MODAL LOGIC 3.2 — COUNTERPART THEORY **LOA 26/5/3**

Achille C. Varzi

1. Outline of CT

- Primitives:

  Wx (x is a possible world)
  Ixy (x is in possible world y)
  Ax (x is actual)
  Cxy (x is a counterpart of y)

- Axioms:

  A1 $\mathrm{I}xy \to \mathrm{W}y$
  (Nothing is in anything except a world)

  A2 $\mathrm{I}xy \quad \mathrm{I}xz \to y{=}z$
  (Nothing is in two worlds)

  A3 $\mathrm{C}xy \to \quad z\mathrm{I}xz$
  (Whatever is a counterpart is in a world)

  A4 $\mathrm{C}xy \to \quad z\mathrm{I}yz$
  (Whatever has a counterpart is in a world)

  A5 $\mathrm{I}xy \quad \mathrm{I}zy \quad \mathrm{C}xz \to x{=}z$
  (Nothing is a counterpart of anything else in its world)

  A6 $\mathrm{I}xy \to \mathrm{C}xx$
  (Anything in a world is a counterpart of itself)

  A7 $\quad x(\mathrm{W}x \quad\quad y(\mathrm{I}yx \leftrightarrow \mathrm{A}y))$
  (Some world contains all and only actual things)

  A8 $\quad x\mathrm{A}x$
  (Something is actual)

2. Remarks

- Comments on the axioms

  — Ad *A1*: The relation I is best interpreted as a mereological relation of parthood, so that 'Ixy' really means "x is part of y": possible worlds are large possible individuals with smaller possible individuals as parts. (As a special case, a world is an improper part of itself.)

— Ad *A2*: Worlds do not overlap; thus, possible individuals in different worlds are never identical (cross-world identity is replaced by the counterpart relation). However, the possible individuals are not all the individuals: cross-world mereological fusions of possible individuals are individuals too, though not *possible* individuals: there is no way for the whole of it to be *actual.*

— Ad *A3–A4*: Only possible individuals are (and have) counterparts. My counterparts are individuals *I would have been*, had the world been otherwise.

— Ad *A5–A6*: The counterpart relation is essentially a cross-world relation, with the only exception that everything qualifies as a counterpart of itself.

— Ad *A7–A8*: There exists a unique actual world. Its description can safely be used:
$@ = \iota x \; \; y(\mathrm{I}yx \leftrightarrow \mathrm{A}y)$

◆ The following principles do *not* generally hold:

R1 $\mathrm{C}xy \rightarrow \mathrm{C}yx$
(Symmetry of the counterpart relation)

R2 $\mathrm{C}xy \quad \mathrm{C}yz \rightarrow \mathrm{C}xz$
(Transitivity of the counterpart relation)

R3 $\mathrm{C}y_1x \quad \mathrm{C}y_2x \quad \mathrm{I}y_1w_1 \quad \mathrm{I}y_2w_2 \quad y_1{\neq}y_2 \rightarrow w_1{\neq}w_2$
(Nothing in any world has more than one counterpart in any other world)

R4 $\mathrm{C}yx_1 \quad \mathrm{C}yx_2 \quad \mathrm{I}x_1w_1 \quad \mathrm{I}x_2w_2 \quad x_1{\neq}x_2 \rightarrow w_1{\neq}w_2$
(No two things in any world have a common counterpart in any other world)

R5 $\mathrm{W}w_1 \quad \mathrm{W}w_2 \quad \mathrm{I}xw_1 \rightarrow \quad y(\mathrm{I}yw_2 \quad \mathrm{C}xy)$
(For any two worlds, anything in one is a counterpart of something in the other)

R6 $\mathrm{W}w_1 \quad \mathrm{W}w_2 \quad \mathrm{I}xw_1 \rightarrow \quad y(\mathrm{I}yw_2 \quad \mathrm{C}yx)$
(For any two worlds, anything in one has some counterpart in the other)

3. Comparison with QML

◆ Translation:

T1 $A \qquad A^{@}$
where A^w (A holds in a world w) is defined recursively as follows:

T2a $A^w = A$, if A is atomic

T2b $(\neg A)^w = \neg A^w$

T2c $(A \quad B)^w = A^w \quad B^w$

T2d $(\quad xA)^w = \quad x(\mathrm{I}xw \rightarrow A^w)$

T2f$_0$ $(\Box A)^w = \quad z(\mathrm{W}z \rightarrow A^z)$
(A holds in every world z)

T2f$_1$ $(\Box A\,x)^w = \quad z \quad y(\mathrm{W}z \quad \mathrm{I}yz \quad \mathrm{C}yx \rightarrow A^z y)$

(*A* holds of every counterpart *y* of *x* in every world *z*)

T2f$_n$ $(\Box A x_1 \ldots x_n)^w = \quad z \quad y_1 \ldots \quad y_n(\mathrm{W}z \quad \mathrm{I}y_1 z \quad \mathrm{C}y_1 x_1 \quad \ldots \quad \mathrm{I}y_n z \quad \mathrm{C}y_n x_n \rightarrow A^z y_1 \ldots y_n)$

- Examples:

E1 $\quad x\mathrm{F}x$

$x(\mathrm{I}x@ \rightarrow \mathrm{F}x)$

(Everything actual is an F)

E2 $\Diamond \quad x\mathrm{F}x$

$w(\mathrm{W}w \quad x(\mathrm{I}xw \quad \mathrm{F}x))$

(Some possible world contains an F)

E3 $\Box \mathrm{F}x$

$z \quad y(\mathrm{W}z \quad \mathrm{I}yz \quad \mathrm{C}yx \rightarrow \mathrm{F}y)$

(Every counterpart of *x*, in any world, is an F)

E4 $\quad x(\mathrm{F}x \rightarrow \Box \mathrm{F}x)$

$x(\mathrm{I}x@ \rightarrow \quad z \quad y(\mathrm{W}z \quad \mathrm{I}yz \quad \mathrm{C}yx \rightarrow \mathrm{F}x))$

(If anything is a counterpart of an actual F, then it is an F)

E5 $\Box \Diamond \mathrm{F}x$

$z_1 \quad y_1(\mathrm{W}z_1 \quad \mathrm{I}y_1 z_1 \quad \mathrm{C}y_1 x \rightarrow \quad z_2 \quad y_2(\mathrm{W}z_2 \quad \mathrm{I}y_2 z_2 \quad \mathrm{C}y_2 y_1 \quad \mathrm{F}y_2)$

(Every counterpart of *x* has a counterpart that is an F)

- Critical principles:

B $\quad A \rightarrow \Box \Diamond A$

Not a theorem (for *A* open) unless R1 (symmetry of C) is assumed

4 $\quad \Box A \rightarrow \Box \Box A$

Not a theorem (for *A* open) unless R2 (transitivity of C) is assumed

BF $\quad x \Box A x \rightarrow \Box \quad x A x$

Not a theorem unless R5 is assumed.

BF' $\quad x \Box A x \rightarrow \Box \quad x A x$

Not a theorem unless R6 is assumed.

BF$_c$ $\quad \Box \quad x A x \rightarrow \quad x \Box A x$

A theorem.

BF$_c$' $\quad \Box \quad x A x \rightarrow \quad x \Box A x$

Not a theorem (obviously).

= $\quad x{=}y \rightarrow \Box(x{=}y)$

Not a theorem unless R3 is assumed

≠ $\quad x \neq y \rightarrow \Box(x \neq y)$

Not a theorem unless R4 is assumed.