Semantics and Ontology in Quantified Modal Logic

Stefano Borgo and Claudio Masolo
Laboratory for Applied Ontology, ISTC-CNR, Trento, Italy
`{borgo, masolo}@loa-cnr.it`

Philosophical logic studies the relationships between philosophical stands and logic as well as the logical and philosophical consequences that can be drawn from these connections. There is a good deal of results in this area spread over the second half of last century and, these days, there seems to be a renewed interest in this topic perhaps due to the introduction of ontological concerns in close areas like applied logic and knowledge representation. In this situation, it is important to provide an organized presentation where major philosophical systems and corresponding logics can be related and compared. Unfortunately, there is no systematic work in this direction (perhaps [1] is an exception). This work aims at a general formal framework where it is possible to study and compare some systems of quantified modal logic from the ontological perspective they embrace. The intention is to use the framework to consistently formalize important philosophical stands, namely, the endurantist, the perdurantist, and the sequentialist positions. Secondly, we want to use such a framework to formally evaluate the different ontological imports of these systems and to relate these results to specific philosophical arguments. Of course, a philosophical stand usually matches several formal systems (alternative options are available within the same philosophical perspective). Therefore, our approach consists in individuating a single formal system as paradigmatic for a philosophical view and to discuss its possible variants for relevant alternatives within that view. Here is the structure of this work.

1. We begin analyzing some semantics for modal logic.

Kripke semantics. The standard frame is $\langle \mathcal{W}, \mathcal{D}, \Re \rangle$ where $\mathcal{D}$ is the set of individuals and $\Re$ is the accessibility relation between worlds. The distinction between constant and varying domain semantics (in the latter case $\mathcal{D}$ is a function $w \mapsto \mathcal{D}(w)$ for $w \in \mathcal{W}$) is considered along the line of [2].

Intensional semantics. The frame here has form $\langle \mathcal{W}, \mathcal{D}, \mathcal{I}, \Re \rangle$, where $\mathcal{D}$ is a function as in Kripke varying domain semantics while $\mathcal{I}$ is a function that associates to every $w \in \mathcal{W}$ a subset $\mathcal{I}(w)$ of all the functions from $\mathcal{W}$ to $\mathcal{D}(w)$. We discuss the assumptions and consequences of including partial or total functions in $\mathcal{I}(w)$. Elements of $\mathcal{I}$ are called *intensional object* while, intuitively, the elements in $\mathcal{D}(w)$ are *stages* of intensional objects in w. Here the variables range over intensional objects and not on $\mathcal{D}$ as in the case of Kripke semantics.

Counterpart semantics. The frame is $\langle \mathcal{W}, \mathcal{D}, \Re, \mathsf{C} \rangle$ where $\langle \mathcal{W}, \mathcal{D}, \Re \rangle$ is a varying domain Kripke frame and C is the counterpart relation assigning to each couple $\langle w, w' \rangle$ a subset of $\mathcal{D}(w) \times \mathcal{D}(w')$. Variables range over $\mathcal{D}$ which intuitively is a set of stages. The satisfaction conditions differ from Kripke semantics on the interpretation of necessity where relation C is used.

2. Our analysis of the previous approaches concentrates on the semantic clauses for (i) predicates, (ii) identity, and (iii) necessity operator since these are crucial to capture the philosophical notion of persistence. The discussion of the relationship between Kripke semantics and endurantism, intensional semantics and perdurantism, counterpart semantics and stage theory is based on the discussion of notions like stage, intensional object, cross-world and world-bound entity as well as their relationships.

3. *The basic framework.* We consider the structure $\langle \mathcal{W}, \mathcal{D}, \Re, \mathsf{E}, \mathsf{S} \rangle$, where $\mathcal{W}$ is the set of *possible worlds*, $\mathcal{D}$ is the set of *possibilia*, $\Re$ is the *accessibility* relation, E is the *existence* relation on $\mathcal{D} \times \mathcal{W}$, and S is the *stage* relation on $\mathcal{D} \times \mathcal{D}$, and characterize it with constraints in FOL. In this framework we can distinguish endurantism, perdurantism, and sequentialism on the basis of existential constraints on possibilia or, more specifically, on stages.

The framework assumes a constant domain and a possibilist quantification ($\forall$ and $\exists$ range over all the possibilia). We discuss the importance of *cross-world possibilia* and *world-bound possibilia* in it. Regarding identity, we are aware that this notion has received several interpretations in the literature and it is crucial in several arguments (e.g., [3]). Here we use it in the strongest sense as to mean "one and the same thing" and discuss how to interpret in the formalism alternative views on identity (e.g., coincidence or the restriction of identity to world-bound entities).

The selected framework itself is compared to alternative choices. For instance, it is known that we can define E and S, by means of a *simpliciter parthood* (P) defined on $\mathcal{D} \cup \mathcal{W}$. In this case, the definition of S does not commit to existential conditions. Instead, E requires the existence of a stage in any possible world in which a possibilia is. From the perdurantist and sequentialist perspectives, this constraint can be accepted but an endurantist, which in his terminology reads P as 'constant parthood', would not embrace it. However, the existential constraint can be weaken to the point that the substitution of E and S by P (within extensional mereology) gives an equivalent (in a sense to be explained) framework.

4. *Towards a uniform interpretation.* As a consequence of the work briefly described above, a first characterization of the ontological options in terms of constraints on $\mathcal{D}$ is provided. In practice, at this point we isolate three structures that differs only on their assumptions on the domain. Then, we fix the Kripke satisfaction conditions for predicates and identity, conditions that are usually adopted by endurantists. For a sequentialist they mean that the predicates and the identity are defined on stages while for a perdurantist they are defined on both stages and cross-world possibilia. Then, we discuss in more details the conditions on the satisfaction clauses that allow us to define perdurantist and sequentialist theories in this framework. Another important point is the relative analysis of the satisfaction conditions for the necessity operator $\square$. Kripke and intensional semantics adopt the same semantic clause for $\square$. The contraints $\mathcal{D}(w) \subseteq \mathcal{D}(w')$ and $\mathcal{I}(w) \subseteq \mathcal{I}(w')$ ensure the conditions behind the counterpart approach. One wonders if starting from the counterpart semantics one can reasonably reconstruct the other approaches as well. This analysis brings out interesting connections which take into account the formal properties of the counterpart relation in connection to the notion of identity in these theories.

There is too much literature to provide all the references in this abstract. Beside what already referred to, here are some authors important for this work: Barcan, Chisholm, Corsi, Fine, Hintikka, Kripke, Lewis, Lowe, Kracht, Kutz, Quine, Sider, Thomson, van Inwagen, Varzi, Wiggins.

References

[1] Charles S. Chihara. *The Worlds of Possibility - Modal Realism and the Semantics of Modal Logic.* Oxford University Press, Oxford, 1998.

[2] Melvin Fitting and Richard L. Mendelsohn. *First-Order Modal Logic*, volume 277 of *Synthese Library - Studies in Epistemology, Logic, Methodology, and Philosophy of Science.* Kluwer Academic Publishers, 1998.

[3] M. Rea, editor. *Material Constitution. A Reader.* Lanham, MD, Rowman & Littlefield, 1997.