Objects, events, qualities

An introduction to formal ontological distinctions (in DOLCE)

Lecture 2 – Properties

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Recap

- Focus on *foundational ontologies*;
  - (few) foundational ontologies that capture main ontological commitments.
- The choice of the formal language is important;
  - the expressivity of the formal language influence the analysis and what we are able to represent of it;
  - the translation to a 'implementation' language can be very problematic.
- Moderate multiplicativism;
  - is not necessary to commit "to be is to be a value of a variable";
  - reductionism is not always the better choice when the formal language is not highly expressive and we aim at *communicating*.

Working assumptions

- An foundational ontology is a FOL theory intended to semantically characterize very general and well founded primitives by ruling out as much as possible non-intended models and aiming at parsimony about the kinds of entities included in the domain even though there is no strong ontological commitment on their existence in reality.

Outline

- Focus on non-temporary properties.
  - I will consider time and change in the next lecture.
- Philosophical theories of properties:
  - *Universalism*,
  - *Trope Theory*,
  - *Resemblance Nominalism*.
- How properties can be structured: philosophical, empirical, and cognitive perspectives on *hierarchies* and *spaces* of properties.
- How properties can be represented in FOL.
- Qualities and spaces in DOLCE.

1 Properties

- Alternative names: *attributes*, *qualities*, *features*, *kinds*, *sorts*, *types*, *universals*.

- Do properties exist?
- Which properties there are?
- Which is the nature of properties?
- How properties can be represented?

⋆ Even though I will consider philosophical theories that in general refer to *objective* properties, I do not want to rule out *concepts*.

2 Properties vs individuals

- Are *properties* and *individuals* (*particulars*) two distinct ontological categories of entities?
  - Are individuals the substrates of their properties, or are they aggregates (*bundles*) of properties?
  - Are properties *repeatable* entities that apply to individuals, or are they abstractions reducible to *bundles of individuals*?

- Following a standard (and multiplicative) approach I consider both *properties* and *individuals* in the domain.
  - I will analyze under which hypotheses properties can be reduced to (bundle of) individuals;
  - but I will **not** consider here theories that reduce individuals to bundles of properties (e.g. in Bertrand Russell).

3 Old problems

- *One over Many*
  How can different individuals be of the same type?
  How a and b can both have (share) the property P?
- *Many over One*
  How the same individual can have different properties?
  How a can have both property P and property Q?
- These two problems are intimately related to the analysis (in terms of *truth-makers*) of the sentence
  - "a has the property P" or, shortly, "a has P".
- Different theories of properties provide different answers/analyses.

PHILOSOPHICAL THEORIES OF PROPERTIES

4 Universalism

- The *individual* a is an *instance* of the *universal* "being P".
- What makes it true that "a has P" is that the universal "being P" is wholly present in a.
  - Categories: *individual* and *universal*.
    Relation: *instantiation* inst : individual $\times$ universal.

5 Universals vs. classes

- Classes are *partially* present in their instances and the instances are the constituents of classes (they depend on instances) while universals are *wholly* present in (but independent from) and they are the constituents of their instances.
- Classes are *extensional* while (in general) universals are *intensional*.
- Classes are *redundant* and *abundant* while universals are *sparse* and *minimal* in order to capture the world's distinctions. E.g. the union of two classes or the complement of a class are still classes, while the conjunction of two universals or the negation of a universal are not necessarily universals.

- **Natural classes**. Properties are *classes*, and the *natural classes* 'correspond' to universals.

6 Universalism: example

- Both the individuals a and b have the property “being crimson” (Crm).

$$\begin{tikzcd}
a \arrow[r, "\text{inst}"] & Crm \\
b \arrow[ur, "\text{inst}"'] &
\end{tikzcd}$$

7 Trope Theory (1/2)

- The "a's P-ness" *trope* (an individual) *inheres* in the individual a and it *exactly resembles* to P-ness tropes that inhere in different individuals (it *belongs* to the equivalence class of P-ness tropes).
- Categories: *individual*, *trope*, and *set*.

  Relations: *inherence* I : trope $\times$ individual;
  resemblance $\approx$: trope $\times$ trope;
  membership $\in$: trope $\times$ set.

- Inherence standardly satisfies the *non migration principle*:
  - $\mathsf{I}(x, y) \wedge \mathsf{I}(x, z) \rightarrow y = z$

  i.e. tropes inhere only in one individual.

8 Trope Theory (2/2)

- Tropes are *individual properties*: if $a \neq b$, then a's P-ness (the way a is P) is different from b's P-ness (the way b is P).
- What makes it true that an individual has a property is that it has a trope inhering in it that resembles other tropes (inhering in different individuals).

- Properties are then devoid of any ontological relevance, however they can be *associated* to equivalence classes of resembling tropes (i.e. abstractions on tropes by means of resemblance).

- Trope Theory addresses change committing to sort of Perdurantism. (we will see in the next lecture)

9 Trope Theory: example

- Both the individuals a and b have the property "being crimson".

$$
\begin{tikzcd}
a & a_{crm} \arrow[l, "\mathrm{I}"'] \arrow[r, dotted, "\in"] \arrow[d, leftrightarrow, "\approx"'] & {|x_{crm}|_{\approx} \equiv Crm} \\
b & b_{crm} \arrow[l, "\mathrm{I}"'] \arrow[ur, dotted, "\in"'] &
\end{tikzcd}
$$

10 Trope Theory vs Universalism

- Parallelism with theories of time and space:
  - *Substantiavalism*: time (space) is a container-like manifold and what happens (exists) is located in it *contingently*.

    Universalism: universals constitute an absolute and independent (from individuals) framework in which individuals are (contingently) ‘located’.
  - *Relationism*: time (space) is derived from relationships between events (physical objects).

    Trope Theory: properties are derived from (exact) resemblance between tropes (they can be associated to classes of exactly resembling tropes).

11 Universalism + Trope Theory

- Universalism and Trope Theory are not incompatible: it is possible to assume that universals are wholly present in tropes that inhere in individuals.
  - Categories: *individual*, *trope*, and *universal*.

    Relations: *inherence* I : trope $\times$ individual;
    instantiation inst : trope $\times$ universal.

- *Exact resemblance* can be defined: two tropes exactly resemble if and only if they are both instances of the same universal.

12 Universalism + Trope Theory: example

- Both the individuals a and b have the property "being crimson".

$$
\begin{tikzcd}
a & a_{crm} \arrow[l, "\mathsf{I}"'] \arrow[r, "\mathsf{inst}"] \arrow[d, dotted, leftrightarrow, "\approx"'] & Crm \\
b & b_{crm} \arrow[l, "\mathsf{I}"'] \arrow[ur, "\mathsf{inst}"'] &
\end{tikzcd}
$$

13 Main philosophical positions on properties

- Both the individuals a and b have the property "being crimson".

Universalism

$$\begin{tikzcd} a \arrow[r, "\mathsf{inst}"] & Crm \\ b \arrow[ur, "\mathsf{inst}"'] & \end{tikzcd}$$

Trope Theory

$$\begin{tikzcd} a & a_{crm} \arrow[l, "\mathsf{I}"'] \arrow[r, dotted, "\in"] \arrow[d, leftrightarrow, "\approx"'] & {|x_{crm}|_{\approx} \equiv Crm} \\ b & b_{crm} \arrow[l, "\mathsf{I}"'] \arrow[ur, dotted, "\in"'] & \end{tikzcd}$$

Universals+Tropes

$$\begin{tikzcd} a & a_{crm} \arrow[l, "\mathsf{I}"'] \arrow[r, "\mathsf{inst}"] \arrow[d, dotted, leftrightarrow, "\approx"'] & Crm \\ b & b_{crm} \arrow[l, "\mathsf{I}"'] \arrow[ur, "\mathsf{inst}"'] & \end{tikzcd}$$

14 Resemblance Nominalism

- Individual a resembles to other individuals (crimson individuals).
- What makes it true that an individual has a property is that it resembles other individuals.
  - Categories: *individuals* and *class*.

    Relations: *resemblance* $\approx$: individual $\times$ individual
    membership $\in$: individual $\times$ class.

- Properties are devoid of any ontological relevance, however they can be *associated* to equivalence classes of resembling individuals, i.e. abstractions on individuals by means of resemblance.

15 Resemblance Nominalism: example

- Both the individuals a and b have the property “being crimson”.

$$\begin{tikzcd} a \arrow[r, dotted, "\in"] \arrow[d, leftrightarrow, "\approx"] & {\color{red}|x|_{\approx} \equiv Crm} \\ b \arrow[ur, dotted, "\in"'] & \end{tikzcd}$$

16 Resemblance Nominalism: problems

- Resemblance Nominalism faces some difficult problems that, to be addressed, require an ontological commitment:
  - *co-extensionality of properties*: how to distinguish extensionally coincident (they correspond to the same class) properties?

    $\Rightarrow$ commitment to *possibilia*
    does not work for *necessarily* co-extensional properties ('being triangular', 'being trilateral') that Rodriguez-Pereyra identifies.
  - *change of properties*: the same object can persist through the change of properties

    $\Rightarrow$ commitment to *temporal slices* of objects (*Perdurantism*)
    (we will see in the next lecture).

17 Resemblance Nominalism vs Trope Theory (1/2)

- *Relationism*: the relations allowing to build time from events are different from the ones used to build space from physical objects.

- In general one can assume that properties can be abstracted from objects by using different resemblance relations: resemblance with respect to a specific aspect of the object.
- It is because Resemblance Nominalism admits only one resemblance relation that it faces problems to distinguish co-extensional properties.
- Trope Theory admits one resemblance relation but it is defined on tropes that already abstract one specific aspect from objects.

18 Resemblance Nominalism vs Trope Theory (2/2)

- A system $\langle D, \equiv^1, \ldots, \equiv^n \rangle$ ($\equiv^i$ are resemblance relations on D)
  - is philosophically weak because all the $\equiv^i$ must to be justified;
  - is stronger than Resemblance Nominalism because of the presence of n different resemblance relations;
  - is weaker than Trope Theory because tropes cannot be reconstructed in it while the $\equiv^i$ can be builded in Trope Theory:

    Let us assume:

    – a trope system $\langle D^*, \mathsf{I}, \equiv \rangle$ where D^* is D extended with tropes;

    – to 'name' the equivalence classes of exactly resembling tropes,

    then, it is possible to define:

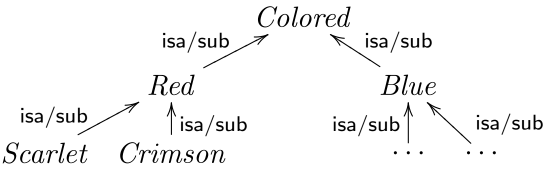
    – $x \equiv^j y$ *iff* $\exists t, s \in |a|^j_{\approx} (\mathsf{I}(t, x) \wedge \mathsf{I}(s, y) \wedge t \equiv s)$

19 *Digression*: abstraction

- $\mathcal{S} = \langle D, \equiv \rangle$ is a generic structure with one *equivalence* relation.
- $\mathcal{S}^e = \langle D^e, =^e \rangle$ is the *abstraction* of $\mathcal{S}$, where
  - D^e is the set of (non-empty) equivalence classes of D;
  - $=^e$ is the identity on D^e.
- Examples:
  - different (punctual) events can be temporally co-localized
    from $\mathcal{E} = \langle E, \equiv_E \rangle$, E set of *events*, $\equiv_E$ *temporal coincidence*
    to $\mathcal{T} = \langle T, =^e \rangle$, T set of *times*.
  - different objects can have the same color
    from $\mathcal{O} = \langle O, \equiv_C \rangle$, O set of *objects*, $\equiv_C$ *color resemblance*
    to $\mathcal{C} = \langle C, =^c \rangle$, C set of *color properties*.

TAXONOMIES OF PROPERTIES

20 Is-a / subsumption relation



- isa/sub is a very general hierarchical relation:
  - ‘Scarlet’ is more specific than (is a subproperty of) ‘Red’;
  - if something is ‘Scarlet’ *then* it is necessarily ‘Red’;
  - if something has property ‘being Scarlet’ it also has ‘being Red’.
- In classical logic isa/sub is usually represented by *entailment*:
  - $Scarlet(x) \rightarrow Red(x)$;
  - $Scarlet^{\mathcal{I}} \subseteq Red^{\mathcal{I}}$ (in semantics).

21 Hierarchies of properties

$$\begin{tikzcd} Colored & & & Animal & \\ Scarlet \arrow[u] & Crimson \arrow[ul] & & Man \arrow[u] & Dog \arrow[ul] \end{tikzcd}$$

- isa/sub provides a purely *extensional* view on hierarchies.
- isa/sub (entailment) can be seen as a *necessary* (but not *sufficient*) condition that all hierarchical relations must satisfy.
- But is it possible, for example, to distinguish the two hierarchical relations depicted in the figure?
- In addition, how different hierarchical relations 'fit' with different theories of properties (especially intensional ones)?

22 Genus-species

- *Genus-species* relations presuppose that subproperties are conjunctions of independent properties, e.g. in

  ‘*Man is a rational animal*’

  ‘being rational’ and ‘being animal’ are independent (one does not entail the other one), and ‘being man’ is the conjunction of them

$$\begin{tikzcd} Animal & & Rational \\ & Man \arrow[ul, "\mathsf{gs}"'] \arrow[ur, "\mathsf{gs}"] & \end{tikzcd}$$

$$Man(x) \triangleq Animal(x) \wedge Rational(x)$$

- *Genus-species* does not seem to hold between ‘scarlet’ and ‘red’: one needs to find a property independent from ‘red’ that in conjuncted with ‘red’ defines ‘scarlet’.

23 Determinate-determinable

$$
\begin{tikzcd}
 & \mathit{Colored} & & & \mathit{Animal} & \\
\mathit{Scarlet} \arrow[ur] & \mathit{Crimson} \arrow[u] & \cdots \arrow[ul] & \mathit{Man} \arrow[ur] & \mathit{Dog} \arrow[u] & \cdots \arrow[ul]
\end{tikzcd}
$$

- Having a *determinate* property entails having a *determinable* property (e.g. $\mathit{Scarlet}$ implies $\mathit{Colored}$) .
- Having a *determinable* property entails having *one and only one* of its (full) *determinates*:
  - no instances of both $\mathit{Scarlet}$ and $\mathit{Crimson}$;
  - a determinable has at least two determinates.

24 Determinate-determinable: Johnson

- To understand ‘color’, one needs ‘scarlet’, ‘crimson’, . . . while ‘animal’ does not require ‘man’, ‘dog’, . . . [*adjectives* vs. *substantives* / *top-down* vs. *bottom-up*]
- To grasp ‘color’ one needs to grasp how different shades of color are different from one another while still being shades of color.
  - A relation between (the instances of) determinates is needed.
- The grasping of *determinables* involves the grasping of certain relations of *similarity* or *intensification* used to generate their determinates.
  - Determinates under the same determinable are different but comparable, e.g., under the determinable ‘color’, ‘scarlet’ is more similar to ‘crimson’ than to ‘turquoise’.

25 Full determinates

$$
\begin{tikzcd}
 & & Colored & & \\
 & Red \arrow[ur, "\mathsf{dD}"] & & Blue \arrow[ul, "\mathsf{dD}"'] & \\
Scarlet \arrow[ur, "\mathsf{dD}"] & Crimson \arrow[u, "\mathsf{dD}"'] & & \cdots \arrow[u, "\mathsf{dD}"] & \cdots \arrow[ul, "\mathsf{dD}"']
\end{tikzcd}
$$

- *Full determinates* admit no more that a difference between any two instances with regard to the relations of intensification by which they are generated (leaves in a hierarchy).
  - E.g., the instances of a specific shade of color (e.g., let us suppose, $Scarlet$) are all the same with respect to color.
  - Shades are colors, but not all colors are shades, since some colors consist of collections of shades ordered by some relation that is included in our grasp of the color (e.g. Red and $Blue$).

26 General determinables

$$\begin{tikzcd}
 & & \textcolor{green}{Colored} & & \\
 & Red \arrow[ur, "\mathsf{dD}"] & & Blue \arrow[ul, "\mathsf{dD}"'] & \\
\textcolor{violet}{Scarlet} \arrow[ur, "\mathsf{dD}"] & \textcolor{violet}{Crimson} \arrow[u, "\mathsf{dD}"'] & & \cdots \arrow[u, "\mathsf{dD}"] & \cdots \arrow[ul, "\mathsf{dD}"']
\end{tikzcd}$$

- Incompatibility of full determinates: as already stated, having a *determinable* entails having *one and only one* of its full *determinates*.
- *Comparability*: instances of different full determinates under a general determinable are (at least qualitatively) comparable.
- Not clear if comparability is defined on properties or instances.
- *General determinables* are maximal with respect to comparability: e.g. all the instances of ‘scarlet’ are comparable with instances of ‘turquoise’ but not with all the instances of ‘1kg’.
- Determinables are generated by *comparability* (*intensification*).

27 Determinate-determinable: Universalism

- Determinables are experimentally derivative from full determinates, they are specified bottom-up *by enumeration* of full determinates.
- To assert that an individual is red is to assert that it has some property, a property that is a member of a certain class of properties: the class of all full determinate shades of red.
- All genuine *universals* are full determinates because instances of an universal need to be *identical* in a certain respect.

28 Predication of determinables: Universalism

$$\begin{tikzcd} a \arrow[r, "\mathsf{inst}"] & \mathit{Scarlet} \arrow[r, "\in"] & \mathit{Red} \arrow[r, "\subset"] & \mathit{Colored} \\ b \arrow[ur, "\mathsf{inst}"'] & & & \end{tikzcd}$$

- Full determinates correspond to universals.
- Determinables correspond to *sets* of full determinates (are they *conceptual* constructions?).
- But *general* determinables collect universals that satisfy unity conditions that require additional relations:
  - resemblance/comparability (with degrees) btw universals [Church];
  - partial identity (defined on parthood) btw universals (comparability can be defined) [Armstrong].

29 Determinates-determinables: Trope theory

- First of all one needs to understand if tropes are maximally specified or not: do the 'the red of the rose r' and 'the color of r' exist?
- Accepting only maximally specified (fully determinate) tropes
  - full determinates correspond to classes of *exactly* resembling tropes;
  - determinables correspond to classes of *inexactly* resembling tropes (e.g. 'the scarlet of r_1' and 'the crimson of r_2'), therefore inexact resemblance (*with degree*) is needed.
- In principle the inexact resemblance does not collect the whole class of tropes, but it stops at the level of the *general determinables* that therefore correspond to *maximal* classes of inexactly resembling tropes.
- This last aspect is quite critical.

30 Predication of determinables: Trope theory

$$
\begin{tikzcd}
a & a_{sc} \arrow[l, "\mathsf{I}"'] \arrow[r, "\in"] \arrow[d, leftrightarrow, "\approx"] & {\color{violet}|Scarlet|_{\approx}} \arrow[dd, "\subseteq"] \\
b & b_{sc} \arrow[l, "\mathsf{I}"] \arrow[ur, "\in"'] \arrow[d, leftrightarrow, "\approx_d"] & \\
c & c_{cr} \arrow[l, "\mathsf{I}"] \arrow[r, "\in"'] \arrow[d, leftrightarrow, "\approx_{d'}"] & |Red|_{\approx_d} \arrow[d, "\subseteq"] \\
d & d_{blu} \arrow[l, "\mathsf{I}"] \arrow[r, "\in"'] & {\color{green}|Colored|_{\approx_{d'}}}
\end{tikzcd}
$$

- *Full determinates* correspond to classes of *exactly* resembling tropes.
- *Determinables* correspond to classes of *inexactly* (at some degree) resembling tropes.
- *General determinables* corresponds to *maximal* classes of inexactly resembling tropes.

SPACES OF PROPERTIES

31 Ontological similarity and comparability

- Universalism and Trope Theory both consider that two entities are *exaclty similar* when and because they 'share a fully determinate property':
  - they share a universal (*universalism*);
  - they have exactly resemblant tropes (*trope theory*).
- In this case, *similarity* is *objective*, *mind* and *language* independent, it is exclusively based on the *ontological nature of entities* providing the *finest* possible analysis.

- Resemblance with degree and partial identity allow to 'abstract' from the objective nature of full determinates.
- Entities sharing a *general determinable* are not *exactly similar* but at least *comparable*.

32 Towards an empirical/epistemological level

- "[J]udgments of similarity (...) are central for a large number of cognitive processes. (...) such judgments reveal the dimensions of our perceptions and their structures." [Gärdenfors, 2000]
- In this case, *similarity* is empirically built on experiments and it is *relative*: it may depend on species, cultures, etc.
- In *science*, the analysis always is conducted at an empirical (or theoretical) level and it depends on the available information, the measurement instruments/methods, the specific theory considered, etc.

- Not only it is possible to abstract from full determinates, but it is possible to consider different full determinates (they are no more *objective*) that can be structured in different ways.
  $\Rightarrow$ **no unique *exact* resemblance relation**.

33 Spaces of properties (1/2)

- Resemblance with *degree* or partial identity introduce a (partial) order among properties.
- To a general determinate can be associated more spaces that depend on culture, instruments of investigation, etc.

- Spaces and concepts may have an *inter-subjective* (vs. objective) nature: they can be the product of (more or less explicit) *social conventions* or the result of some (evolutionary) *cognitive processes* typical of a kind of agents.
- Spaces and concepts may exist *in time*: they can be created, adopted, and destroyed by (communities of) intentional agents.

34 Spaces of properties (2/2)

- Properties can be structured in *spaces* on the basis of the *intensification* relation between full determinates:
  - spaces can have a topological or geometrical structure (more expressive relations are needed).
- This is particularly evident in the case of empirical properties (**concepts**?) that can be structured according to particular empirical or epistemological point of views.
- In this cases, the determinates of a general determinable can be arranged in different *spaces* that can contain:
  - different full determinates (different exact resemblance/identity);
  - different determinables (granularities / degrees of resemblance);
  - different structures (different comparability/intensification).

35 Spaces with the same full determinates

- Full determinates are ‘objective’ but they can be *contextually* organized in different spaces.

$$
\begin{tikzcd}
a \arrow[r, "::"] & \color{violet}{Scarlet} \arrow[r, "\mathsf{pos}"] \arrow[ddr, "\mathsf{pos}"'] & DarkRed \arrow[r, "\mathsf{dD}"] & S^1_1 \arrow[dr] & \\
& & \vdots & \vdots & \color{teal}{Colored} \\
& & Red \arrow[r, "\mathsf{dD}"] & S^1_n \arrow[ur] &
\end{tikzcd}
$$

  where :: stands for **inst** in Universalism or for a composition of **I** and $\in$ in Trope theory, and **pos** is a general relation to represent the fact that $Scarlet$ has a specific position in a space.

- To the same general deteminable, different spaces with different (non fully) determinates can be associated, e.g. $DarkRed$ is not considered in space S^1_n.

36 Spaces with different full determinates

- Both full determinates and structures of spaces are *contextual*.

$$
\begin{tikzcd}
a \arrow[r, "::"] \arrow[ddr, "::"'] & \color{violet}{Scarlet} \arrow[r, "\mathsf{dD}"'] & Red \arrow[r, "\mathsf{dD}"'] & S^1_1 \arrow[dr] & \\
& & \vdots & \vdots & \color{green}{Colored} \\
& \color{violet}{DarkRed} \arrow[r, "\mathsf{dD}"] & Red \arrow[r, "\mathsf{dD}"] & S^1_n \arrow[ur] &
\end{tikzcd}
$$

- If we assume that for each general determinable there is a space S^* with maximal granularity (as defined by, say, a *refinement relation*), then the atomic regions of S^* can be taken to be the the "objective" full determinates.

- To individuate all the spaces relative to the same domain or dimension (general determinable) an additional relation is necessary.

37 Conceptual spaces: Peter Gärdenfors (1/2)

- *Dimensions* correspond to "the different ways stimuli are judged to be similar or different", e.g. temperature, weight, pitch, and brightness.
- A point in a dimension represents a specific property (e.g. a temperature) and the association of two objects to the same point represents the experimental fact that the two objects are completely *similar* with respect to that dimension (e.g. temperature).
- Points can be ordered (e.g. temperatures can be "low" or "high") therefore each dimension is endowed with a (pseudo) metric that represents the level of similarity between stimuli.

38 Conceptual spaces: Peter Gärdenfors (2/2)

- A set of dimensions is *integral* if an object that is located in one dimension, is necessarily located in all the other dimensions.
  - E.g., {*hue*, *brightness*} is integral: an object that has a particular hue necessarily has also a particular brightness and vice versa.
- *Domains* are maximal sets of integral dimensions.
  - E.g. {*hue*, *chromaticness*, *brightness*} form a domain (color) because it is integral but *hue*, *chromaticness* and *brightness* are separable from any dimension that does not belong to this set.
- A property corresponds to a *region* in a domain.
- *Conceptual spaces* are collections of one or more domains, and their regions represent *concepts* (points in conceptual spaces correspond to the more specific concepts).

REPRESENTING PROPERTIES AND SPACES IN FIRST ORDER LOGIC

39 Properties as predicates (1/2)

- $\mathsf{Red}(x) \wedge \mathsf{Orange}(y) \wedge \mathsf{Blue}(z) \wedge \mathsf{Sim}_{\mathsf{C}}(x, y, z)$

- (Some) Universalists
  - refuse extensionality (universals are not just sets of entities);
  - Boolean (logical) combination of universals are not universals.

- (Some) Conceptualists
  - assume that concepts are created, they can disappear and depend on societies or groups of agents that use them: *properties of properties*.

- In FOL, one can:
  - *reify* properties into the domain of quantification, and
  - introduce a non extensional relation of *instantiation*.

40 Properties as predicates (2/2)

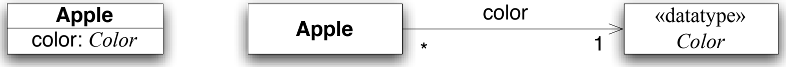
- (Some) Tropicalists
  have less problems to accept extensionality and Boolean closure, because they just *associate* (and do not identify) properties to sets of tropes, however they require a deeper analysis in terms of *inherence* and tropes:
  - $\mathsf{Red}(x) \triangleq \exists r_t(\mathsf{I}(r_t, x) \wedge \mathsf{Red}_\mathsf{T}(r_t))$
  - $\mathsf{Sim}_\mathsf{C}(x, y, z) \triangleq \exists c_t, c'_t, c''_t(\mathsf{Color}_\mathsf{T}(c_t) \wedge \mathsf{Color}_\mathsf{T}(c'_t) \wedge \mathsf{Color}_\mathsf{T}(c''_t) \wedge \mathsf{I}(c_t, y) \wedge \mathsf{I}(c'_t, x) \wedge \mathsf{I}(c''_t, z) \wedge \mathsf{Sim}_\mathsf{T}(c_t, c'_t, c''_t))$
  - $\mathsf{Crimson}_\mathsf{T}(c_t) \wedge \mathsf{Crimson}_\mathsf{T}(c'_t) \rightarrow c_t \approx c'_t$ (for full determinates)

  where
  - P_T indicates a class of tropes, and
  - Sim_T is a similarity relation defined on tropes.

41 Attributes

- $\mathsf{Color}(x, \mathsf{red}) \wedge \mathsf{Color}(y, \mathsf{orange}) \wedge \mathsf{Color}(z, \mathsf{blue}) \wedge \mathsf{Sim}_{\mathsf{P}}(\mathsf{red}, \mathsf{orange}, \mathsf{blue})$
  - $\mathsf{Red}(x) \triangleq \mathsf{Color}(x, \mathsf{red})$
  - $\mathsf{Sim}_{\mathsf{C}}(x, y, z) \triangleq \exists c_p, c'_p, c''_p (\mathsf{Color}(x, c_p) \wedge \mathsf{Color}(y, c'_p) \wedge \mathsf{Color}(z, c''_p) \wedge \mathsf{Sim}_{\mathsf{P}}(c_p, c'_p, c''_p))$

- In UML, function color: Apple $\rightarrow$ *Color*



where *Color* is a *datatype*, i.e. a class of *values* (not *objects*).

- To impose the functional requirement
  - $\mathsf{Color}(x, y) \wedge \mathsf{Color}(x, z) \rightarrow y = z$

42 Attribute functions (1/2)

- Each *function/datatype* corresponds to a general determinable.
- Each *value* corresponds to a full determinate.
- The functional view admit only full determinates are in the domain of quantificationis and is, in general, not extensional:

  $\mathsf{Color}(x, \mathsf{crimson}) \leftrightarrow \mathsf{Length}(x, \mathsf{1m})$ does not entail $\mathsf{crimson} = \mathsf{1m}$.
- Predication on values allows to express dependences, structures, time stamp, etc.
- Bunge-Weber-Wand provided an universalistic interpretation to attributes, however a trope-theoretical interpretation is possible (where classes of tropes that correspond to attribute values are needed):
  - $\mathsf{Color}(x, \mathsf{crimson}) \leftrightarrow \exists c_t(\mathsf{I}(c_t, x) \wedge \mathsf{Crimson}_\mathsf{T}(c_t))$

43 Attribute functions (2/2)

- To represent non-full determinates, one needs:

1 · to include non-full determinates in the 'values' discarding the functionality constraints:

$\mathsf{Color}(x, \mathsf{crimson}) \wedge \mathsf{Color}(x, \mathsf{red}) \wedge \mathsf{crimson} \neq \mathsf{red}$,

$\mathsf{Color}(x, \mathsf{crimson}) \rightarrow \mathsf{Color}(x, \mathsf{red})$; or

2 · to treat non-full determinables as predicates defined in terms of disjunctions of full determinates, e.g.

$\mathsf{Red}(x) \triangleq \mathsf{Color}(x, \mathsf{crimson}) \vee \ldots \vee \mathsf{Color}(x, \mathsf{scarlet})$.

44 *Digression*: full determinates vs. values

- What is the ontological nature of values?
  1. Can the same value be used for different attributes? For example, can '$1m$' be used for *height* and *length*?
  2. Do '$1m$' and '$100cm$' refer to two different values?
- Full determinates are specific properties, therefore 'being $1m$ high' and 'being $1m$ long' are just two different properties.
- The same full determinate can be 'measured' in different ways: 'being $1m$ high' and 'being $100cm$ high' refer to the same property but to different *measurement systems*.
- 'm' and 'cm' can refer to different granularities or measurement's precisions.

45 Reification of properties and instantiation

- $\mathsf{inst}(x, \mathsf{red}) \wedge \mathsf{Color}(\mathsf{red}) \wedge \mathsf{inst}(y, \mathsf{orange}) \wedge \mathsf{Color}(\mathsf{orange}) \wedge \mathsf{inst}(z, \mathsf{blue}) \wedge \mathsf{Color}(\mathsf{blue}) \wedge \mathsf{Sim}_{\mathsf{P}}(\mathsf{red}, \mathsf{orange}, \mathsf{blue})$
  - $\mathsf{Color}(x, y) \triangleq \mathsf{inst}(x, y) \wedge \mathsf{Color}(y)$
  - $\mathsf{Sim}_{\mathsf{C}}(x, y, z) \triangleq \exists c_p, c'_p, c''_p(\mathsf{Color}(x, c_p) \wedge \mathsf{Color}(y, c'_p) \wedge \mathsf{Color}(z, c''_p) \wedge \mathsf{Sim}_{\mathsf{P}}(c_p, c'_p, c''_p))$
- Compatible both with Universalism and Trope Theory.
- Similar to attributes but:
  - general determinables correspond to unary predicates;
  - full determinates and determinables are both in the domain;
  - full-determinate can be distinguished from determinates: $\mathsf{Color}(\mathsf{red}) \wedge \mathsf{Color}(\mathsf{crimson}) \wedge \forall x(\mathsf{inst}(x, \mathsf{cimson}) \rightarrow \mathsf{inst}(x, \mathsf{red}))$;
  - the *instantiation* relation (inst) needs to be characterized.

PROPERTIES AND SPACES IN DOLCE AND DOLCE-CORE

46 Properties and spaces in DOLCE

- The intuition if very close to the last framework we considered:
  - Both determinables and full-determinates are in the domain of quantification;
  - A sort of *instantation* relation (called *quale* (**ql**) in DOLCE) is considered.

- However DOLCE introduces some novelties.

47 Properties as regions

- General determinables do not correspond to predicates but, as in the case of determinables and full-determinates, to *spatial regions*.
- The determinable-determinate relation is represented by means of a *classical extensional mereology* based on *parthood simpliciter* (P):
  - full determinates correspond to *atomic regions* (called *qualia*);
  - general determinables correspond to regions called *spaces* that include their determinates, e.g. $\mathsf{P}(\mathsf{crimson}, \mathsf{red}) \wedge \mathsf{P}(\mathsf{red}, \mathsf{color})$;
  - structural constraints can be introduced among regions;
  - correspondence btw mereological operators and logical ones.
- DOLCE admits only one space for each general determinable.
- Spaces in DOLCE are similar to conceptual spaces of Gärdenfors, but properties do not need to correspond to self-connected regions.

48 Individual qualities (1/2)

- DOLCE represents the dimension of classification, the specific aspect along with individuals are compared (that is associated to a space/group of spaces), by introducing *individual qualities*.
- We will analyze the differences between individual qualities and tropes. For the moment, just note that:
  - $\sim$ to tropes, they inhere in a specific object: the ‘weight of John’ is different from the ‘weight of Sam’;
  - $\neq$ from tropes, they correspond to global determinables (not to full determinates): an individual quality (e.g. ‘weight of John’) can be associated to different full determinates at different times.
- The inherence relation is called *quality* (`qt`) in DOLCE.

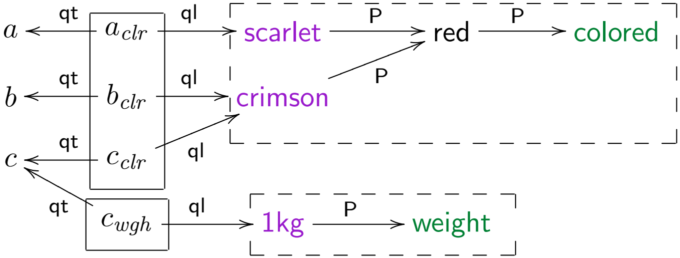
49 Individual qualities (2/2)

- As in the case of tropes, dimensions/aspects of classification can be identified by means of *comparability* relations on individual qualities.
- DOLCE follows an alternative (but equivalent) solution that assumes n disjoint predicates QT_i (called *quality kinds*), one for each dimension/aspect (e.g. Q_{color}, Q_{weight}).
- Spaces can be characterized on the basis of the Q_i predicates as the regions x that are maximal with respect to the following property:

  $$\forall q, q', r, r' (\mathsf{P}(r, x) \wedge \mathsf{P}(r', x) \wedge \mathsf{ql}(r, q) \wedge \mathsf{ql}(r', q') \wedge Q_i(q) \rightarrow Q_i(q'))$$

  i.e. they collect all the regions that classify individual qualities of the same kind (comparable individual qualities).

50 General schema in DOLCE



$$\mathsf{Color}(x, \mathsf{red}) \triangleq \exists q, y(\mathsf{qt}(q, x) \wedge \mathsf{ql}(y, q) \wedge \mathsf{P}(y, \mathsf{red}) \wedge \mathsf{P}(\mathsf{red}, \mathsf{colored}))$$

- a_{clr}, the color individual quality that inheres in a, represents the dimension/aspect of a that we are classifying.

51 *Digression*: naïve linguistic evidences

- Individual qualities allow also for a more 'direct' semantics of some NL expressions.
  - This rose is red.
  - Red is a color.
  - This rose has a color.
  - The color of this rose turned to brown in one week.
  - Red is opposite to green and close to brown.
  - The patients temperature is increasing.
  - The doctor measured the patient's temperature.

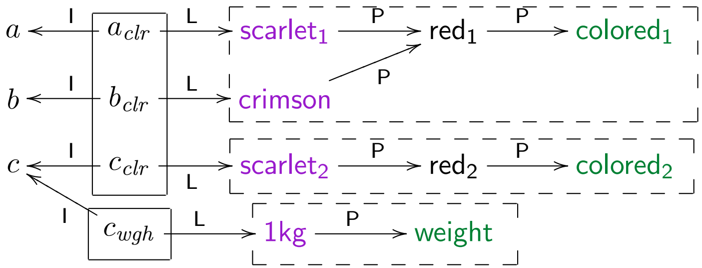
52 *Digression*: International System of Units

- A similar approach is adopted by the SI, where:
  - quantities in the *particular* sense correspond to individual qualities;
  - quantities in the *general* sense correspond to general determinables.

53 Properties and spaces in DOLCE-CORE

- More standard terminology:
  - *inherence* (I) instead of *quale* (ql);
  - *location* (L) instead of *quality* (qt).
- DOLCE-CORE modifies DOLCE to associate different spaces to the same dimension/aspect of classification (i.e. different spaces can be associated to the same quality kind Q_i): *cognitive/empiral move*.
- Different spaces correspond to different viewpoints or conventions, thus they are disjoint even when associated to the same dimension.
  - The previous maximality condition does not work.
  - A finite set of disjoint primitive predicates $S^i_1, \ldots S^i_n$ that correspond to different spaces is associated to each quality kind Q_i: $S^i_j(x) \wedge \mathsf{P}(r, x) \wedge \mathsf{L}(r, q) \rightarrow Q_i(q)$.

54 General schema in DOLCE-CORE



- Regions inside different spaces associated to the same quality kind can be compared (e.g. refinement relation) in a purely extensional way looking at the individual qualities located in them.

- If an unique maximal refined space exists, its atoms can be seen as an empirical surrogate for objective properties.

55 Towards an empirical approach: measurement

- By giving a central role to measurement (instruments, reference systems, and calibration) individual qualities are no more necessary because the specific way (and therefore the dimension along with) objects are compared is determined by measurement systems.

- Simpler framework: objects are directly located in spaces (however one needs measurement instruments linked to spaces)

$$
\begin{tikzcd}
a \arrow[r, "\mathsf{L}"] & \text{scarlet} \arrow[r, "\mathsf{P}"] & \text{red} \arrow[r, "\mathsf{P}"] & \text{colored} \\
b \arrow[r, "\mathsf{L}"'] & \text{crimson} \arrow[ur, "\mathsf{P}"'] & &
\end{tikzcd}
$$

- This move provides an empirical basis to this general framework.

56 Properties in DOLCE and DOLCE-CORE

- This is not the whole story:
  - time and change need to be taken into account;
  - properties that are not organized in spaces need to be taken into account.
- These are the topic of the next lecture!