces (EQTS)

We have seen that qualities depend on their host and, informally, capture one single aspect of the host like color, weight, shape and the like. Furthermore, two qualities are clustered together in a quale whenever their hosts are *identical relatively to that aspect.* Since this clustering of qualities into qualia is preserved in the spaces $S_i^j$, perhaps one can discharge qualia altogether without losing information and expressive power. In this section we pursue this second approach.

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divided into a number of times during which the quale associated to the quality does not change. No mereological assumption is imposed on these times.

[6] If spaces are built out of atoms, relation $\mathsf{refin_S}$ is a partial order.

Let us consider again the categories $ED$, $T$, $QT$, $QT_1, \ldots, QT_n$ and the spaces $S_i^1, \ldots, S_i^{m_i}$ (for $1 \le i \le n$) as given before. The relation $\mathsf{inh}$ and the predicate $RG$ are characterized as in the previous section.

### 4.1 Temporalized position of qualities

In order to capture the changes in the qualities of endurants, the *position* relation involves now a temporal parameter. We call this the *temporalized position of qualities* ($\mathsf{pos_{QT}}$). Expression $\mathsf{pos_{QT}}(x, y, t)$ stands for "$x$ is a position of the quality $y$ at time $t$". The axiomatization of $\mathsf{pos_{QT}}$ reflects in part that of relations $\mathsf{abs}$ and $\mathsf{pos_{QL}}$ in the previous approach. In particular,

(A19) corresponds to (A8); (A20) to (A16); axioms (A21) to (A13); (A22) to (A15).

*Domain restriction*
(A18) $\mathsf{pos_{QT}}(x, y, t) \to RG(x) \land QT(y) \land T(t)$
*Correspondence*
(A19) $\mathsf{pos_{QT}}(x, y, t) \to \bigwedge_i (\bigvee_j S_i^j(x) \leftrightarrow QT_i(y))$
*Other constraints*
(A20) $\mathsf{pos_{QT}}(x|S_i^j, y, t) \land \mathsf{pos_{QT}}(x'|S_i^j, y, t) \to x = x'$
(A21) $\mathsf{pos_{QT}}(x, y, t) \to \mathsf{PRE}(y, t)$
(A22) $QT_i(x) \land \mathsf{PRE}(x, t) \to \bigwedge_j (\exists y(\mathsf{pos_{QT}}(y|S_i^j, x, t)))$

Also recall that, being temporal, $\mathsf{pos_{QT}}$ has the dissective property.

### 4.2 Are qualia necessary?

We changed the EQQS system of section 3 in as much as needed to avoid the introduction of qualia. At this point, it is natural to ask if the new system EQTS is somewhat weaker than EQQS, that is, if there is some situation that we can capture in the latter system but not in the first.

In EQQS, a qualia identifies a sort of (temporary) equivalence relation among qualities: qualities that are indistinguishable unless we refer to their hosts, are all associated to the same quale. In a sense, qualia provide the ontological status of qualities: two endurants with qualities that match to the same quale are "the same" with respect to that quality type. Within this approach, two spaces $S_i^j$ and $S_i^k$ may be seen as different ways to organize the qualia in subclass $QL_i$ and, indirectly, to represent coarse granularities on the qualities in $QT_i$. This result is obtained by positioning different qualia in the same region.

But, beside identifying this ontological status of qualities, are qualia necessary to compare endurants or to evaluate them? Let us consider the case of refinement. Is it possible to define in EQTS a relation of refinement as done in EQQS? Taking into account the temporal parameter in $\mathsf{pos_{QT}}$, the definition of refinement on regions can be formulated as follows (on spaces, it suffices to take (D4) with $\mathsf{refin}$ as given below):

(D5) $\mathsf{refin}(i,x,j,y,k) \triangleq S_i^j(x) \wedge S_i^k(y) \wedge \forall z,t(\mathsf{pos_{QT}}(x,z,t) \rightarrow \mathsf{pos_{QT}}(y,z,t)) \wedge \forall z,t(\mathsf{pos_{QT}}(y,z,t) \rightarrow \exists w(\mathsf{pos_{QT}}(x,w,t)))$

As we have seen, in EQQS qualia provide the finest granularity in evaluating the category of qualities while spaces $S_i^j$ add extra conditions by further grouping qualia and by furnishing ordering, topological, or even metric relations. In EQTS we have only the spaces $S_i^j$ but it is still possible to define a notion of "maximal granularity" by imposing, for each quality type $QT_i$, the existence of a space that, according to definition (D4), refines all the spaces associated to $QT_i$. Let $\exists!k$ to mean "there exists a unique index $k$", then

(A23) $\bigwedge_i \exists!k(\bigwedge_j \mathsf{refin_S}(i,k,j))$

Let us write $S_i^*$ for the space isolated by axiom (A23). One sees that, with the introduction of $S_i^*$, EQQS and EQTS are equivalent in expressive power.[7] Thus, an equivalence between the two systems can be established formally through extra assumption (A23). Still, the two approaches differ ontologically as the extra category in EQQS shows. We will come back to this issue later.

## 5 Endurants, Qualia, and Spaces (EQLS)

We have seen that, under some hypotheses, dropping the category of qualia one maintains the same expressive power of EQQS. Do we reach the same result removing qualities instead of qualia?

Quality kinds are in a one-to-one correspondence with quale kinds, therefore it seems quite natural to use this correspondence to "bypass" qualities by: ($i$) introducing a (temporalized) *abstraction* relation between endurants and qualia ($\mathsf{abs_{ED}}$), and ($ii$) maintaining the *position* relation $\mathsf{pos_{QL}}$ already introduced in section 3. The axioms characterizing $\mathsf{abs_{ED}}$ follow closely those given for the other position relation, namely $\mathsf{pos_{QT}}$ (as before, we do not list dissectivity):

*Domain restriction*
(A24) $\mathsf{abs_{ED}}(x,y,t) \rightarrow QL(x) \wedge ED(y) \wedge T(t)$
*Other constraints*
(A25) $\mathsf{abs_{ED}}(x|QL_i,y,t) \wedge \mathsf{abs_{ED}}(x'|QL_i,y,t) \rightarrow x = x'$
(A26) $\mathsf{abs_{ED}}(x,y,t) \rightarrow \mathsf{PRE}(y,t)$

In a sense, qualities carve up particular aspects of an object. "The weight of John" is not comparable to a quale (the weight of John can change in time,

[7] The careful reader might observe that space $S_i^*$ could present topological or geometrical properties that cannot be present in the class $QL_i$. This is irrelevant if space $S_i^*$ is present in both systems.

the quale cannot; also it is specific of John while a quale can be contemporarily associated to several endurants). The quality combines John specificity and the weight kind (dimension), thus it can have properties that are not directly ascribable to a quale, nor to the positions of qualia in the spaces $S_i^j$. Let's consider, for example, the sentence "the weight of John is good now". It can be interpreted in at least two different ways: ($i$) the weight-quale associated to John at this moment is "good"; and ($ii$) the weight-quality of John is at this moment "good". In the first case, "good" is an absolute property since it applies to a quale, i.e., to an atemporal entity. If it happens that the weight of Mary is mapped to the same quale at the same time (or even at another time $t$), then necessarily "the weight of Mary is good now (at time $t$, respectively)" holds as well.

In the second case "good" is applied to the weight-aspect of John, which is specific to John. With this interpretation the expression "the weight of Mary is good now" may be false no matter where the weight of Mary is mapped now. Thus, one cannot capture the particular link between qualities and their hosts via qualia unless introducing a class of qualia-properties like "John goodness", "Mary goodness", etc. which seems ontologically unpalatable. There is an alternative solution though. It is possible to reconstruct qualities of kind $QT_i$ as couples[8] $\langle e, Q_i^{|e|} \rangle$, were $e$ is a specific endurant, and $Q_i^{|e|}$ is the set of all the qualia of kind $QL_i$ that are linked to $e$:

(D6) $Q_i^{|e|} = \{q \mid QL_i(q) \wedge \exists t(\mathsf{abs_{ED}}(x, e, t))\}$

Now, one can define $\mathsf{inh}$ and $\mathsf{abs}$ by taking $\langle e, Q_i^{|e|} \rangle$ as argument of $QT_i$ (the definition is given for quality subtypes; it is easily extended to the general case):

(D7) $\mathsf{inh}(\langle e, Q_i^{|e|} \rangle | QT_i, e')$ iff $e = e'$
(D8) $\mathsf{abs}(q|QL_i, \langle e, Q_i^{|e|} \rangle, t)$ iff $q \in Q_i^{|e|} \wedge \mathsf{abs_{ED}}(q, e, t)$

## 6 Endurants and Spaces (ES)

Can one reject qualities and qualia altogether? Here we take a step further and consider a system that adopts the category of endurants and the spaces $S_i^j$ only. In this approach, an endurant is positioned in the spaces which are associated to different aspects of that endurant. For example, an endurant for which color is defined will be related by a *position* relation to spaces $S_h^j$ where $h$ is the index of the spaces classifying colors. In the previous systems, the quality types $QT_i$ (or the quale types $QL_i$) determined the different aspects of an endurant. Now we need a new mechanism to capture this distinction. Typically, one would introduce an additional level of classes: on the one hand regions are partitioned in spaces,

[8] Another possibility is to introduce qualities as sums of an endurant and the qualia (of a specific kind) associated to it. In infinite domains this means to adopt *general extensional mereology* (GEM) in every $QL_i$. Note that GEM is stronger than CEM [3, 4].

on the other hand spaces are "clustered" in generalized spaces (see definition (D2)) with each generalized space capturing an aspect of the endurants. Then, it is possible to set a correspondence between these generalized spaces and the quality kinds of EQtS (or the qualia kinds of EQlS). That is, given the qualities types, let us say just color and length, one assumes that there exist the "color space" $GS_1$, which is the union of all the color spaces $S_1^j$ (recall that all the spaces $S_i^j$ are disjoint) and the "length space" $GS_2$, the union of all the length spaces $S_2^j$. $GS_1$ and $GS_2$ are what we called the generalized spaces. In short,

- In EQtS
  (a) there are $n$ non empty and disjoint sets of qualities $QT_1, \ldots, QT_n$;
  (b) each $QT_i$ is associated to the spaces $S_i^1, \ldots, S_i^{m_i}$.
- In EQlS
  (a) there are $n$ non empty and disjoint sets of qualia $QL_1, \ldots, QL_n$;
  (b) each $QL_i$ is associated to the spaces $S_i^1, \ldots, S_i^{m_i}$.
- In ES
  (a) there are $n$ non empty and disjoint generalized spaces $GS_1, \ldots, GS_n$;
  (b) each $GS_i$ is partitioned into the spaces $S_i^1, \ldots, S_i^{m_i}$.

### 6.1 Temporalized position of endurants

To link endurants and regions we consider a temporalized position relation over endurants ($\mathsf{pos_{ED}}$). The axiomatization of $\mathsf{pos_{ED}}$ is similar to the axiomatization of $\mathsf{pos_{QT}}$, the only major difference being axiom (A32) stating that an endurant positioned in a space at $t$, has always a position in that space:

*Domain restriction*
(A27) $\mathsf{pos_{ED}}(x, y, t) \rightarrow RG(x) \wedge ED(y) \wedge T(t)$
*Other constraints*
(A28) $\mathsf{pos_{ED}}(x|S_i^k, y, t) \wedge \mathsf{pos_{ED}}(x'|S_i^k, y, t) \rightarrow x = x'$
(A29) $\mathsf{pos_{ED}}(x, y, t) \rightarrow \mathsf{PRE}(y, t)$
(A30) $ED(x) \wedge \mathsf{PRE}(x, t) \rightarrow \bigvee_i (\exists y(\mathsf{pos_{ED}}(y|GS_i, x, t)))$
(A31) $ED(x) \wedge \exists y(\mathsf{pos_{ED}}(y|GS_i, x, t)) \rightarrow \bigwedge_j (\exists z(\mathsf{pos_{ED}}(z|S_i^j, x, t)))$
(A32) $ED(x) \wedge \exists y(\mathsf{pos_{ED}}(y|GS_i, x, t)) \wedge \mathsf{PRE}(x, t') \rightarrow \exists z(\mathsf{pos_{ED}}(z|GS_i, x, t'))$

### 6.2 Alternatives and expressive power

Instead of introducing generalized spaces $GS_i$, it is possible to consider a (binary) similarity relation between regions. The intended meaning of such a relation is "regions $x$ of $S_i^j$ and $y$ of $S_i^k$ are similar if they are positions of the same aspect of the same endurant". With this relation at our disposal, it is easy to reconstruct the sets $GS_i$.

Another possibility is the introduction of an additional parameter in $\mathsf{pos}_{\mathsf{ED}}$ that isolates the specific aspect we are considering. Technically this is not problematic but we need to introduce this additional category of entities that is ontologically obscure (there are no individual qualities but only "names" or "reification of kinds" of qualities).

Finally, we can take advantage of our previous work to reconstruct in ES the missing notions. In short, we can reconstruct qualia as regions in minimal spaces as done in EQTS, and following the strategy taken in EQLS we can reconstruct qualities (and the relations $\mathsf{inh}$ and $\mathsf{abs}$) as couples $\langle e, R_i^{|e|}\rangle$, where $e$ is an endurant and $R_i^{|e|}$ is the set of the positions that $e$ has (in time) in the minimal space $R_i^*$.

# 7 The nature of qualia

## 7.1 Qualia and tropes

The distinction between qualia and tropes (see [5] for a review on tropes) is important in ontology since these entities provide different ways to represent and explain qualitative change. In this paper we have adopted qualities as basic entities but this should not be considered as a rejection of tropes. Tropes supply a different view and can be formalized in a similar way. Here we discuss their advantages and drawbacks.

In trope theories, qualitative change is expressed in terms of *substitution* of tropes: when an endurant $a$ changes in time (say, it is red at time $t_1$ and yellow at time $t_2$), this means that a trope inherent in $a$ at $t_1$ disappears and a new trope is created. Thus, differently from qualities which are associated to different qualia over time, tropes do not change. It is their coming out and disappearing in time that explains the changes we observe in endurants: endurants change by acquiring some tropes while losing others. Tropes represent the different properties an endurant has at/during $t$ and, at each time, an endurant can possess only one trope for each property. We encountered a similar restriction on qualia: only one quale instance of a specific quale type is allowed at a time. Similarly, the tropes of the endurant at each time $t$ need to be instances of different (kinds of) universals [6]. Thus, the different tropes that inhere in the same endurant $a$ at time $t$ must be related to different aspects of the endurant.

In the theories EQQS and EQTS, qualities persist through change, i.e. the color of an endurant survives to the change from red to yellow and no entity disappears or is created. The change is represented by a relational change which explains why the aspects of the endurant vary in time.[9] Sometimes it is claimed that trope theory better explains changes in endurants since there is something

[9] In EQLS and in ES change is explained analogously. In this cases, the involved relations ($\mathsf{abs}_{\mathsf{ED}}$ and $\mathsf{pos}_{\mathsf{ED}}$, respectively) apply directly to endurants.

happening that motivates the change: a trope that was *in* the endurant is *substituted* by a new one (an explanation resembling a substitution of "parts").

Note that, from the point of view of expressiveness, there is no real difference in adopting qualities or tropes. This is not so if we look at the ontological nature of these entities.

It is possible to formalize the trope approach by taking the class of tropes to be partitioned in $n$ types[10] $TR_1, \ldots, TR_n$ and by associating $TR_i$ to the non-empty spaces $S_i^1, \ldots, S_i^{m_i}$. Then, it suffices to use the inherence and the (non temporalized) position relations. For lack of space, we do not list axioms. Anyway, the formalization is quite similar to the one given for direct inherence ($\mathsf{inh}$) and exact position ($\mathsf{pos_{QL}}$) in the EQQS approach. An important difference is about axiom (A10). For tropes, an additional condition constraining temporal coincidence of tropes is necessary, i.e. it is possible that two different tropes of the same type inhere in one endurant, but only at different times:

(D9) $x \sim_T y \triangleq \forall t(\mathsf{PRE}(x,t) \leftrightarrow \mathsf{PRE}(y,t))$
(A33) $\mathsf{inh}(x|TR_i, y) \wedge \mathsf{inh}(x'|TR_i, y) \rightarrow (x \sim_T y \leftrightarrow x = x')$

Note that, like qualia, two tropes with different positions in a space $S_i^j$ could be associated to the same position in another space $S_i^k$. Furthermore, tropes are not dependent on spaces.

Tropes are similar to qualia in as much as they provide the finest distinction on aspects of endurants but differently from qualia they are in time, and their temporal extension represents the time during which the property they "embody" is valid.

Finally, note that tropes are often related to a realistic approach in ontology which results in more constrained systems while, as we have seen, qualities and qualia do not force such a strong stand and their systems are quite flexible allowing the reconstruction of several notions starting from different sets of categories. From a practical point of view, this very fact can make a system based on tropes unfit for integration or interaction with systems that are less ontologically committed.

### 7.2 Qualia and their positions in spaces

The EQQS and EQtS approaches consider qualia as ontological markers of *aspect similarity*. This means that two endurants with qualities mapped (at one specific time) to the same quale are, at that time and limited to this quality type, ontologically indistinguishable. This is not a matter of empirical or epistemological equivalence: independently form the instruments or cognitive processes used to analyze the endurants, these endurants are identical with respect to that

[10] Formally, universals are here considered as predicates. One can reify universals in the domain and add an "instance-of" relation to capture the same notions [6].

particular aspect. As pointed out earlier, this is a case of *relative identity*. On the contrary, positions in spaces capture the empirical/epistemological level: the qualia can be organized, ordered, regrouped in very different ways depending on the space structure. Therefore each space supplies a "point of view" on qualia.

From a metaphysical viewpoint the distinction is quite interesting, but it loses importance when applications are considered. In this case, the analysis of the endurants is always conducted at an empirical or theoretical level. For example, in the case of engineering (domain) models, the available information on the domain and the available measurement instruments determine the spaces to consider. Any other (finer) distinction is irrelevant. This is also the case of scientific theories that aim to describe "reality": the spaces and their structures depend on the theories scientists are considering, and, to some extent, on the measurement methods they employ. Analogously, in cognitive science the spaces $S_i^j$ are built according to the behavior of subjects in experiments. We will come back to this in sections 8 and 9.

With or without qualia, the spaces $S_i^j$ and their structure can depend on $(i)$ culture (e.g. people in different societies classify colors, shapes, etc. in different ways); $(ii)$ instruments of investigation or scientific theories; $(iii)$ interpretations of experiments; etc. In general, they are created, adopted, and destroyed in time by (communities of) intentional agents. It follows that the spaces with their internal structure (mereological, topological, geometrical, metrical etc.) have a definite temporal extension, and therefore may or may not be present at a given time. Here we do not analyze this aspect further.

When qualia are part of the ontological framework, spaces can assume a more abstract role. We can see them as different "structures" that apply to any quale type along the lines of Klein's notion of geometry (in this case, a space is identified to a set of transformations and each class $QL_i$ furnishes a domain of application). We will not pursue this approach in this paper. Also, note that by introducing the category of qualia we gain uniformity across domains. If at time $t$ the color qualities of two endurants map to the same quale, then at $t$ their color qualities are indistinguishable in any domain, let it be psychology, astronomy, or linguistics. Neither a change of spaces $S_i^j$ nor the introduction of new measurement methods can affect this basic fact. On the other hand, this might be seen as a lack of flexibility in the system. Once the ontological level of qualia is characterized, the resulting ontology is not compatible with other ontologies (or frameworks) that adopt a larger set of qualia.

## 8 Qualities and conceptual spaces

In [2], Peter Gärdenfors models the "representations" used in cognitive science by introducing the notion of *conceptual space*. Conceptual spaces are collections of related *domains* each of which is a collection of (integral and separable) *dimensions* like, for example, temperature, weight, pitch, and brightness.

The theory of conceptual spaces is based on the notion of *similarity*: "Judgments of similarity (...) are central for a large number of cognitive processes. (...) such judgments reveal the dimensions of our perceptions and their structures" ([2], p.5). Dimensions correspond to "the different ways stimuli are judged to be similar or different" ([2], p.6), and, in this sense, they are taken to represent the various qualities of endurants. A point in a dimension may represent, for example, a particular temperature. Then, the association of two endurants to the same point represents the experimental fact that the two endurants are completely *similar* with respect to temperature. Points can be ordered (e.g. a tone can be "low" or "high") and it is generally assumed that each dimension is endowed with a mathematical structure: the level of similarity between stimuli is therefore embedded in the metric (or pseudo metric) relation defined on the dimensions. A set of dimensions is *integral* if an endurant that has a "position" inside one dimension, necessarily has a position inside all the other dimensions. For example, {*hue*, *brightness*} is integral because if an endurant has a particular hue it necessarily has also a particular brightness (and viceversa). A set of dimension is *separable* if it is not integral like {*hue*, *size*}.

In Gärdenfors terminology, domains are maximal sets of integral dimensions. For example the three-color dimensions *hue*, *chromaticness*, and *brightness* form a domain because the set {*hue*, *chromaticness*, *brightness*} is integral, but *hue*, *chromaticness* and *brightness* are separable from any dimension that does not belong to this set. Domains can be used to assign properties to endurants, i.e., to classify endurants: a particular property corresponds to a *region* in a domain. The separability constraint allows to assign properties (regions in a domain) independently from other properties (regions in other domains). This captures the experimental fact that the weight of an endurant is independent from the endurant's color.

Finally, conceptual spaces are defined as collections of one or more domains and concepts are represented as complex regions in conceptual spaces. A point in a conceptual space constraints the properties of endurants at the maximal level of detail.

Clearly, conceptual spaces are thought to be *theoretical entities* and they are "static in the sense that they only describe the *structure* of representations" ([2], p.31). Furthermore, they are often understood as part of a *relativistic approach*: their structure depends on the underlying culture, on measurement methods and sensors (in the case of scientific conceptual spaces), or on interpretation of the behavior of subjects (in the case of phenomenal conceptual spaces). This is another reason to conclude that conceptual spaces do not match the ontological import of qualities nor that of qualia: ontologically the theory of conceptual spaces seems close to the ES approach.

## 9 Qualities and the International System of Units

The NIST guide to the International System of Units (SI)[11] distinguishes between:

- a quantity in the *general* sense: a property ascribed to phenomena, bodies, or substances that *can* be quantified for, or assigned to, a particular phenomenon, body, or substance (e.g. *mass* and *electric charge*); and
- a quantity in the *particular* sense: a quantifiable or assignable property ascribed to a particular phenomenon, body, or substance (e.g. *the mass of the moon* and *the electric charge of the proton*).

Also, the SI introduces the notion of *physical quantity* as a quantity that can be used in the mathematical equations of science and technology. A *unit* is a particular physical quantity, defined and adopted by convention, with which other particular quantities of the same kind are compared. The result of this comparison is a number. The *value* (or magnitude) of a physical quantity is the "product" of a number (the numerical value) and an unit. For instance, we can represent the fact that the tower of Pisa is 55 meter high and that its weight is 14.453 tonnes by: $h_{PT} = 55\,m; w_{PT} = 14.453.000\,Kg$. This representation explicitly refers to the particular height ($h_{PT}$) and weight ($w_{PT}$) of the tower of Pisa. Of course, another building might have the "same" height-quantity (in the SI general sense), i.e., $h_b = 55\,m$. However, $h_b$ and $h_{PT}$ are different quantities in the *particular* sense. They are different because ascribed to different endurants and yet they have the same *value*: $55\,m$. The identity $h_{PT} = h_b$ in SI is understood as a shortcut for $value(h_{PT}) = value(h_b)$. In this sense, quantities in the SI *particular* sense are similar to our qualities. Furthermore, the notion of ascription is similar to that of inherence as captured by relation inh.

Unfortunately, SI does not give much information on how to interpret an expression like $55\,m = 180\,ft$. That is, one could take $55\,m$ and $180\,ft$ to be the *same* position in a given space (including both units) or to be two *distinct* positions in different spaces which are connected by some correspondence relation. The interpretation changes depending on the way one introduces spaces and, more specifically, on their relationship with units of measure. For instance, the two values $55\,m$ and $180\,ft$ must be considered ontologically different entities if we take a value to be the combination of a numerical value (55 in one case, 180 in the other) and a unit (rispectively, $m$ and $ft$) since, clearly, their components *are* different. Alternatively, one can say that there is only one space (and so one position) and that the two units simply provide different ways to identify the unique position in that space.

The SI considers a set of *base* (general) physical quantities (length, mass, time, electric current, thermodynamic temperature, amount of substance, and

[11] http://physics.nist.gov/cuu/index.html

luminous intensity) in terms of which all the other physical quantities can be expressed by means of an equation[12]. (An similar equation is given for units.) For example, velocity is expressed in terms of length and time by the following equation: $v = l \cdot t^{-1}$ (and the SI derived unit of velocity is $m \cdot s^{-1}$). Note that these equations do not involve quantities in the *particular* sense but just quantities in the *general* sense. The goal is to state how the values of complex quantities can be reduced to the values of the base quantities. This is similar to the distinction between dimensions and domains in the Gärdenfors approach (see section 8). For Gärdenfors the link between the dimensions of a domain and the domain itself is directly coded into the structure of the space. This fact explains why, for instance, the color domain has the shape of a double cone and not of a 3D cube. The existential dependence of complex quantities with respect to the *base* quantities in SI corresponds to the relationship between dimensions and domains in Gärdenfors' approach. However, SI is very restrictive in combining basic quantities to obtain different spaces so that all possible spaces are essentially multi-dimensional cubes. In conclusion, the SI approach seems closer to the EQLS system.

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[12] This equations has the form $Q = l^{\alpha} \cdot m^{\beta} \cdot t^{\gamma} \cdot I^{\delta} \cdot T^{\varepsilon} \cdot n^{\zeta} \cdot I_v^{\eta} \cdot \sum_{k=1}^{K} a_k$.