MODAL LOGIC 1.1 — INTRODUCTION **LOA 28/4/3**

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1. What is modal logic?

◆ Basic ideas:

— Modal logic is an extension of ordinary logic

— It is concerned with logical facts (e.g., inferences) that involve modalities, i.e., qualifications of propositions.

— Example: the following statement is true.

Alessandro is in Trento

But we may want to qualify this truth: the statement is true, but could have been false (things could have been otherwise)

— On the other hand, the following statement is not just true: it is necessarily true (could not have been false)

Alessandro is Alessandro

You might want to say that this is a logical fact: self identities are logically true (true under every interpretation of the non-logical constants). Then consider

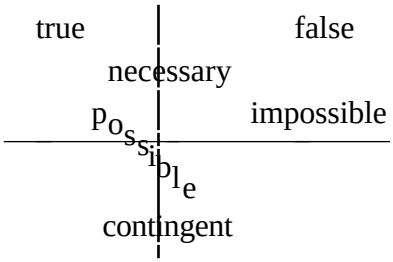
Hesperus is Phosphorus
Alessandro is a person
Nothing is in two places at the same time

These are not true in every logically possible model. Yet, arguably, they are true in every possible world. (E.g., since 'Hesperus' and 'Phosphorus' pick out the same object, that object could not be different from itself.)

Modal logic should not be confused with model theory.

◆ (Of course, there is plenty of room for controversy:
- could I have been fatter?
- could I have been a woman?
- could I have been that chair?
- could I have been a mosquito?
- could I have been the French Revolution?
- could I have been the number 7?)

◆ The basic picture (mostly from ARISTOTLE, De Interpretatione, chapters 12-13)



◆ Modal logic is interested in the <u>interrelationships</u> between these modalities.

— Not interested in their nature
- logical
- metaphysical
- physical
- sociological

  etc.

— Not an explanation (on pain of circularity), but a help in understanding

2. Two questions

◆ Is this the business of <u>logic</u>?

— One could argue it is just a matter of <u>semantics</u> (explain the meaning of 'necessary' etc.)
But the same could be said of ordinary logic (explain the meaning of 'not', 'and', 'all', etc.)

— One could argue it is a matter of <u>theorizing</u> (e.g., axiomatic characterization of necessity, etc., or axiomatic characterization of identity, parthood, and other notions)

(I would be happy to change the lable to <u>Theory of modality</u>, or something like that.)

◆ Couldn't we deal with modalities without invoking anything else than <u>standard logic</u>?
Four possible strategies:

1) Modalities as <u>truth-functional connectives</u>

- However, there are only four possible truth functions

p	f_1	f_2	f_3	f_4
T	T	F	T	F
F	F	T	T	F

and none is adequate to model e.g. necessity:

f_1 yields	$\Box p \Leftrightarrow p$	(determinism)
f_2 yields	$\Box p \Leftrightarrow \neg p$	
f_3 violates	$\Box p \quad p$	and makes $\Box$ a trivial operator anyhow
f_4 validates	$\Box p \quad \neg p$	" "

in general, from the T of *p* we don't know anything about the T of $\Box p$
in general, from the F of *p* we don't know anything about the F of $\Diamond p$

- Besides, there are many other cases of non-truth-functional connectives

  It is well-known that p
  According to Mary, p
  It is surprisingly the case that p
  p *because* q
  p, *and as a consequence* q (non according to Davidson)

2) Move everything to the <u>metalanguage</u>

- Modalities as metalogical properties of sentences (necessity = validity, modulo a certain selection of admissible models)
- But then we may want to <u>formalize the metalanguage</u>...

3) Modalities as <u>metalinguistic predicates</u> of sentences (cp. <u>truth-predicate</u>) (Carnap 1937)

It is necessary that ... ⇔ *'....' is necessarily true*

- <u>Quine</u> 1963: the lowest "grade of modal involvement":

  Necessity resides in the way in which we say things, and not in the things we talk about [p. 176]

- But <u>Montague</u> 1963 shows this is no good—cp. Tarski's problems with truth predicate.
- Only recently this strategy has been reconsidered: see <u>Schwartz</u> 1992.

4) Translation using <u>quantifiers</u>

Necessarily p ⇔ $w\ p(w)$
Possibly p ⇔ $w\ p(w)$

But this involves a number of <u>complications</u>

- quantify over possible world
- propositions become predicates of worlds
- problems when one moves on to quantified modal logic (ariety)

(Come back to this later.)

3. Need for a logic (Theory) of modality

- So: think of modal logic as involving non-truth-functional connectives.
  - — Notation: $\Box$, $\Diamond$ or N, M.
- What is the logic of this extended language?
  - — We need not only laws such as

$$\frac{\Box p \rightarrow \Box q \quad \Box p}{\Box q}$$

  - — but also specific principles such as

$$\frac{\Box(p \rightarrow q) \quad \Box p}{\Box q}$$

- The choice of valid principles may be a controversial matter. It depends
  - — on one's views on what is necessary etc.
  - — on the specific interpretation of $\Box$ and $\Diamond$.
- Necessity and possibility are alethic modalities (modifications of truth = aletheia), but there are also
  - — Epistemic, deontic, temporal (or tense), spatial modalities.
  - — Indeterminacy
  - — Provability
    - $\Box A \Leftrightarrow A$ is provable (relative to some one formal system, e.g., Peano Arithmetic)
    - Gödel's second incompleteness theorem would read: $\neg \Box \rightarrow \neg \Box \neg \Box$
- And there are other uses, too. For example:
  - — Intuitionism ($\neg\neg A \nvdash A$)

MacKinsey-Tarski's map

$I(p)$	$\Box p$	
$I(\neg A)$	$\Box \neg I(A)$	(negation as impossibility)
$I(A \rightarrow B)$	$\Box(I(A) \rightarrow I(B))$	

[Note: This means that intuitionism—a restriction of classical logic—may be interpreted as an extension of it.]