

CHAPTER 6 – PRODUCTION MODELS

6.1 BASIC ASSUMPTION ABOUT THE EVOLUTION OF THE BIOMASS OF A NON EXPLOITED STOCK

The production models (also called general production models, global models, sintetic models or Lotka-Volterra models) consider the stock globally, that is, they do not take into consideration the structure of the stock by age or size.

The total biomass of a *non exploited* stock cannot grow beyond a certain limit. The value of that limit depends, for each resource, on the available space, on the feeding facilities, on the competition with other species, etc. In conclusion, it depends on the capacities of the ecosystem to maintain the stock. That size limit of the biomass will be designated by *Carrying Capacity*, k .

The total biomass of a non exploited fishery resource has the tendency to increase with the time towards its carrying capacity, k , with a non constant absolute rate, $\text{air}(B_t)$. The rate, $\text{air}(B_t)$, is small when the biomass is small, increases when the biomass grows and is again small when the biomass gets close to the carrying capacity. Changes, including reductions, can occur in the biomass due to fluctuations of the natural factors, but, in any case, the tendency will always be an increase towards its carrying capacity.

The instantaneous rates $\text{air}(B_t)$ or $\text{rir}(B_t)$ are therefore not constant.

In order to formulate the basic assumption of a model for the evolution of the non-exploited biomass, one can adopt a function H of B_t , as was done with the basic assumption of the individual growth, and define it:

$$\underset{\text{natural}}{\text{rir}}[H(k) - H(B_t)] = -r \quad \text{with } r \text{ constant}$$

r is the intrinsic growth rate of B_t . The relative instantaneous rate, $\text{rir}(B_t)$, of the non-exploited biomass can therefore be deduced as.

$$\underset{\text{natural}}{\text{rir}}(B_t) = \frac{r[H(k) - H(B_t)]}{\left\{ B_t \cdot \frac{dH}{dB_t} \right\}}$$

6.2 EXPLOITED STOCK

When the stock is exploited, the rate of variation of the biomass due to all causes, that is, the total $\text{rir}(B_t)$, can be separated into two components: natural $\text{rir}(B_t)$ due to all causes but not fishing and $\text{rir}(B_t)$ due to fishing:

$$\text{rir}(B_t)_{\text{total}} = \text{rir}(B_t)_{\text{natural}} + \text{rir}(B_t)_{\text{fishing}}$$

In an interval of time T_i and with a constant fishing level, it will be:

$$\text{rir}(B_t)_{\text{fishing}} = F_i = \text{constant}$$

and:

$$\text{rir}(B_t)_{\text{total}} = f(B_t)_{\text{natural}} - F_{\text{ifishing}}$$

The natural rate, $\text{rir}(B_t)$, (which, according to the basic assumption of the natural evolution of the biomass, B , is supposed to be a function of the biomass, B_t) is usually designated as $f(B_t)$.

Comments

1. Historically, the production models were the first to be used on the analyses of the evolution of biological populations, Lotka-Volterra (1925-1928).
2. Schaefer (1954) applied a production model to a fish stock subject to fishing.
3. The carrying capacity, k , has been designated in fisheries biology, as B_∞ and also as B_0 . Currently the symbol k is preferred (notice that this symbol is different from the symbols, K , of the individual growth models and of the relation S-R).
4. The basic assumption about natural $\text{rir}(B_t)$ previously presented, can be mathematically formulated in different ways..
5. The production models can only be used in fisheries to analyse the effects of fishing level changes and not of changes in the exploitation pattern, because the models consider the biomass in a global way and do not take into consideration the age or size stock structure.

6.3 VARIATION OF THE BIOMASS IN THE INTERVAL T_i

The “total”, “natural” and “by fishing” instantaneous rates can be approximated by the relative mean rates, $\text{rmr}(B_t)$. In fact, it can be said that $\text{rmr}(B_t) \cong \text{rir}(B_t)$ relative to the mean \bar{B}_i . (This relation is exact in the case of the exponential model) .

The following general expression in terms of instantaneous rates

$$\text{rir}(B_t)_{\text{total}} = f(B_t)_{\text{natural}} - F_{\text{ifishing}}$$

can then be approximated, replacing the rates by the respective mean rates in relation to the mean biomass during the interval T_i :

$$\text{rmr}(B_t) = f(\bar{B}_i) - F_i \quad \text{in relation to } \bar{B}_i$$

or

$$\frac{1}{\bar{B}_{i \text{ total}}} \cdot \frac{\Delta B_i}{T_i} \cong f(\bar{B}_i) - F_{\text{ifishing}}$$

and

$$\Delta B_i \cong f(\bar{B}_i) \cdot \bar{B}_i \cdot T_i - F_i \cdot \bar{B}_i \cdot T_i$$

The variation of the biomass due to all causes of mortality is then decomposed into the variation due to natural mortality and the variation due to the fishing mortality:

$$\Delta B_{i \text{ total}} \cong f(\bar{B}_i) \cdot \bar{B}_i \cdot T_i - Y_{i \text{ fishing}}$$

The value of the biomass, B_{i+1} , at the end of the interval T_i is:

$$B_{i+1} = B_i + \bar{B}_i \cdot f(\bar{B}_i) \cdot T_i - Y_i$$

6.4 LONG-TERM PROJECTIONS (LT) (EQUILIBRIUM CONDITIONS)

The situation of equilibrium at the interval T_i implies that the biomass of the stock, at the end of the interval T_i (usually 1 year) is equal to the biomass at the beginning of the same interval, $B_{i+1}=B_i$, or the variation of the biomass is zero $\Delta B_i=0$.

Bringing the instantaneous rates closer to the mean rates, when the stock is in equilibrium, during T_i , then $\Delta B_i=0$ and $\text{rmr}(B_i)=0$. Thus, a equilibrium condition will be:

$$f(\bar{B}_i) = F_i$$

Then the equilibrium conditions, referred to with the subindex E are:

$$f(\bar{B}_E) = F_E$$

$$Y_E = F_E \cdot \bar{B}_E \cdot T$$

6.5 BIOMASS AND FISHING LEVEL INDICES

In practice the values of \bar{B}_i e F_i are not always available and so, one has to look for quantities that are associated with the biomass, B , and the fishing level, F , preferably quantities (called *indices*) proportional to those parameters.

Let \bar{U} be an index of the mean biomass, \bar{B} , then during the interval of time T we have:

$$\bar{U} = q \cdot \bar{B}$$

and let f be the index of fishing mortality coefficient, F_i , then during the interval of time T :

$$f = \text{const} \cdot F \cdot T$$

$$\text{from} \quad Y = F \cdot \bar{B} \cdot T \quad \text{and} \quad \bar{U} = q \cdot \bar{B}$$

$$\text{will have} \quad Y = F \cdot (1/q) \cdot \bar{U} \cdot T$$

thus, to have $Y = f \cdot \bar{U}$ is necessary to be $f = \frac{1}{q} \cdot FT$

A very common index of \bar{B} is the catch per unit effort (cpue). The index of F will be the fishing effort in an appropriate unit, in order to be proportional to the fishing level.

The constant of proportionality, q , is designated as the capturability or catchability coefficient and indicates the fraction of the biomass that is caught by unit of effort.

6.6 BIOLOGICAL TARGET REFERENCE POINTS (TRP)

Long-term (or equilibrium) biological reference points can also be defined for these models.

F_{MSY} is the value of F that makes the long-term capture, Y , maximum.

F_{MSY} is different of F_{max} . In fact F_{MSY} maximizes the Catch in weight, while F_{max} , maximizes the Catch in weight per Recruit. Notice that the value of F_{max} cannot be calculated with production models, because the age structure of the stock and the recruitment, R are considered implicit in the basic assumptions of the model.

The biological reference points depend on the basic assumptions of the model, therefore the value of F_{MSY} of the structural models is different from the value of F_{MSY} of the production models because the relation S - R , as well as the natural mortality coefficient, M , are implicit in the production models.

To compare results of the two types of models one has to take into consideration that each model is based on different basic assumptions.

For the same reasons, $F_{0.1}$ of the production models is a different concept to $F_{0.1}$ of the structural models.

$F_{0.1}$, $B_{0.1}$ and $Y_{0.1}$ of the productions models could be calculated directly from the basic assumptions but it is preferable to obtain those characteristics using the constant relations between the reference points 0.1 and MSY (Cadima, 1991).

6.7 TYPES OF PRODUCTION MODELS

The most common production models in fishery stocks assessment are the Schaefer model (1954), the Fox model (1970) and the Pella and Tomlinson model (1969), the latter is also designated as GENPROD (name of the computer program that the authors elaborated for the application of their model). Fox mentions that the elaboration of his model was based on an idea from Garrod (1969).

Each one of these models corresponds to one particular function of $H(B_t)$ of the basic assumption.

6.7.1 SCHAEFER MODEL

The function $H(B_t)$ of the basic assumption of this model is:

$$H(B_t) = B_t^{-1}$$

Relative instantaneous rate, $\text{rir}(B_t)$, due to natural causes

The general basic assumption of the Schaefer model is:

$$\text{rir}_{\text{natural}}[k^{-1} - B^{-1}] = -r$$

and then, the instantaneous rate of variation of the “natural” biomass can be mathematically deduced as:

$$\text{rir}(B_t)_{\text{natural}} = r \cdot \left(1 - \frac{B_t}{k} \right)$$

Equilibrium conditions

The relative mean rate, $\text{rmr}(B_t)$, in relation to \bar{B} , will be:

$$\text{tmr}(B_t)_{\text{relative to } \bar{B}_E} = f(\bar{B}_E) = r \cdot \left(1 - \frac{\bar{B}_E}{k} \right)$$

and, as in equilibrium, $f(\bar{B}_E) = F_E$, the equilibrium conditions can then be expressed as:

$$\begin{aligned} B_E &= k \cdot (1 - F_E / r) \\ Y_E &= F_E \cdot \bar{B}_E \cdot T_i \end{aligned}$$

Notice that \bar{B}_E is linear with F_E and for $F_E = 0$, $\bar{B}_E = k$ = carrying capacity = virgin biomass.

Graphically, the relation between \bar{B}_E and F_E shows a straight line with interception equal to k and slope equal to $-k/r$.

Target point, F_{MSY}

The Schaefer equilibrium conditions during one year are:

$$Y_E = F_E \cdot \bar{B}_E \qquad \bar{B}_E = k \cdot (1 - F_E / r)$$

Y maximum will occur when $dY/dF=0$, then derivating the previous expression of Y_E in order to F and making it equal to zero the target point F_{MSY} will be:

Target point, F_{MSY} (Schaefer)

$$F_{\text{MSY}} = r/2 \qquad B_{\text{MSY}} = k/2 \qquad Y_{\text{MSY}} = rk/4$$

In fact, the derivative

$$dY/dF = \bar{B} + F (dB/dF)$$

or
$$dY/dF = k(1 - F/r) + F (-k/r) = k - 2k.F/r$$

and then,
$$F_{MSY} = r/2$$

the relations of the remaining characteristics are obtained by substituting this result in the equilibrium conditions.

Target point, $F_{0.1}$

The ratio between $F_{0.1}$ and F_{MSY} is constant and equal to 0.90, so:

Target point, $F_{0.1}$ (Schaefer)

$$F_{0.1}/F_{MSY} = 0.90 \qquad B_{0.1}/B_{MSY} = 1.10 \qquad Y_{0.1}/Y_{MSY} = 0.99$$

In fact, as seen before, $dY/dF = k - 2k.F/r$ and, as $F_{0.1}$ corresponds to $dY/dF = 0.1k$, so:

$$0.1k = k - 2kF_{0.1}/r$$

or
$$0.90 = 2 F_{0.1}/r$$

or
$$0.90 = F_{0.1}/F_{MSY}$$

Abundance indices, \bar{U} , and fishing level indices, f

As seen in Section 6.5, the indices \bar{U} and f , are assumed to be proportional to \bar{B} and F , so the equilibrium condition can be written as:

$$\bar{U}_E = a + b.f_E \text{ and } Y_E = f_E \cdot \bar{U}_E \qquad (a, b \text{ are constants}).$$

The target point, f_{MSY} , is obtained by equating to zero the derivative of Y_E in order to f_E :

Target point, f_{MSY} (Schaefer)

$$f_{MSY} = -a/(2b) \qquad \bar{U}_{MSY} = a/2 \qquad Y_{MSY} = -a^2/(4b)$$

In the production models, the ratios $f_{0.1}/f_{MSY}$ e $\bar{U}_{0.1}/\bar{U}_{MSY}$ are equal to the ratios $F_{0.1}/F_{MSY}$ and $B_{0.1}/B_{MSY}$. With Schaefer's model we will then have:

Target point, $f_{0.1}$ (Schaefer)

$$f_{0.1}/f_{MSY} = 0.90 \qquad \bar{U}_{0.1}/\bar{U}_{MSY} = 1.10 \qquad Y_{0.1}/Y_{MSY} = 0.99$$

From $\bar{U} = q \cdot \bar{B}$, and $FT = q \cdot f$, the previous expressions of F_{MSY} and f_{MSY} , one can also obtain the relations between the parameters k and r and the coefficients a , b and q :

$$k = a/q \quad r = -a q / (bT) \quad kr = -a^2 / (bT)$$

When the value of the interval T is 1 year, T will not appear in these expressions. It is possible to calculate the parameters k and r , knowing the values of the capturability coefficient, q . Notice that the product $k \cdot r$ does not depend on q .

6.7.2 FOX MODEL

For the Fox production model the function $H(B_t)$ will be:

$$H(B_t) = \ln(B_t)$$

Relative instantaneous rate, $rir(B_t)$, due to natural causes

For the Fox model, from the expression of the general basic assumption, we have:

$$rir_{\text{natural}} [\ln k - \ln B] = -r$$

and then, as previously referred to, the instantaneous rate of variation of the “natural” biomass can be mathematically deduced from that expression and written as:

$$rir(B_t)_{\text{natural}} = r \cdot \ln(k / B_t)$$

Equilibrium conditions

The equilibrium condition of the biomass can be expressed by:

$$r \cdot \ln(k / \bar{B}_E) = F_E$$

Then, the equilibrium conditions will be:

$$\bar{B}_E = k \cdot e^{-F_E / r} \quad Y_E = F_E \cdot \bar{B}_E \cdot T_i$$

Notice that $\ln(\bar{B}_E)$ is linear with F_E and that, for $F_E = 0$, $\bar{B}_E = k$ = virgin biomass or carrying capacity. The relation between $\ln(\bar{B}_E)$ and F_E is linear, with interception equal to $\ln k$, and slope = $-1/r$.

Target point, F_{MSY}

Derivating Y_E in order to F and equating the derivative to zero, F_{MSY} , B_{MSY} and Y_{MSY} will be:

$$F_{MSY} = r \quad B_{MSY} = k/e \quad Y_{MSY} = rk/e$$

Target point, $F_{0.1}$

In this model the ratio between $F_{0.1}$ and F_{MSY} is constant and equal to 0.7815. So, it can be written:

$$F_{0.1}/F_{MSY} = 0.7815 \quad B_{0.1}/B_{MSY} = 1.2442 \quad Y_{0.1}/Y_{MSY} = 0.9724$$

These results are obtained in a similar way to those for the Schaefer model. The equation to solve will be:

$$e^{-F_{0.1}/r} \cdot (1 - F_{0.1}/r) = 0.1$$

which requires iterative methods to find the value of $F_{0.1}/r$. The solution is $F_{0.1}/r = 0.7815$ that is equal to $F_{0.1}/F_{MSY}$.

Abundance indices, \bar{U} , and fishing level indices, f

For the Fox model the equilibrium condition can be written as:

$$\bar{U}_E = e^{a+b \cdot f_E} \quad \text{or} \quad \ln \bar{U}_E = a + b \cdot f \quad (a, b \text{ are constants})$$

and

$$Y_E = f_E \cdot \bar{U}_E$$

Target point, f_{MSY}

The target point, f_{MSY} , can be obtained by equating to zero the derivative of Y_E in order to f_E :

$$f_{MSY} = -1/b \quad \bar{U}_{MSY} = e^a/e \quad Y_{MSY} = -e^a/b e$$

Target point, $f_{0.1}$

In the Fox model, the ratios $f_{0.1}/f_{MSY}$ et $\bar{U}_{0.1}/\bar{U}_{MSY}$ are equal to $F_{0.1}/F_{MSY}$ and $\bar{B}_{0.1}/\bar{B}_{MSY}$ and then:

$$f_{0.1}/f_{MSY} = 0.7815 \quad \bar{U}_{0.1}/\bar{U}_{MSY} = 1.2442 \quad Y_{0.1}/Y_{MSY} = 0.9724$$

From $\bar{U} = q \cdot \bar{B}$ and from $FT = q \cdot f$, the following can be deduced:

$$k = e^a/q, \quad r = -q/(bT) \quad kr = -e^a/(bT)$$

When the value of the interval T is one year, T will not appear in those expressions. The last expression allows the calculation of the product $k \cdot r$. To calculate k and r separately it is necessary to know the value of the coefficient of capturability, q .

6.7.3 PELLA AND TOMLINSON MODEL (GENPROD)

For this production model the function $H(B_t)$ will be:

$$H(B_t) = B_t^{-p}$$

Relative instantaneous rate, $\text{rir}(B_t)$, due to natural causes

The expression of the basic assumption of the GENPROD model, will be :

$$\text{rir}_{\text{natural}}(k^{-p} - B^{-p}) = -r$$

therefore

$$\text{rir}(B_t)_{\text{natural}} = (r/p) [1 - (k/\bar{B})^{-p}]$$

Equilibrium conditions

In equilibrium conditions, F_E will be:

$$F_E = f(\bar{B}_E) = (r/p) [1 - (k/\bar{B})^{-p}]$$

Then, the equilibrium conditions can be expressed as:

$$\bar{B}_E = k [1 - p F_E / r]^{1/p} \quad Y_E = F_E \cdot \bar{B}_E \cdot T_i$$

Notice that the relation between $(\bar{B}_E)^p$ and F_E is linear with intercept equal to k and the slope equal to $-pk/r$, in conclusion, for $F_E = 0$, $\bar{B}_E = k$ = carrying capacity=virgin biomass.

Target point, F_{MSY}

Derivating Y_E in order to F and equating to zero, we will have:

Target point, F_{MSY} (Pella and Tomlinson)

$$F_{MSY} = r \left(\frac{1}{1+p} \right)^{\frac{1}{p}} \quad B_{MSY} = k \left(\frac{1}{1+p} \right)^{\frac{1}{p}} \quad Y_{MSY} = r \cdot k \left(\frac{1}{1+p} \right)^{\frac{1}{p}} \left(\frac{1}{1+p} + 1 \right)^{\frac{1}{p}}$$

Target point, $F_{0.1}$

The ratio between $F_{0.1}$ and F_{MSY} is constant for each value of p and can be obtained in a similar way to the previous cases. The equation to solve by iterative methods is:

$$X = 1 - 0.1 \cdot [1 - p / (1+p) \cdot X]^{1/p} \quad \text{where} \quad X = F_{0.1} / F_{MSY}$$

$$\text{And also} \quad B_{0.1} / B_{MSY} = [1 + p - p / (1+p) \cdot X]^{(1+1/p)}$$

$$Y_{0.1} / Y_{MSY} = [F_{0.1} / F_{MSY}] \cdot [B_{0.1} / B_{MSY}]$$

The following Table summarizes the most important results:

	parameter p	$F_{0.1}/F_{MSY}$	$B_{0.1}/B_{MSY}$	$Y_{0.1}/Y_{MSY}$
Fox	0.0 *	0.781521	1.244182	0.972355
	0.2	0.819995	1.193441	0.978616
	0.4	0.848355	1.158613	0.982915
	0.6	0.869888	1.133469	0.985991
	0.8	0.886657	1.114599	0.988268
Schaefer	1.0 *	0.900000	1.100000	0.990000
	1.2	0.910816	1.088420	0.991350
	1.4	0.919724	1.079045	0.992424
	1.6	0.927165	1.071323	0.993293
	1.8	0.933457	1.064867	0.994008
	2.0	0.938835	1.059401	0.994602
	2.2	0.94377	1.054720	0.995704
	2.4	0.947516	1.050674	0.995531
	2.6	0.951059	1.047146	0.995898
	2.8	0.954188	1.044045	0.996216
	3.0	0.956969	1.041302	0.996494

Notice that $(F_{0.1}/F_{MSY}) + (B_{0.1}/B_{MSY}) \cong 2$. From this result it can be said that when $F_{0.1}$ is smaller than F_{MSY} by a certain percentage, the equivalent relation of the biomasses will be bigger by the same percentage.

Abundance indices, \bar{U} , and fishing level indices, f

For the Pella and Tomlinson model, the equilibrium conditions can be written as:

$$\bar{U} = (a + b.f)^{1/p} \quad \text{or} \quad \bar{U}^p = (a + b.f) \quad (a, b \text{ are constants}).$$

$$Y_E = f_E \cdot \bar{U}_E$$

The target point, f_{MSY} , can be obtained by equating to zero the derivative of Y_E in order to f_E :

Target point, f_{MSY}

$$f_{MSY} = -a/(b(1+1/p)) \quad \bar{U}_{MSY} = [a/(1+p)]^{1/p} \quad Y_{MSY} = -(p/b).(a/(1+p))^{(1+1/p)}$$

The ratios $f_{0.1}/f_{MSY}$ and $\bar{U}_{0.1}/\bar{U}_{MSY}$ will be equal to the ratios $F_{0.1}/F_{MSY}$ and $\bar{B}_{0.1}/\bar{B}_{MSY}$, respectively. These last ratios can be observed in the previous Table.

The values of k , r e kr can also be obtained from $\bar{U} = q \cdot \bar{B}$, and from $F.T = q.f$

$$k = a^{1/p}/q \quad r = -apq/(bT) \quad kr = (-p/bT)a^{(1+1/p)}$$

When the size of the interval is one year, T will not appear in those expressions. The last expression in the previous table allows the calculation of the product k.r. The separate values of k and r can be calculated if the value of the coefficient of capturability, q is known.

Comments

1. The Pella and Tomlinson model has been criticized in its practical application because sometimes it produces better adjustments with non reliable values of the parameter p, resulting in extremely high values of F_{MSY} .
2. It is also important to notice that p, the additional parameter of that model, is written with different symbols depending on the authors.
3. The values of the biological reference points relative to F, estimated by Schaefer's model, are more restrictive than the corresponding values estimated by the Fox or GENPROD production models.

6.8 SHORT-TERM PROJECTIONS

6.8.1 GENERAL METHODS

Long-term projections have been estimated in fisheries since the 50's using these production models but in practice, it was only in the 90's that methods were developed for short-term projections. These methods are based in the Schaefer, Fox and Pella et Tomlinson expressions for the non-exploited biomass.

By applying production models as referred to in Section 6.3, the variation of the biomass for 1 year can be expressed, in a general way, as:

$$\Delta B_{i\text{total}} \cong f(\bar{B}_i) \cdot \bar{B}_i \cdot T_i - Y_{i\text{fishing}}$$

or
$$B_{i+1} = B_i + \bar{B}_i \cdot f(\bar{B}_i) \cdot T_i - Y_i$$

where B_i = biomass at the beginning of the year i

B_{i+1} = biomass at the end of the year i

\bar{B}_i = mean biomass during the year i

Y_i = catch in weight during the year i

$f(\bar{B}_i)$ is the approximation of the mean rate of "natural" variation of the biomass, relative to \bar{B}_i during the year i.

The expression of the variation of the biomass is the basis for most of the methods for short-term projections. Computer programs were prepared for the application of these

methods, which also determine long-term projections, biological reference points, etc. Some examples are CEDA and BYODIN, Rosenberg *et al.* (1990), and Punt and Hilborn (1996) respectively.

Theoretically those methods suppose that \bar{B}_i et Y_i are known for a period of years. The function $f(\bar{B}_i)$ can be that of Schaefer, Fox or Pella and Tomlinson.

To determine the parameters r and k it would be necessary to adopt one of the expressions of $f(\bar{B}_i)$ and the value B_1 in the first interval of the period of years.

In practice, the values of the annual mean biomasses are not available, only the associated quantities, usually assumed to be proportional to the mean biomasses, that is indices $U_i = q \cdot \bar{B}_i$ so, the parameters to be estimated are r , k and q (see Chapter 7).

The object function, of the least squares method, to be minimized is $\Phi = \sum (U_{\text{obs}} - U_{\text{mod}})^2$ that is, the sum of the squares of the residuals between the observed values and the estimated values (*designated by error of the process*). However, when the relation $U = q \cdot \bar{B}_i$ is not determinant, but it is supposed to have an error, *designated as observation error*, then it is preferable to adopt the object function $\Phi = \sum (\ln U_{\text{obs}} - \ln U_{\text{mod}})^2$ (Punt and Hilborn, 1996).

6.8.2 PRAGER METHOD (1994)

Prager (1994) adopted the Schaefer model and used the relative *instantaneous* rate of the variation of the biomass in the initial basic expression (not the *mean* rate approximation) that is,

$$\text{rir}(B_t) = r[1 - B_t / k] \cdot B_t - F_t \cdot B_t$$

He integrated this expression during the year i and obtained the relation between B_{i+1} and B_i

He also calculated the mean biomass, \bar{B}_i , integrating B_t during the year i . Finally the catch in weight is calculated as:

$$Y_i = F_i \cdot \bar{B}_i.$$

The estimation of the parameters can then be made using the least squares method. The computer program prepared for this estimation is called ASPIC (Prager, 1995).

6.8.3 YOSHIMOTO AND CLARKE METHOD (1993)

The short-term projections are derived from the basic assumption of the production models,

$$\text{rir}(B_t)_{\text{total}} = \text{rir}(B_t)_{\text{natural}} - (F_t)_{\text{fishing}}$$

or, representing $\text{rir}(B_t)_{\text{natural}}$ by $f(B_t)$:

$$\text{rir}(B_t)_{\text{total}} = f(B_t) - (F_t)_{\text{fishing}}$$

Integrating this expression during the interval of time T_i and considering that:

$$\text{rir}(B_t) = \text{air} \ln(B_t) \quad \text{and} \quad F_t = F_i = \text{constant:}$$

$$\int_{t_i}^{t_{i+1}} \text{air}(\ln B_t) dt =$$

$$\ln(B_{i+1}) - \ln(B_i) = f(B_t)_{\text{cum}_i} - F_i T_i = \bar{f}(B_t) T_i - F_i T_i$$

For the next interval T_{i+1} (which is the interval where one intends to project the stock and the catch):

$$\ln(B_{i+2}) - \ln(B_{i+1}) = \bar{f}(B_{i+1}) \cdot T_{i+1} - F_{i+1} \cdot T_{i+1}$$

Calculating the simple arithmetic mean of the two previous expressions and considering that :

$$1/2[\ln(B_{i+1}) + \ln(B_{i+2})] - 1/2[\ln(B_i) + \ln(B_{i+1})] = \ln(\bar{B}_{i+1}^*) - \ln(\bar{B}_i^*)$$

where \bar{B}_i^* is the geometric mean of B_i and B_{i+1} , and \bar{B}_{i+1}^* is the geometric mean of B_{i+1} and B_{i+2} ,

Therefore, the mean of the two expressions will be:

$$\ln(\bar{B}_{i+1}^*) - \ln(\bar{B}_i^*) = (1/2) \{ \bar{f}(B_i) T_i + \bar{f}(B_{i+1}) T_{i+1} \} - (1/2) \{ F_i T_i + F_{i+1} T_{i+1} \}$$

The natural $\text{rir}(B_t)$ of the Fox model is, as mentioned before $f(B_t) = r (\ln k - \ln B_t)$, so the approximation, $\bar{f}(B_i)$ can be written as:

$$r[\ln k - \ln(\bar{B}_i^*)]$$

where \bar{B}_i^* is the geometric mean of B_i and B_{i+1} .

Therefore, the previous expression relative to the geometric means, can be re-written as:

$$\begin{aligned} \ln(\bar{B}_{i+1}^*) - \ln(\bar{B}_i^*) &= \\ &= (1/2) \{ r(\ln k - \ln(\bar{B}_i^*)) T_i + r(\ln k - \ln(\bar{B}_{i+1}^*)) T_{i+1} \} - (1/2) \{ F_i T_i + F_{i+1} T_{i+1} \} \end{aligned}$$

To simplify, and as the intervals of T_i are usually constant (and equal to one year), one can use T instead of T_i and T_{i+1} and the expression will be:

$$\ln(\bar{B}_{i+1}^*) - \ln(\bar{B}_i^*) = (rT/2) \{ \ln k - \ln(\bar{B}_i^*) + \ln k - \ln(\bar{B}_{i+1}^*) \} - (1/2) \{ F_i T + F_{i+1} T \}$$

or reorganizing the terms of this expression, it will be:

$$(1 + rT/2) \cdot \ln(\bar{B}_{i+1}^*) = rT \ln k + (1 - rT/2) \ln(\bar{B}_i^*) - (T/2) (F_i + F_{i+1})$$

Finally, the expression can be written as follows:

$$\ln(\bar{B}_{i+1}^*) = 2rT/(2+rT) \cdot \ln k + (2-rT)/(2+rT) \cdot \ln(\bar{B}_i^*) - T/(2+rT) \cdot (F_i + F_{i+1})$$

As seen in the long-term projections (or equilibrium), it is more common to have biomass indices, \bar{U} , and fishing level indices, f , rather than \bar{B} et F values.

Using the indices $\bar{U} = q \bar{B}$ and $qf = FT$

the Yoshimoto and Clarke expression (1993) can be written as :

$$\ln \bar{U}_{i+1} = \frac{2rT}{2+rT} \cdot \ln(qk) + \frac{2-rT}{2+rT} \cdot \ln \bar{U}_i - \frac{q}{2+rT} (f_i + f_{i+1})$$

It is useful, in practice, to write this expression in the following way:

$$\ln \bar{U}_{i+1} = b_1 + b_2 \cdot \ln \bar{U}_i + b_3 \cdot (f_i + f_{i+1})$$

where:

$$b_1 = \frac{2rT}{2+rT} \ln(qk)$$

$$b_2 = \frac{2-rT}{2+rT}$$

$$b_3 = -\frac{q}{2+rT}$$

From the coefficients b_1 , b_2 and b_3 one can estimate the parameters q , r and k (keep in mind that in the long-term projections it was not possible to obtain q separately) as :

$$q = -4b_3/(1+b_2)$$

$$rT = 2(1-b_2) / (1+b_2)$$

$$k = ((1+b_2)/(-4b_3)) \cdot e^{b_1/(1-b_2)}$$

Comments

1. The fact of having developed, in this manual, the Yoshimoto and Clarke model for the short-term projections, does not mean a special preference for this model over other models for the short-term projections.
2. Yoshimoto and Clarke designated their expression by the integrated expression of Fox, as it is based on the direct integration of the basic assumption.

3. Notice that $\bar{f}(B_i)$ and $\bar{f}(B_{i+1})$ are, in general, different from $f(\bar{B}_i)$ and $f(\bar{B}_{i+1})$. However, the means of $f(\cdot)$ may be considered equal to $f(\text{means of } B)$ if another type of mean of B is used.

- Definition of \bar{B}_i^* through a function

Consider n values B_i and the simple arithmetic mean of $f(B_i)$, that is,

$$\overline{f(B)} = (1/n) \cdot \sum f(B_i)$$

- Let \bar{B}^* be a value such as $f(\bar{B}^*) = \overline{f(B)}$.

\bar{B}^* is designated as the mean of B_i through the function f .

EXAMPLES

4. If $f(B) = \ln(B)$ then $\ln(\bar{B}^*) = (1/n) \cdot \sum \ln(B_i)$ and \bar{B}^* is the mean of the values B_i through the logarithm function, also designated as geometric mean of the values B_i
5. If $f(B) = B^{-1}$ then $(\bar{B}^*)^{-1} = (1/n) \cdot \sum (B_i)^{-1}$ and \bar{B}^* is designated as harmonic mean of the values B_i
6. If $f(B) = B^{-p}$ then $(\bar{B}^*)^{-p} = (1/n) \cdot \sum (B_i)^{-p}$ and \bar{B}^* is designated as the mean of order $(-p)$ of the values B_i .
7. Another approach (Cadima & Pinho, 1995) of the integrated equation of Fox can be:

$$\ln \bar{U}_{i+1} = b_1 + b_2 \cdot \ln \bar{U}_i + b_3 \cdot (f_i + f_{i+1})$$

where:

$$\begin{aligned} b_1 &= (1 - e^{-rT}) \ln(qk) & k &= \frac{1 - b_2}{-2 \cdot b_3 \cdot \ln b_2} \cdot e^{\frac{b_1}{1-b_2}} \\ b_2 &= e^{-rT} & rT &= -\ln b_2 \\ b_3 &= -q(1 - e^{-rT}) / (2rT) & q &= 2 \cdot \ln b_2 \cdot \frac{b_3}{1 - b_2} \end{aligned}$$

This last approach of the integrated Fox model can be deduced from the basic assumption of the model, during the interval T_i :

$$r_{ir}(B_t)_{\text{total}} = r \cdot (\ln k - \ln B_t)_{\text{naturalFox}} - (F_i)_{\text{fishing}}$$

Taking into account the properties of rates and assuming r , k and F_i constant during T_i interval, the absolute instantaneous rate of $[r \cdot (\ln k - \ln B_t) - F_i]$ will be:

$$\text{air}[r \cdot (\ln k - \ln B_t) - F_i] = -r \cdot \text{air}(\ln B_t) = -r \cdot r_{ir}(B_t)$$

So substituting $r_{ir}(B_t)$ by the Fox expression mentioned before, one can write:

$$\text{air}[r \cdot (\ln k - \ln B_t) - F_i] = -r \cdot [r \cdot (\ln k - \ln B_t) - F_i]$$

or
$$\frac{\text{air}[r.(\ln k - \ln B_t) - F_i]}{[r.(\ln k - \ln B_t) - F_i]} = -r$$

Finally, by the definition of r the expression will be:

$$r[r.(\ln k - \ln B_t) - F_i] = -r$$

showing that $[r.(\ln k - \ln B_t) - F_i]$ follows an exponential model, during the interval T_i with r constant.

So, the final value of $(r.\ln k - r.\ln B_t - F_i)$, can be expressed as:

$$(r.\ln k - r.\ln B_{i+1} - F_i) = (r.\ln k - r.\ln B_i - F_i). e^{-r.T_i}$$

or

$$\ln B_{i+1} = (1 - e^{-r.T_i}).\ln k + e^{-r.T_i}.\ln B_i - (1 - e^{-r.T_i}).F_i/r$$

At the following interval, T_{i+1} , the expression would be:

$$\ln B_{i+2} = (1 - e^{-r.T_{i+1}}).\ln k + e^{-r.T_{i+1}}.\ln B_{i+1} - (1 - e^{-r.T_{i+1}}).F_{i+1}/r$$

then, the mean of the two previous expressions, considering $T_i = T_{i+1} = T$, will be:

$$\ln \bar{B}_{i+1}^* = (1 - e^{-r.T}).\ln k + e^{-r.T}.\ln \bar{B}_i^* - ((1 - e^{-r.T}) / 2r).(F_i + F_{i+1})$$

where \bar{B}_i^* = geometric mean of B_i and B_{i+1} and \bar{B}_{i+1}^* = geometric mean of B_{i+1} and B_{i+2} .

Using the indices $\bar{U}_i = q. \bar{B}_i^*$ and $qf_i = F_i T$ the expression will be:

$$\ln \bar{U}_{i+1} = (1 - e^{-r.T}).\ln (qk) + e^{-r.T}.\ln \bar{U}_i - ((1 - e^{-r.T})/2rT).q(f_i + f_{i+1})$$

which is the initial expression of comment n° 7.

CHAPTER 7 – ESTIMATION OF PARAMETERS

In the previous chapters, several models used in stock assessment were analysed, the respective parameters having been defined. In the corresponding exercises, it was not necessary to estimate the values of the parameters because they were given. In this chapter, several methods of estimating parameters will be analysed. In order to estimate the parameters, it is necessary to know the sampling theory and statistical inference.

This manual will use one of the general methods most commonly used in the estimation of parameters – the least squares method. In many cases this method uses iterative processes, which require the adoption of initial values. Therefore, particular methods will also be presented, which obtain estimates close to the real values of the parameters. In many situations, these initial estimates also have a practical interest. These methods will be illustrated with the estimation of the growth parameters and the S-R stock-recruitment relation.

The least squares method is presented under the forms of Simple linear Regression, multiple linear model and non linear models (method of Gauss-Newton).

Subjects like residual analysis, sampling distribution of the estimators (asymptotic or empiric Bootstrap and jackknife), confidence limits and intervals, etc., are important. However, these matters would need a more extensive course.

7.1 SIMPLE LINEAR REGRESSION – LEAST SQUARES METHOD

Model

Consider the following variables and parameters:

Response or dependent variable	= Y
Auxiliary or independent variable	= X
Parameters	= A,B

The response variable is linear with the parameters

$$Y = A + BX$$

Objective

The objective of the method is to estimate the parameters of the model, based on the observed pairs of values and applying a certain criterium function (the observed pairs of values are constituted by selected values of the auxiliary variable and by the corresponding observed values of the response variable), that is :

Observed values x_i and y_i for each pair i , where $i=1,2,...,i,...,n$
 Values to be estimated A and B and $(Y_1, Y_2, ..., Y_i, ..., Y_n)$ for the n observed pairs of values

(Estimates values: \hat{A} and \hat{B} (or a and b) and $(\hat{Y}_1, \hat{Y}_2, ..., \hat{Y}_i, ..., \hat{Y}_n)$)

Object function (or criterium function)

$$\Phi = \sum_{i=1}^n (y_i - Y_i)^2$$

Estimation method

In the least squares method the estimators are the values of A and B which minimize the object function. Thus, one has to calculate the derivatives $\partial\Phi/\partial A$ e $\partial\Phi/\partial B$, equate them to zero and solve the system of equations in A and B.

The solution of the system can be presented as :

$$\bar{x} = (1/n) \cdot \sum x$$

$$\bar{y} = (1/n) \cdot \sum y$$

$$S_{xx} = \sum (x - \bar{x})(x - \bar{x})$$

$$S_{xy} = \sum (x - \bar{x})(y - \bar{y})$$

$$b = S_{xy}/S_{xx}$$

$$a = \bar{y} - b \cdot \bar{x}$$

Notice that the observed values y , for the same set of selected values of X , depend on the collected sample. For this reason, the problem of the simple linear regression is usually presented in the form :

$$y = A + BX + \varepsilon$$

where ε is a random variable with *expected value* equal to zero and *variance* equal to σ^2 .
 So, the expected value of y will be Y or $A+BX$ and the variance of y will be equal to the variance of ε .

The terms deviation and residual will be used in the following ways:

Deviation is the difference between y_{observed} and $y_{\text{mean}} (\bar{y})$ i.e., deviation = $(y - \bar{y})$

while

Residual is the difference between y_{observed} and $Y_{\text{estimated}} (\hat{Y})$, i.e., residual = $(y_i - \hat{Y}_i)$.

To analyse the adjustment of the model to the observed data, it is necessary to consider the following characteristics:

Sum of squares of the residuals:

$$SQ_{\text{residual}} = \sum (y - \hat{Y})^2$$

This quantity indicates the residual variation of the observed values in relation to the estimated values of the response variable of the model, which can be considered as the variation of the observed values that is not explained by the model.

Sum of squares of the deviations of the estimated values of the response variable of the model:

$$SQ_{\text{model}} = \sum (\hat{Y} - \bar{y})^2$$

This quantity indicates the variation of the estimated values of the response variable of the model in relation to its mean, that is the *variation* of the response variable explained by the model.

Total sum of squares of the deviations of the observed values equal to:

$$SQ_{\text{total}} = \sum (y - \bar{y})^2$$

This quantity indicates the total variation of the observed values in relation to the mean

It is easy to verify the following relation:

$$SQ_{\text{total}} = SQ_{\text{model}} + SQ_{\text{residual}}$$

or

$$1 = \frac{SQ_{\text{model}}}{SQ_{\text{total}}} + \frac{SQ_{\text{residual}}}{SQ_{\text{total}}}$$

or

$$1 = r^2 + (1 - r^2)$$

where

r^2 (coefficient of determination) is the percentage of the total variation that is explained by the model and

$1-r^2$ is the percentage of the total variation that *is not explained* by the model.

7.2 MULTIPLE LINEAR REGRESSION – LEAST SQUARES METHOD

Model

Consider the following variables and parameters:

Response or dependent variable	= Y
Auxiliary or independent variables	= $X_1, X_2, \dots, X_j, \dots, X_k$
Parameters	= $B_1, B_2, \dots, B_j, \dots, B_k$

The response variable is linear with the parameters

$$Y = B_1X_1 + B_2X_2 + \dots + B_kX_k = \sum B_jX_j$$

Objective

The objective of the method is to estimate the parameters of the model, based on the observed n sets of values and by applying a certain criterium function (the observed sets of values are constituted by selected values of the auxiliary variable and by the corresponding observed values of the response variable), that is:

Observed values $X_{1,i}, X_{2,i}, \dots, X_{j,i}, \dots, X_{k,i}$ and y_i for each set i , where $i=1, 2, \dots, i, \dots, n$

Values to be estimated $B_1, B_2, \dots, B_j, \dots, B_k$ et $(Y_1, Y_2, \dots, Y_i, \dots, Y_n)$

The estimated values can be represented by :

$$\hat{B}_1, \hat{B}_2, \dots, \hat{B}_j, \dots, \hat{B}_k \text{ (ou } b_1, b_2, \dots, b_j, \dots, b_k) \text{ et } \hat{Y}_1, \hat{Y}_2, \dots, \hat{Y}_i, \dots, \hat{Y}_n$$

Object function (or criterium function)

$$\Phi = \sum_{i=1}^n (y_i - Y_i)^2$$

Estimation method

In the *least squares method* the estimators are the values of B_j which minimize the object function.

As with the simple linear model, the procedure of minimization requires equating the partial derivatives of Φ to zero in order to each parameter, B_j , where $j=1, 2, \dots, k$. The system is preferably solved using matrix calculus.

Matrix version

Matrix $X_{(n,k)}$ = Matrix of the n observed values of each of the k auxiliary variables

Vector $y_{(n,1)}$ = Vector of the n observed values of the response variable

Vector $Y_{(n,1)}$ = Vector of the values of the response variable given by the model (unknown)

Vector $B_{(k,1)}$ = Vector of the parameters

Vector \hat{B} or $b_{(k,1)}$ = Vector of the estimators of the parameters

Model

$$Y_{(n,1)} = X_{(n,k)} \cdot B_{(k,1)} \text{ ou } Y = X \cdot B + \varepsilon$$

Object function

$$\Phi_{(1,1)} = (y - Y)^T \cdot (y - Y) \text{ ou } \Phi_{(1,1)} = (y - X \cdot B)^T \cdot (y - X \cdot B)$$

To calculate the least squares estimators it will suffice to put the derivative $d\Phi/dB$ of Φ in order to vector B , equal to zero. $d\Phi/dB$ is a vector with components $\partial\Phi/\partial B_1$, $\partial\Phi/\partial B_2$, ..., $\partial\Phi/\partial B_k$. Thus:

$$d\Phi/dB_{(k,1)} = -2 \cdot X^T \cdot (y - X \cdot B) = 0$$

or
$$X^T y - (X^T \cdot X) \cdot B = 0$$

and
$$b = \hat{B} = (X^T \cdot X)^{-1} \cdot X^T y$$

The results can be written as:

$$b_{(k,1)} = (X^T \cdot X)^{-1} \cdot X^T y$$

$$\hat{Y}_{(n,1)} = X \cdot b \quad \text{or} \quad \hat{Y}_{(n,1)} = X (X^T \cdot X)^{-1} \cdot X^T y$$

$$\text{residuals}_{(n,1)} = (y - \hat{Y})$$

Comments

In statistical analysis it is convenient to write the estimators and the sums of the squares using idempotent matrices. Then the idempotent matrices L , $(I - L)$ and $(I - M)$ with

$L_{(n,n)} = X (X^T \cdot X)^{-1} \cdot X^T$, I = unity matrix and $M_{(n,n)} = \text{mean}_{(n,1)}$ matrix = $1/n [1]$ where $[1]$ is a matrix with all its elements equal to one, are used.

It is also important to consider the sampling distributions of the estimators assuming that the variables ε_i are independent and have a normal distribution.

A summary of the main properties of the expected value and variance of the estimators is presented :

	$E[c_1+c_2.u] = c_1+c_2.E[u]$	$V[c_1+c_2.u] = c_2.V[u].c_2^T$
1 –	Random variable, ε Expected value of ε Variance of ε	ε_n . (independent) $E[\varepsilon] = 0$. $V[\varepsilon]_{(n,n)} = E[\varepsilon.\varepsilon^T] = I.\sigma^2$
2 –	Observed response variable y Expected value of y Variance of y	$y = Y + \varepsilon$ $E[y] = Y = X.B$. $V[y]_{(n,n)} = V[\varepsilon]_{(n,n)} = I.\sigma^2$
3 –	Estimator of B Expected value of \hat{B} Variance of \hat{B}	$\hat{B} = (X^T.X)^{-1}.X^T.y$ $E[\hat{B}] = B$ $V[\hat{B}]_{(k,k)} = (X^T.X)^{-1}.\sigma^2$
4 –	Estimator of Y of the model Expected value of \hat{Y} Variance of \hat{Y}	$\hat{Y} = X. \hat{B} = L.y$ $E[\hat{Y}] = Y$. $V[\hat{Y}] = L.\sigma^2$
5 –	Residual e Expected value of e Variance of e	$e = y - \hat{Y} = (I-L).y$ $E[e] = 0$ $V[e] = (I-L).\sigma^2$

6 – Sum of squares

6.1 - Residual Sum of squares = $SQ_{\text{residual}(1,1)} = (y - \hat{Y})^T (y - \hat{Y}) = y^T (I-L)y$

This quantity indicates the residual variation of the observed values in relation to the estimated values of the model, that is, the variation not explained by the model.

6.2 - Sum of squares of the deviation of the model = $SQ_{\text{model}(1,1)} = (\hat{Y} - \bar{y})^T (\hat{Y} - \bar{y}) = y^T (L-M)y$

This quantity indicates the variation of the estimated response values of the model in relation to the mean, that is, the variation explained by the model.

6.3 - Total Sum of the squares of the deviations = $SQ_{total(1.1)} = (y - \bar{y})^T (y - \bar{y}) = y^T (I - M) y$

This quantity indicates the total variation of the observed values in relation to the mean. It is easy to verify the following relation:

$$SQ_{total} = SQ_{model} + SQ_{residual} \quad \text{or}$$

$$1 = \frac{SQ_{model}}{SQ_{total}} + \frac{SQ_{residual}}{SQ_{total}}$$

or $1 = R^2 + (1 - R^2)$

where:

R^2 is the percentage of the total variation that is *explained* by the model. In matrix terms it will be:

$$R^2 = [y^T (L - M) y] \cdot [y^T (I - M) y]^{-1}$$

$1 - R^2$ is the percentage of the total variation that is not explained by the model.

The ranks of the matrices (I-L), (I-M) and (L-M) respectively equal to (n-k), (n-1) and (k-1), are the degrees of freedom associated with the respective sums of squares.

7.3 NON-LINEAR MODEL – METHOD OF GAUSS-NEWTON – LEAST SQUARES METHOD

Model

Consider the following variables and parameters:

Response or dependent variable = Y

Auxiliary or independent variable = X

Parameters = $B_1, B_2, \dots, B_j, \dots, B_k$

The response variable is non-linear with the parameters

$Y = f(X; B)$ where B is a vector with the components $B_1, B_2, \dots, B_j, \dots, B_k$

Objective

The objective of the method is to estimate the parameters of the model, based on the n observed pairs of values and by applying a certain criterium function (the observed sets of values are constituted by selected values of the auxiliary variable and by the corresponding observed values of the response variable), that is:

Observed values x_i and y_i for each pair i , where $i=1,2,\dots,i,\dots,n$

Values to be estimated $B_1, B_2, \dots, B_j, \dots, B_k$ and $(Y_1, Y_2, \dots, Y_i, \dots, Y_n)$ form the n pairs of observed values.

(Estimates = $\hat{B}_1, \hat{B}_2, \dots, \hat{B}_j, \dots, \hat{B}_k$ or $b_1, b_2, \dots, b_j, \dots, b_k$ and $\hat{Y}_1, \hat{Y}_2, \dots, \hat{Y}_i, \dots, \hat{Y}_n$)

Object function or criterium function

$$\Phi = \sum_{i=1}^n (y_i - Y_i)^2$$

Estimation criterium

The estimators will be the values of B_j for which the object function is minimum.
(This criterium is called the least squares method).

Matrix version

It is convenient to present the problem using matrices.
So:

Vector $X_{(n,1)}$ = Vector of the observed values of the auxiliary variable

Vector $y_{(n,1)}$ = Vector of the observed values of the response variable

Vector $Y_{(n,1)}$ = Vector of the values of the response variable given by the model

Vector $B_{(k,1)}$ = Vector of the parameters

Vector $b_{(k,1)}$ = Vector of the estimators of the parameters

Model

$$Y_{(n,1)} = f(X; B)$$

Object function

$$\Phi_{(1,1)} = (y - Y)^T \cdot (y - Y)$$

In the case of the non linear model, it is not easy to solve the system of equations resulting from equating the derivative of the function Φ in order to the vector B , to zero. Estimation by the least squares method can, based on the Taylor series expansion of function Y , use iterative methods.

Revision of the Taylor series expansion of a function

Here is an example of the expansion of a function in the Taylor series in the case of a function with one variable.

The approximation of Taylor means to expand a function $Y = f(x)$ around a selected point, x_0 , in a power series of x :

$$Y = f(x) = f(x_0) + (x-x_0).f'(x_0)/1! + (x-x_0)^2 f''(x_0)/2! + \dots + (x-x_0)^i f^{(i)}(x_0)/i! + \dots$$

where $f^{(i)}(x_0) = i^{\text{th}}$ derivatives of $f(x)$ in order to x , at the point x_0 .

The expansion can be approximated to the desired power of x . When the expansion is approximated to the power 1 it is called a linear approximation, that is,

$$Y \cong f(x_0) + (x-x_0).f'(x_0)$$

The Taylor expansion can be applied to functions with more than one variable. For example, for a function $Y = f(x_1, x_2)$ of two variables, the linear expansion would be:

$$Y \approx f(x_{1(0)}, x_{2(0)}) + (x_1 - x_{1(0)}) \cdot \frac{\delta f(x_{1(0)}, x_{2(0)})}{\delta x_1} + (x_2 - x_{2(0)}) \cdot \frac{\delta f(x_{1(0)}, x_{2(0)})}{\delta x_2}$$

which may be written, in matrix notation, as

$$Y = Y_{(0)} + A_{(0)} \cdot (x - x_{(0)})$$

where $Y_{(0)}$ is the value of the function at the point $x_{(0)}$, with components $x_{1(0)}$ and $x_{2(0)}$, and $A_{(0)}$ is the matrix of derivatives whose elements are equal to the partial derivatives of $f(x_1, x_2)$ in order to x_1, x_2 at the point $(x_{1(0)}, x_{2(0)})$.

To estimate the parameters, the Taylor series expansion of function Y is made in order to the parameters B and not to the vector X .

For example, the linear expansion of $Y = f(x, B)$ in B_1, B_2, \dots, B_k , would be:

$$Y = f(x; B) = f(x; B_{(0)}) + (B_1 - B_{1(0)}) \frac{\partial f}{\partial B_1}(x; B_{(0)}) + \dots + (B_2 - B_{2(0)}) \frac{\partial f}{\partial B_2}(x; B_{(0)}) + \dots + (B_k - B_{k(0)}) \frac{\partial f}{\partial B_k}(x; B_{(0)})$$

or, in matrix notation, it would be :

$$Y_{(n,1)} = Y_{(0) (n,1)} + A_{(0) (n,k)} \cdot \Delta B_{(0) (k,1)}$$

where

A = matrix of order (n, k) of the partial derivatives of the matrix $f(x; B)$ in order to the vector B at the point $B_{(0)}$ and

$$\Delta B_{(0)} = \text{vector } (B - B_{(0)}).$$

Then, the object function will be:

$$\Phi = (y - Y)^T \cdot (y - Y) = (y - Y_{(0)} - A_{(0)} \cdot \Delta B_{(0)})^T (y - Y_{(0)} - A_{(0)} \cdot \Delta B_{(0)})$$

To obtain the minimum of this function it is more convenient to differentiate Φ in order to the vector ΔB than in relation to vector B and put it equal to zero. Thus:

$$0 = -2(A_{(0)})^T (y - Y_{(0)} - A_{(0)} \cdot \Delta B_{(0)}) = -2A_{(0)}^T (y - Y_{(0)}) + 2 A_{(0)}^T A_{(0)} \cdot \Delta B_{(0)}$$

or
$$A_{(0)}^T A_{(0)} \cdot \Delta B_{(0)} = A_{(0)}^T (y - Y_{(0)})$$

Therefore:

$$\Delta B_{(0)} = (A_{(0)}^T \cdot A_{(0)})^{-1} \cdot A_{(0)}^T \cdot (y - Y_{(0)})$$

If $\Delta B_{(0)}$ is “equal to zero” then the estimate of B is equal to $B_{(0)}$.

(In practice, when we say “equal to zero” in this process, we really mean smaller than the approximation vector one has to define beforehand).

If $\Delta B_{(0)}$ is not “equal to zero” then the vector $B_{(0)}$ will be replaced by :

$$B_{(1)} = B_{(0)} + \Delta B_{(0)}$$

And the process will be repeated, that is, there will be another iteration with $B_{(0)}$ replaced by $B_{(1)}$ (and $A_{(0)}$ replaced by $A_{(1)}$). The iterative process will go on until the convergence at the desired level of approximation is reached.

Comments

1. It is not guaranteed that the process always converges. Sometimes it does not, some other times it is too slow (even for computers!) and some other times it converges to another limit!!
2. The above described method is the Gauss-Newton method which is the basis of many other methods. Some of those methods introduce modifications in order to obtain a faster convergence like the Marquardt method (1963), which is frequently used in fisheries research. Other methods use the second order Taylor expansion (Newton-Raphson method), looking for a better approximation. Some others, combine the two modifications.
3. These methods need the calculation of the derivatives of the functions. Some computer programs require the introduction of the mathematical expressions of the derivatives, while others use sub-routines with numerical approximations of the derivatives.
4. In fisheries research, there are methods to calculate the initial values of the parameters, for example in growth, mortality, selectivity or maturity analyses.
5. It is important to point out that the convergence of the iterative methods is faster and more likely to approach the true limit when the initial value of the vector $B_{(0)}$ is close to the real value.

7.4 ESTIMATION OF GROWTH PARAMETERS

The least squares method (non-linear regression) allows the estimation of the parameters K , L_∞ and t_0 of the individual growth equations.

The starting values of K , L_∞ and t_0 for the iterative process of estimation can be obtained by simple linear regression using the following methods :

Ford-Walford (1933-1946) and Gulland and Holt (1959) Methods

The Ford-Walford and Gulland and Holt expressions, which were presented in Section 3.4, are already in their linear form, allowing the estimation of K and L_∞ with methods of simple linear regression on observed L_i and T_i . The Gulland and Holt expression allows the estimation of K and L_∞ even when the intervals of time T_i are not constant. In this case, it is convenient to re-write the expression as:

$$\Delta L/T_i = K \cdot L_\infty - K \cdot \bar{L}$$

Stamatopoulos and Caddy Method (1989)

These authors also present a method to estimate K , L_∞ and t_0 (or L_0) using the simple linear regression. In this case the von Bertalanffy equation should be expressed as a linear relation of L_t against e^{-Kt} .

Consider n pairs of values t_i , L_i where t_i is the age and L_i the length of the individual i where $i=1,2, \dots, n$.

The von Bertalanffy equation , in its general form is (as previously seen):

$$L_\infty - L_t = (L_\infty - L_a) \cdot e^{-K(t-t_a)}$$

It can be written as:

$$L_t = L_\infty - (L_\infty - L_a) \cdot e^{-Kt} \cdot e^{+Kt_a}$$

The equation has the simple linear form, $y = a + bx$, where:

$$\begin{aligned} y &= L_t & a &= L_\infty & b &= - (L_\infty - L_a) \cdot e^{+Kt_a} \\ x &= e^{-Kt} \end{aligned}$$

If one takes $L_a = 0$, then $t_a = t_0$, but, if one considers $t_a = 0$, then $L_a = L_0$.

The parameters to estimate from a and b will be L_∞ , t_0 or L_0 .

The authors propose adopting an initial value $K_{(0)}$, of K , and estimating $a_{(0)}$, $b_{(0)}$ and $r^2_{(0)}$ by simple linear regression between y ($= L_t$) and $x (= e^{-K_{(0)}t})$. The procedure may be repeated for several values of K , that is, $K_{(1)}$, $K_{(2)}$, One can then adopt the regression that results in the larger value of r^2 , to which K_{\max} , a_{\max} and b_{\max} correspond. From the values of a_{\max} , b_{\max} and K_{\max} one can obtain the values of the remaining parameters.

One practical process towards finding K_{\max} can be:

- (i). To select two extreme values of K which include the required value, for example $K=0$ and $K=2$ (for practical difficulties, use $K = 0.00001$ instead of $K = 0$).
- (ii). Calculate the 10 regressions for equally-spaced values of K between those two values in regular intervals.
- (iii). The corresponding 10 values of r^2 will allow one to select two new values of K which determine another interval, smaller than the one in (i), containing another maximum value of r^2 .
- (iv). The steps (ii) and (iii) can be repeated until an interval of values of K with the desired approximation is obtained. Generally, the steps do not need many repetitions.

7.5 ESTIMATION OF M – NATURAL MORTALITY COEFFICIENT

Several methods were proposed to estimate M , and they are based on the association of M with other biological parameters of the resource. These methods can produce approximate results.

7.5.1 RELATION OF M WITH THE LONGEVITY, t_λ

Longevity: Maximum mean age t_λ of the individuals in a non-exploited population.

Duration of the exploitable life: $t_\lambda - t_r = \lambda$ (Figure 7.1)

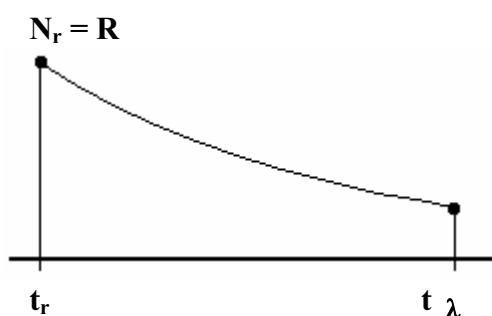


Figure 7.1 Duration of the exploitable life

Tanaka (1960) proposes "NATURAL" Survival Curves (Figure 7.2) to obtain the values of M from longevity.

A cohort practically vanishes when only a fraction, p , of the recruited individuals survives. In that case, $N_\lambda = R \cdot e^{-M \cdot \lambda}$, and it can be written:

$$p = \frac{N_\lambda}{R} = e^{-M \cdot \lambda} \quad \text{and so} \quad M = -(1/\lambda) \cdot \ln p$$

Different values of the survival fraction produce different survival curves of M in function of λ .

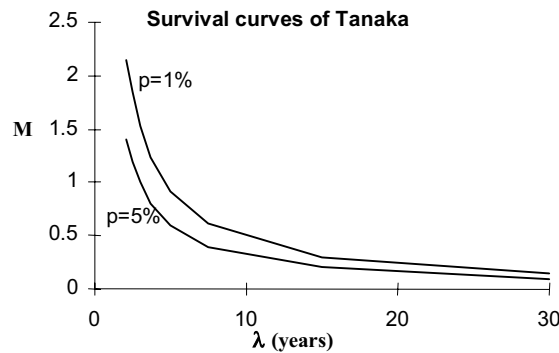


Figure 7.2 Survival curves by Tanaka

Any value of p can be chosen, for instance, $p = 5\%$, (*i.e.* one in each twenty recruits survives until the age t_λ) as variable value of the survival curves.

7.5.2 RELATION BETWEEN M AND GROWTH

Beverton and Holt Method (1959)

Gulland (1969) mentions that Beverton and Holt verified that species with a larger mortality rate M also presented larger values of K . Looking for a simple relation between these two parameters, they concluded approximately that:

$$1 \leq \frac{M}{K} \leq 2 \quad \text{for small pelagic fishes}$$

$$2 \leq \frac{M}{K} \leq 3 \quad \text{for demersal fishes}$$

Pauly Method (1980)

Based on the following considerations:

1. Resources with a high mortality rate cannot have a very big maximum size;
2. In warmer waters, the metabolism is accelerated, so the individuals can grow up to a larger size and reach the maximum size faster than in colder waters.

Based on data of 175 species, Pauly adjusted multiple linear regressions of transformed values of M against the corresponding transformed values of K , L_{∞} and temperature, T , and selected one that was considered to have a better adjustment, that is, the following empirical relation:

$$\ln M = -0.0152 - 0.0279 \ln L_{\infty} + 0.6543 \ln K + 0.463 \ln T^{\circ}$$

with the parameters expressed in the following units:

$$M = \text{year}^{-1}$$

$$L_{\infty} = \text{cm of total length}$$

$$K = \text{year}^{-1}$$

$$T^{\circ} = \text{surface temperature of the waters in } ^{\circ}\text{C}$$

Pauly highlights the application of this expression to small pelagic fishes and crustaceans. The Pauly relation uses decimal logarithms to present the first coefficient different from the value -0.0152 which was given in the previous expression, written with natural logarithms.

7.5.3 RELATION BETWEEN M AND REPRODUCTION

Rikhter and Efanov Method (1976)

These authors analysed the dependency between M and the age of first (or 50 percent) maturity. They used data from short, mean and long life species, and suggested the following relation of M with the, t_{mat} , age of 1st maturity:

$$M = \frac{1.521}{(t_{\text{mat}50\%})^{0.720}} - 0.155$$

(Units)

$$\left. \begin{array}{l} t_{\text{mat}50\%} \rightarrow \text{year} \\ M \rightarrow \text{year}^{-1} \end{array} \right\}$$

Gundersson Method (1980)

Based on the assumption that the natural mortality rate should be related to the investment of the fish in reproduction, beyond the influence of other factors, Gundersson established several relations between M and those factors.

He proposed, however, the following simple empirical relation, using the Gonadosomatic Index (GSI) (estimated for mature females in the spawning period) in order to calculate M :

$$M = 4.64 \times \text{GSI} - 0.37$$

7.5.4 KNOWING THE STOCK AGE STRUCTURE, AT BEGINNING AND END OF YEAR, AND CATCHES IN NUMBER, BY AGE, DURING THE YEAR

The natural mortality coefficients M_i , at age i can be calculated from the catch, C_i , in numbers, and the survival numbers, N_i and N_{i+1} at the beginning and end of a year, by following the steps:

$$\text{calculate} \quad E_i = \frac{C_i}{N_i - N_{i+1}}$$

$$\text{calculate} \quad Z_i = \ln N_i - \ln N_{i+1}$$

$$\text{calculate} \quad M_i = Z_i \cdot (1 - E_i)$$

The several values of M obtained in each age could be combined to calculate a constant value, M , for all ages.

Paloheimo Method (1961)

Let us consider the supposition that F_i is proportional to f_i for several years i , that is

$$F_i = q \cdot \frac{f_i}{T_i} \quad \text{for } T_i = 1 \text{ year, } F_i = q \cdot f_i,$$

$$\text{then:} \quad Z_i = q \cdot f_i + M$$

So, the linear regression between Z_i and f_i has a slope $b = q$ and an intercept $a = M$.

7.6 ESTIMATION OF Z – TOTAL MORTALITY COEFFICIENT

There are several methods of estimating the total mortality coefficient, Z , assumed to be *constant* during a certain interval of ages or years.

It is convenient to group the methods, according to the basic data, into those using ages or those using lengths.

7.6.1 METHODS USING AGE DATA

The different methods are based on the general expression of the number of survivors of a cohort, at the instant t , submitted to the total mortality, Z , during an interval of time, that is:

$$N_t = N_a \cdot e^{-Z(t-t_a)}$$

Z is supposed to be constant in the interval of time (t_a, t_b) .

Taking logarithms and re-arranging the terms, the expression will be:

$$\ln N_t = \text{Cte} - Z \cdot t$$

where Cte is a constant ($= \ln N_a + Zt_a$).

This expression shows that the logarithm of the number of survivors *is linear* with the age, being the slope equal to $-Z$.

Any constant expression which does not affect the determination of Z will be referred to as Cte.

1. If Z can be considered constant inside the interval (t_a, t_b) and, having available abundance data, \bar{N}_i , or indices of abundance in number, \bar{U}_i in several ages, i , *then*, the application of the simple linear regression allows one to estimate the total mortality coefficient Z .

In fact

$$\bar{N}_i = N_i \cdot \frac{1 - e^{-ZT_i}}{ZT_i} \quad \text{so } \bar{N}_i = N_i \cdot \text{Constant}$$

and, as

$$N_i = N_a \cdot e^{-Z(t_i - t_a)}$$

then, by substitution:

$$\bar{N}_i = \text{Cte} \cdot e^{-Zt_i} \quad (T_i = \text{const} = 1 \text{ year})$$

and also

$$\ln \bar{N}_i = \text{Cte} - Zt_i$$

The simple linear regression between $\ln \bar{N}_i$ and t_i allows the estimation of Z (notice that the constant, Cte is different from the previous one. In this case only the slope matters to estimate Z).

2. If ages are not at constant intervals, the expression could be approximated and expressed in terms of the t_{centrali} . For T_i variable, it will be:

$$\bar{N}_i \approx N_i \cdot e^{-ZT_i/2}$$

and, as
$$N_i = N_a \cdot e^{-Z \cdot (t_i - t_a)}$$

it will be
$$\bar{N}_i \approx \text{Cte} \cdot e^{-Zt_{\text{centrali}}}$$

and finally:
$$\ln \bar{N}_i \approx \text{Cte} - Z \cdot t_{\text{centrali}}$$

3. When using indices \bar{U}_i , the situation is similar because $\bar{U}_i = q \cdot \bar{N}_i$, with q constant, and then, also:

$$\ln \bar{U}_i = Cte - Zt_i$$

The simple linear regression between $\ln \bar{U}_i$ and t_i allows one to estimate Z .

4. If the intervals are not constant, the expression should be modified to:

$$\ln \bar{U}_i \approx Cte - Z \cdot t_{\text{centrali}}$$

Simple linear regression can be applied to obtain Z , from catches, C_i , and ages, t_i , supposing that F_i is constant.

$C_i = F_i \bar{N}_i T_i$ and so, $\ln C_i = Cte + \ln \bar{N}_i$ when T_i is constant. So:

$$\ln C_i = Cte - Z \cdot t_i$$

5. If the intervals are not constant, the expression should be modified to:

$$\ln C_i / T_i \approx Cte - Z \cdot t_{\text{centrali}}$$

6. Let V_i be the cumulative catch from t_i until the end of the life, then:

$$V_i = \sum C_k = \sum F_k N_{\text{kcum}},$$

Where the sum goes from the last age until age i ,

As F_k and Z_k are supposed to be constant $\sum N_{\text{kcum}} = N_i / Z$ and so:

$$V_i = FN/Z \quad \text{and} \quad \ln V_i = Cte + \ln N_i$$

Therefore:

$$\ln V_i = Cte - Z \cdot t_i$$

7. Following Beverton and Holt (1956), Z can be expressed as :

$$Z = \frac{1}{\bar{t} - t_a}$$

Then, it is possible to estimate Z from the mean age \bar{t}

This expression was derived, considering the interval (t_a, t_b) as (t_a, ∞) .

7.6.2 METHODS USING LENGTH DATA

When one has available data by length classes instead of by age, the methods previously referred to can still be applied. For that purpose, it is convenient to define the relative age.

Using the von Bertalanffy equation one can obtain the age t in function of the length, as:

(the expression is written in the general form in relation to t_a and not to t_0)

$$t = t_a - (1/K) \cdot \ln[(L_\infty - L_t) / (L_\infty - L_a)]$$

or

$$t = t_a - \frac{1}{K} \cdot \ln\left(1 - \frac{L_t - L_a}{L_\infty - L_a}\right)$$

(This equation is referred to by some authors as the inverse von Bertalanffy equation).

The difference $t - t_a$ is called relative age, t^* .

$$\text{So: } t^* = -(1/K) \cdot \ln[(L_\infty - L_t) / (L_\infty - L_a)] \quad \text{or} \quad t^* = -(1/K) \ln[1 - (L_t - L_a) / (L_\infty - L_a)]$$

For $t_a = t_0$, $L_a = 0$ and:

$$t^* = -\frac{1}{K} \cdot \ln\left(1 - \frac{L_t}{L_\infty}\right)$$

t^* is called a relative age because the absolute ages, t , are related to a constant age, t_a .

In this way, the duration of the interval T_i can either be calculated by the difference of the absolute ages or by the difference of the relative ages at the extremes of the interval:

$$T_i = t_{i+1} - t_i = t_{i+1}^* - t_i^*$$

Also:

$$t_{\text{centrali}}^* = t_{\text{centrali}} + \text{Cte}$$

$$\bar{t}^* = \bar{t} + \text{Cte}$$

So, the previous expressions still hold when the *absolute* ages are replaced by the *relative* ages:

$$\ln \bar{N}_i = \text{Cte} - Z \cdot t_{\text{centrali}}^*$$

$$\ln \bar{U}_i = \text{Cte} - Z \cdot t_{\text{centrali}}^*$$

$$\ln V_i = \text{Cte} - Z \cdot t_i^*$$

$$\ln C_i/T_i = \text{Cte} - Z \cdot t_{\text{centrali}}^*$$

Finally, the expression would also be :

$$Z = \frac{1}{t^*}$$

Beverton and Holt (1957) proved that :

$$Z = K \cdot \frac{L_{\infty} - \bar{L}}{\bar{L} - L_a}$$

\bar{L} must be calculated as the mean of the lengths weighted with abundances (or their indices) or with the catches in numbers.

Comments

1. The application of any of these methods must be preceded by the graphical representation of the corresponding data, in order to verify if the assumptions of the methods are acceptable or not and also to determine the adequate interval, (t_a , t_b).
2. These formulas are proved with the indications that were presented, but it is a good exercise to develop the demonstrations as they clarify the methods.
3. It is useful to estimate a constant Z, even when it is not acceptable, because it gives a general orientation about the size of the values one can expect.
4. The methods are sometimes referred to by the names of the authors. For example, the expression $\ln V_i = Cte - Z \cdot t_i^*$ is called the Jones and van Zalinge method (1981).
5. The mean age as well as the mean length in the catch can be calculated from the following expressions:

$$\bar{t} = \frac{-(t_{\text{centrali}} \cdot C_i)}{-C_i} \quad \text{with } C_i = \text{catch in number in the age class } i$$

$$\bar{L} = \frac{-(L_{\text{centrali}} \cdot C_i)}{-C_i}$$

where C_i = catch in number in the length class i

$$t^* = \frac{-(t^*_{\text{centrali}} \cdot C_i)}{-C_i}$$

with C_i = catch in number in the age class.

The relative age should be $t^* = - (1/K) \cdot \ln[(L_{\infty} - L_t) / (L_{\infty} - L_a)]$

Summary of the Methods to Estimate the Total Mortality Coefficient, Z

Assumption: Z is constant in the interval of ages, (t_a, t_b)

T Constant

$$\ln \bar{N}_i = Cte - Z \cdot t_i$$

$$\ln \bar{U}_i = Cte - Z \cdot t_i$$

$$\ln C_i = Cte - Z \cdot t_i$$

$$\ln V_i = Cte - Z \cdot t_i$$

$$\left. \begin{matrix} \textcircled{B} \\ \textcircled{C} \\ \textcircled{D} \\ \textcircled{E} \end{matrix} \right\} \frac{V_i}{TM_i} = \frac{1}{k} C_k$$

T_i variable

$$\ln \bar{N}_i = Cte - Z \cdot t_{\text{central}_i}$$

$$\ln \bar{U}_i = Cte - Z \cdot t_{\text{central}_i}$$

$$\left. \begin{matrix} \textcircled{B} \\ \textcircled{C} \\ \textcircled{D} \\ \textcircled{E} \end{matrix} \right\} t_{\text{central}_i} = \frac{t_i}{2} + \frac{T_i}{2}$$

$$\ln \frac{\bar{C}_i}{T_i} = Cte - Z \cdot t_{\text{central}_i}$$

$$\ln V_i = Cte - Z \cdot t_i$$

$$Z = \frac{1}{\bar{t} - t_a} \quad (t_b = \infty) \text{ (Beverton and Holt equation of Z)}$$

Supposition: Z is constant in the interval of lengths, (L_a, L_b)

$$\text{Relative age } t_i^* = -\frac{1}{K} \cdot \ln \left(\frac{\textcircled{B} L_\infty - L_t}{\textcircled{C} L_\infty - L_a} \right)$$

$$\ln N_i = Cte - Z \cdot t_i^*$$

$$\ln \bar{N}_i = Cte - Z \cdot t_{\text{central}_i}^*$$

$$t_{\text{central}_i}^* = \frac{t_i^* + t_{i+1}^*}{2}$$

$$\ln \bar{U}_i = Cte - Z \cdot t_{\text{central}_i}^*$$

$$\ln \frac{C_i}{T_i} = Cte - Z \cdot t_{\text{central}_i}^*$$

$$T_i = t_{i+1}^* - t_i^* \text{ (Gulland and Holt equation)}$$

$$\ln V_i = Cte - Z \cdot t_i^*$$

(Jones and van Zalinge equation)

$$Z = K \cdot \frac{L_\infty - \bar{L}}{\bar{L} - L_a}$$

(t_b^{*} = ∞) (Beverton and Holt equation of Z)

7.7 ESTIMATION OF THE PARAMETERS OF THE STOCK-RECRUITMENT (S-R) RELATION

The least squares method (non-linear model) can be used to estimate the parameters, α and k , of any of the S-R models.

The initial values of the Beverton and Holt model (1957) can be obtained by re-writing the equation as:

$$(R/S)^{-1} \text{ or } \frac{S}{R} = \frac{1}{\alpha} + \frac{1}{\alpha k} \cdot S$$

and estimating the simple linear regression between $y (= S/R)$ and $x (=S)$ which will give the estimations of $1/\alpha$ and $1/(\alpha k)$. From these values, it will then be possible to estimate the parameters α and k . These values can be considered as the initial values in the application of the non-linear model.

In the Ricker model (1954) the parameters can be obtained by re-writing the equation as:

$$\ln \frac{R}{S} = \ln \alpha - \frac{1}{k} \cdot S$$

and applying the simple linear regression between $y (= \ln R/S)$ and $x (=S)$ to estimate $\ln \alpha$ and $(-1/k)$. From these values, it will be possible to estimate the parameters (α and k) of the model, which can be considered as the initial values in the application of the non-linear model.

It is useful to represent the graph of y against x in order to verify if the marked points are adjustable to a straight line before applying the linear regression in any of these models.

In the models with the flexible parameter, c , like for example, the Deriso model (1980), the equation can be re-written as:

$$\left(\frac{R}{S} \right)^c = \alpha^c - c \cdot \alpha^c \cdot \frac{S}{k}$$

For a given value of c the linear regression between $y (= (R/S)^c)$ and $x (=S)$ allows the estimation of the parameters α and k .

One can try several values of c to verify which one will have a better adjustment with the line y against x ; for example, values of c between -1 and 1.

The values thus obtained for α , k and c , can be considered as initial values in the application of the iterative method, to estimate the parameters α , k and c of the non-linear Deriso model.

7.8 ESTIMATION OF THE MATRIX [F] AND OF THE MATRIX [N] – COHORT ANALYSIS – AC and LCA

7.8.1 COHORT ANALYSIS BY AGE- (AC)

The cohort analysis is a method to estimate the fishing mortality coefficients, F_i , and the number of survivors, N_i , at the beginning of each age, from the annual structures of the stock catches, in number, over a period of years.

More specifically, consider a stock where the following is known:

Data

age, i , where $i = 1, 2, \dots, k$
year, j , where $j = 1, 2, \dots, n$

Matrix of catches [C] with

$C_{i,j}$ = Annual catch, in number, of the individuals with the age i and during the year j

Matrix of natural mortality [M] with

$M_{i,j}$ = natural mortality coefficient, at the age i and in the year j .

Vector [T] where

T_i = Size of the age interval i (in general, $T_i = T = 1$ year)

Objective

To estimate

matrix [F]
and
matrix [N].

In the resolution of this problem, it is convenient to consider these estimations separately; one interval of age i (part 1); all the ages during the life of a cohort (part 2); and finally, all the ages and years (part 3).

PART 1 (INTERVAL T_i)

Consider that the following characteristics of a cohort, in an interval T_i are known :

C_i = Catch in number

M_i = Natural mortality coefficient

T_i = Size of the interval

Adopting a value of F_i , it is then possible to estimate the number of survivors at the beginning, N_i , and at the end, N_{i+1} , of the interval.

In fact, from the expression:

$$C_i = \frac{F_i}{F_i + M_i} \cdot N_i \cdot (1 - e^{-(F_i + M_i) \cdot T_i})$$

one can calculate N_i which is the only unknown variable in the expression.

To calculate N_{i+1} one can use the expression $N_{i+1} = N_i \cdot e^{-(F_i + M_i) \cdot T_i}$ where the values N_i , F_i and M_i were previously obtained.

PART 2 (DURING THE LIFE)

Suppose now that the catches C_i of each age i , of a cohort during its life, the values of M_i and the sizes of the interval T_i are known.

Adopting a certain value, F_{final} , for the Fishing Mortality Coefficient in the last class of ages, it is possible, as mentioned in part 1, to estimate all the parameters (related to numbers) *in that last age group*. In this way, one will know the number of survivors at the beginning and end of the last age.

The number at the beginning of that last class of ages, is also the number N_{last} at the end of the previous class, that is, N_{final} is the initial number of survivors of the class before last.

Using the C_i expression, resulting from the combination of the two expressions above :

$$C_i = \frac{F_i}{F_i + M_i} \cdot N_{\text{final}} \cdot (e^{+(F_i + M_i) \cdot T_i} - 1)$$

one can estimate F_i in the previous class, which is the only unknown variable in the expression. The estimation may require iterative methods or trial and error methods.

Finally, to estimate the number N_i of survivors at the beginning of the class i , the following expression can be used :

$$N_i = N_{\text{final}} \cdot e^{(F_i + M_i) \cdot T_i}$$

Repeating this process for all previous classes, one will successively obtain the parameters in all ages, until the first age.

In the case of a completely caught cohort, the number at the end of the last class is zero and the catch C has to be expressed as :

$$C_{\text{final}} = \frac{F_{\text{final}}}{(F_{\text{final}} + M_i)} \cdot N_{\text{final}}$$

Pope Method

Pope (1972) presented a simple method to estimate the number of survivors at the beginning of each age of the cohort life, starting from the last age.

It is enough to apply successively in a backward way, the expression:

$$N_i \approx (N_{i+1} e^{MT/2} + C_i) \cdot e^{MT/2}$$

Pope indicates that the approximation is good when $MT \leq 0.6$

Pope's expression is obtained, supposing that the catch is made exactly at the central point of the interval T_i (Figure 7.3).

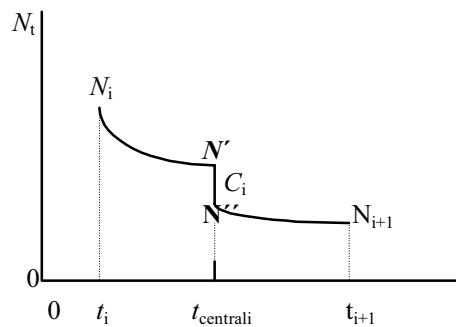


Figure 7.3 Number of survivors during the interval $T_i = t_{i+1} - t_i$ with the catch extracted at the central point of the interval

Proceeding from the end to the beginning one calculates successively:

$$N'' = N_{i+1} e^{+MT_i/2}$$

$$N' = N'' + C_i$$

$$N_i = N' \cdot e^{+MT/2}$$

substituting N' by $N'' + C_i$, the expression will be:

$$N_i = (N'' + C_i) \cdot e^{MT/2}$$

Finally, substituting N'' by $N_{i+1} \cdot e^{+MT_i/2}$, it will be:

$$N_i \approx \left(N_{i+1} e^{MT/2} + C_i \right) \cdot e^{MT/2}$$

Part 3 (period of years)

Let us suppose now that the Catch matrix [C], the natural mortality [M] matrix and the vector size of the intervals [T], are known for a period of years.

Let us also assume that the values of F in the last age of all the years represented in the matrices and the values of F of all the ages of the last year were adopted. These values will be designated by F_{terminal} (Figure 7.4)

Ages	Years				
	2000	2001	2002	2003	
1	C	C	C	C	F_{terminal}
2	C	C	C	C	F_{terminal}
3	C	C	C	C	F_{terminal}
F_{terminal} F_{terminal} F_{terminal} F_{terminal}					

Figure 7.4 Matrix of catch, [C], with F_{terminal} in the last line and in the last column of the matrix C. The shadowed zones exemplify the catches of a cohort

Notice that in this matrix the elements of the diagonal correspond to values of the same cohort, because one element of a certain age and a certain year will be followed, in the diagonal, by the element that is a year older.

From parts 1 and 2 it will then be possible to estimate successively Fs and Ns for all the cohorts present in the catch matrix.

Comments

1. The values of $M_{i,j}$ are considered constant and equal to M, when there is no information to adopt other values.
2. When data is referred to ages, the values T_i will be equal to 1 year.
3. The last age group of each year is, sometimes grouped ages(+). The corresponding catches are composed of individuals caught during those years, with several ages. So, the cumulative values do not belong to the same cohorts, but are survivors of several previous cohorts with different recruitments and submitted to different fishing patterns. It would not be appropriate to use the catch of a group (+) and to apply cohort analysis. Despite this fact, the group (+) is important in order to calculate the annual totals of the catches in weight, Y, of total biomasses, B, and the spawning stock biomass. So, it is usual to start with the cohort analysis on the age immediately before the group (+) and use the group (+) only to calculate the annuals Y, B and (SP). The value of F in that group (+) in each year, can be estimated as being the same fishing mortality coefficient as the previous age or, in some cases, as being a reasonable value in relation to the values of F_i in the year that is being considered.
4. A difficulty in the technical application appears when the number of ages is small or when the years are few. In fact, in those cases, the cohorts have few age classes represented in the Matrix [C] and the estimations will be very dependent on the adopted values of $F_{\text{terminals}}$.

5. The cohort analysis (CA) has also been designated as: VPA (Virtual Population Analysis), Derzhavin method, Murphy method, Gulland method, Pope method, Sequential Analysis, etc. Sometimes, CA is referred to when the Pope formula and the VPA are used in other cases. Megrey (1989) presents a very complete revision about the cohort analyses.
6. It is also possible to estimate the remaining parameters in an age i , related to numbers, that is, N_{cumi} , \bar{N}_i , D_i , Z_i and E_i . When the information on initial individual or mean weights matrices $[w]$ or $[\bar{w}]$ are available, one can also calculate the matrices of annual catch in weight $[Y]$, of biomasses at the beginning of the years, $[B]$, and of mean biomasses during the years $[\bar{B}]$. If one has information on maturity ogives in each year, for example at the beginning of the year, spawning biomasses $[SP]$ can also be calculated. Usually, only the total catches Y , the stock biomasses (total and spawning) at the beginning and the mean biomasses of the stock (total and spawning) in each year are estimated.
7. The elements on the first line of the matrix $[N]$ can be considered estimates of the recruitment to the fishery in each year.
8. The fact that the $F_{terminals}$ are adopted and that these values have influence on the resulting matrix $[F]$ and matrix $[N]$, forces the selection of values of $F_{terminals}$ to be near the real ones. The agreement between the estimations of the parameters mentioned in the points 6. and 7. and other independent data or indices (for example, estimations by acoustic methods of recruitment or biomasses, estimations of abundance indices or cpue's, of fishing efforts, etc) must be analysed.
9. The hypothesis that the exploitation pattern is constant from year to year, means that the fishing level and the exploitation pattern can be separated, or $F_{sepi} = F_j \times s_i$. This hypothesis can be tested based on the matrix $[F]$ obtained from the cohort analysis.

It is usual to call this separation VPA-Separable (SVPA).

We have $\sum_i F_{i,j} = F_{totj}$

and $\sum_j F_{i,j} = s_{toti}$

and $\sum_{i,j} F_{i,j} = F_{tot}$

Then, if $F_{ij} = F_j \cdot s_i$ one can prove that $F_j \cdot s_i = (F_{totj} \cdot s_{toti}) / F_{tot}$.

If the estimated values of F_{ij} are the same as the previous $F_{sepi} = F_j \cdot s_i$ then the hypothesis is verified. This comparison can be carried out in two different ways, the simplest is to calculate the quotients F_{sepi} / F_{ij} . If the hypothesis is true this quotient is equal to one. If the hypothesis is not verified it is always possible to consider other hypotheses with the annual vector $[s]$ constant in some years only, mainly the last years.

10. It is usual to consider an interval of ages, where it can be assumed that the individuals caught are "completely recruited". In that case, the interval of ages corresponds to exploitation pattern constant (for the remaining ages, not completely recruited, the exploitation pattern should be smaller). For that interval of ages, the means of the values

of $F_{i,j}$ in each year are then calculated. Those means, \bar{F}_j , are considered as fishing levels in the respective years. The exploitation pattern in each cell, would then be the ratio $F_{i,j} / \bar{F}_j$. The s_i , for the period of years considered, can be taken as the mean of the relative pattern of exploitation calculated before. Alternatively, they can also be taken as referring to s_i of an age chosen for reference.

7.8.2 LENGTH COHORT ANALYSIS - (LCA)

The technique of the cohort analysis, applied to the structure of the catches of a cohort during its life, can be made with non constant intervals of time, T_i . This means that the length classes structure of the catches of a cohort during its life, can also be analysed.

The methods of analysis of the cohort in those cases is called the LCA (Length Cohort Analysis). The same techniques; Pope method, iterative method, etc, of the CA for the ages, can be applied to the LCA analysis (the intervals T_i 's can be calculated from the relative ages).

One way to apply the LCA to the length annual catch compositions, will be: to group the catches of length classes belonging to the same age interval in each year. The technique CA can then be applied directly to the resulting age composition of the catches by age of the matrix [C]. This technique is known as “slicing” the length compositions. To “slice”, one usually inverts the von Bertalanffy length growth equation and estimates the age t_i for each length L_i (sometimes using the relative ages t_i^*) (Figure 7.5). It is possible that when grouping the length classes of the respective age interval, there are length classes composed by elements that belong to two consecutive age groups. In these cases, it will be necessary to “break” the catch of these extreme classes into two parts and distribute them to each of those ages. In the example of Figure 7.5, the catches of the length class (24-26] belong to age 0 and to age 1. So, it is necessary to distribute that catch to the two ages. One simple method is to attribute to age 0 the fraction $(1.00 - 0.98)/(1.06 - 0.98) = 0.25$ of the annual catch of that length class and to age 1 the fraction $(1.06 - 1.00)/(1.06 - 0.98) = 0.75$. The method may not be the most appropriate one, because it is based on the assumption that, in the length classes, the distribution of the individuals by length is uniform. So, it is necessary to use the smallest possible interval of length classes, when applying this distribution technique.

Another way to do the length cohort analysis is to use the catches in the length classes of the same age group. It is possible to follow the cohorts in the matrix [C], through the length classes belonging to a same age, in a certain year, with the length classes of the next age, in the following year, etc. In this way, the different cohorts existing in the matrix will be separated and the evolution of each one of them will be by length classes, not by age (see Figure 7.5).

Group			Years			
Age	Relative age	Length Classes	2000	2001	2002	2003
0	1.03	20-	41	30	17	49
	1.54	22-	400	292	166	472
	1.98	24-	952	699	400	1127
1	2.06	26-	1766	1317	757	2108
	2.30	28-	2222	1702	985	2688
	2.74	30-	2357	1872	1093	2902
	2.88	32-	2175	1091	1067	2739
2	3.00	34-	1817	948	1416	1445
	3.42	36-	1529	812	1270	1250
	3.64	38-	1251	684	980	1053
	3.83	40-	1003	560	702	710
	3.96	42-	787	290	310	558
3	4.01	44-	595	226	179	834
	4.25	46-	168	70	71	112

Cohort of the year 2000

Figure 7.5 Example of a matrix [C] with the catches of the cohort shadowed, written in bold, recruited at year 2000, “sliced” by length classes,

The LCA R. Jones method (1961), of analysing a length composition during the life of a cohort can then be applied. The different values of T_i are calculated as $T_i = t_{i+1}^* - t_i^*$, where t_i^* e t_{i+1}^* are the relative ages corresponding to the extremes of the length interval i . The vector [N] can also be obtained as the number of initial survivors in each length class of the cohort, and in each age class.

Comments on cohort analyses

1. Certain models, called *integrated models*, combine all the available information (catches, data collected on research cruises, effort and cpue data, etc) with the matrix [C], and integrate in a unique model, in order to optimize the previously defined criterium function. A model integrating CA and the hypothesis of constant exploitation pattern was developed and called SVPA, separable VPA, because the Fishing level and Exploitation pattern are “separable”.
2. Fry (1949) considered the cumulative catches of a cohort by age during its life, from the end to the beginning, as an image of the number of survivors at the beginning of each age (which the author designated as “virtual population”):

$$\sum_{k=final}^i C_k = V_i = N_i \text{ virtual}$$

In the fishery that Fry studied, M was practically equal to zero.

If M is different from zero it can also be said that the number N_i of survivors at the beginning of the interval i will be :

$$N_i = \frac{i}{k=\text{final}} D_k$$

where D_k represents the number of total deaths at the interval k .

Adopting the initial values, $E_{k(0)}$, for the exploitation rates, E , in all the classes, one can calculate the total deaths:

$$D_{k(0)} = C_k / E_{k(0)}.$$

$N_{i(0)}$ can be calculated as the cumulative total deaths from the last class up to the i^{th} class, that is:

$$N_{i(0)} = \frac{i}{k=\text{ult}} D_{k(o)}$$

Then the expression will be:

$$Z_{i(1)} \cdot T_i = \ln(N_{i+1(0)} / N_{i(0)})$$

and:

$$F_{i(1)} \cdot T_i = E_{i(0)} \cdot Z_{i(1)} \cdot T_i$$

Comparing $E_{i(1)}$ with $E_{i(0)}$, the new values of E will be:

$$E_{i(1)} = F_{i(1)} \cdot T_i / (F_{i(1)} \cdot T_i + M_i \cdot T_i)$$

One can then estimate the values of E with the desired approximation by an iterative method, repeating the five calculations (of D_i , N_i , $Z_i T_i$, $F_i T_i$ and E_i), using $E_{i(1)}$ instead of $E_{i(0)}$.

In the last class, the number, N_{last} , can be taken as equal to the number of deaths, D_{last} , and in this case, N_{last} will be calculated as :

$$N_{\text{last}} = D_{\text{last}} = C_{\text{last}} / E_{\text{last}}$$

3. Finally, the results of CA and of LCA give a perspective view of the stock in the previous years. That information is useful for the short and long-term projections. Usually, data concerning the catches is not available for the year in which the assessment is done and so it is necessary to project the catches and the biomasses to the beginning of the present year before calculating the short-term projection.
4. When the relative ages are calculated, it is usual to adopt zero as the age t_a corresponding to the value of L_a , taken as the lower limit of the first length class represented in the catches.

CHAPTER 8 – EXERCISES

8.1 MATHEMATICAL REVISION

1. Calculate:

A)	10^4	$16^{-\frac{1}{2}}$	8427^0	$0.01^{0.5}$
B)	$\frac{4^3}{4^5}$	$(3^{-4})^4$	$5^2 + 4^2$	$2^2 \times 2^5$
C)	$\log 1000$	$\log 0.01$	$\left. \begin{array}{l} \log \frac{10^4}{10^{-3}} \\ \log \frac{10^4}{10^{-3}} \end{array} \right\}$	
D)	$\ln e$	$\ln \frac{1}{e}$	$\ln e^{-5}$	$e^{\ln e}$
E)	$\frac{0}{0}$	$\frac{A}{0}$	$\frac{0}{A}$	$\frac{\infty}{\infty}$
	$\frac{0}{0}$	$\frac{A}{0}$	$\frac{0}{A}$	$\frac{\infty}{\infty}$

2. Verify that

a) $a = e^{\ln a}$	b) $a = 10^{\log a}$
c) $\frac{e^x - 1}{x} \approx 1$ for $-0.01 < x < +0.01$	d) $\frac{e^x - 1}{x} \approx e^{\frac{x}{2}}$ for $-0.5 < x < +0.5$

3. Solve the following expressions applying natural logarithms to both members of the equality:

a) $y = a \cdot x^5$	b) $y = a \cdot e^{-b(x+2 \cdot c)}$	c) $y - a = b \cdot e^{-c(x-b)}$
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Note: a, b, e, c are constants; e is the basis of natural logarithms ($e = 2.7183..$); x and y are variables.

4. Determine the value of x in the following expressions:

a) $e^{-x} = 5.2$	b) $10^x = 5.5$	c) $y - a = b \cdot e^{c(x-b)}$
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5. Calculate the derivatives of the following expressions:

a) $y=13$	g) $y=5^x$	m) $y=(4+2x)^3$
b) $y=3-8x$	h) $y=e^{-3 \cdot x}$	n) $y=(x-6)^2$
c) $y=x^5$	i) $y=\ln x$	o) $y=a \cdot (3-e^{-b \cdot x})^3$
d) $y=x^{2/7}$	j) $y=\ln(5x+4)$	p) $y=(4x+3) \cdot (e^x-4)$
e) $y=x^{-3}$	k) $y=1/x$	
f) $y=e^{3 \cdot x}$	l) $y=(2+4x)/(3-x)$	

6. Calculate the indefinite integrals of the following functions:

- | | | |
|----------------------------|--|-------------------------------------|
| a) $f(x) = 0$ | f) $f(x) = \frac{1}{x}$ | k) $f(x) = e^{-0.5 \cdot x}$ |
| b) $f(x) = 5.34$ | g) $f(x) = \frac{-5}{2-5 \cdot x}$ | l) $f(x) = 3 \cdot e^{2 \cdot x+1}$ |
| c) $f(x) = x^6$ | h) $f(x) = \frac{1+2 \cdot x}{40+x+x^2}$ | m) $f(x) = x \cdot e^x$ |
| d) $f(x) = 1+3 \cdot x$ | i) $f(x) = e^x$ | n) $f(x) = \ln x$ |
| e) $f(x) = 4 \cdot x^{-3}$ | j) $f(x) = e^{0.2 \cdot x}$ | o) $f(x) = x \cdot \ln x$ |

7. Calculate the area under the function

- a) $f(x) = 2 + 5x$ between $x = 1$ and $x = 4$
- b) $f(x) = e^{3 \cdot x}$ between $x = 0$ and $x = 1$
- c) $f(x) = \frac{2}{5+2x}$ between $x = \frac{1}{2}$ and $x = \frac{3}{2}$
- d) $f(x) = 1 + 3x$ between $x = -2$ and $x = 2$

8. Calculate the value of $y_{\text{cumulative}}$ with

- a) $y = e^{-2x}$ between $x = 0$ and $x = 0.8$
- b) $f(x) = \frac{2}{1+2x}$ between $x = 0$ and $x = 2$
- c) $f(x) = 2 \cdot x^3$ between $x = 0$ and $x = 1$

9. Calculate the Mean Value of y with

- a) $y = 3 \cdot e^{-7x}$ between $x = 0$ and $x = 1$
- b) $y = 4 \cdot (1 - e^{-0.2x})$ between $x = 1$ and $x = 3$
- c) $y = 2 - x$ between $x = 0$ and $x = 1.2$

10. Calculate the integral of

- a) $f(x) = 2 \cdot e^{-0.5x}$ with the initial condition $x = 1 \heartsuit F(x) = 4$ where $F(x) = \int f(x) \cdot dx$
- b) $f(x) = \frac{3}{x}$ with the initial condition $F(1) = 2$
- c) $\frac{dy}{dx} = 0.2 \cdot y$ with the initial condition $x = 0 \heartsuit y = 10$
- d) $\frac{dy}{dx} = \frac{3}{1 + 3x}$ with the initial condition $x = 0 \heartsuit y = 0$

8.2 RATES

(2.2)

Consider the function, $y = 40 - 35 \cdot e^{-0.2x}$ at the interval $(0,10)$

1. Calculate:

- a) The values of y for $x = 0,1,2,3,4,5,6,7,8,9,10$;
- b) Represent graphically the function y at the interval $(0,10)$ of x ;
- c) The variation, Δy , corresponding to the interval $(1,2)$ of x ;
- d) The absolute mean rate of variation of y , $amr(y)$, at the intervals $(1,7)$, $(2,5)$, $(5,6)$ and $(8,9)$ of x ;
- e) The absolute instantaneous rate of variation of y , $air(y)$, at the points $x=3$ and $x=4$;
- f) Calculate the relative mean rate of variation of y , $r.m.r.(y)$, at the interval $(8,9)$ in relation to the value of y corresponding to the initial point, to the final point and to the central point of that interval;
- g) Calculate a relative instantaneous rate of variation of y , $r.i.r.(y)$ at the central point of the interval $(8,9)$.

2. Calculate the *air*(y) of the following functions:
 - a) $y = 1 + 10x$
 - b) $y = x^3 - 2x + 3$
 - c) $y = e^x$
 - d) $y = \ln x$
3. Calculate the *rir*(y) of the following functions:
 - a) $y = 4 + x$
 - b) $y = e^x$
 - c) $y = 6 \cdot e^{2x}$
 - d) $y = a \cdot x$ with $a = \text{constant}$
4. Calculate the *air* of the *air*(y) of $y = 3x^2 - 4x - 12$
5. Given the function, $y = 3 \cdot e^{-1.8x}$ verify that $\text{rir}(y) = \text{air}(\ln y)$

8.3 SIMPLE LINEAR MODEL

(2.3)

Consider a model that relates the characteristic y with time t , where the basic assumption is:

$$\text{air}(y) = -3, \text{ for } 0 < t < \infty$$

Adopt the initial condition: for $t = 0$, $y = 30$

1. How would you designate this model?
Write the general expression for the value of the characteristic y at the instant t ;
2. Calculate the value of y when $t = 0, 1, 2, 3, 4, 5, 6$ and draw the graph of y against t .
3. Considering the interval of time, Δt , from $t = 2$ to $t = 4$
 - a) Calculate the variation of y during the mentioned interval Δt ;
 - b) Calculate the central value of y in the interval Δt ;
 - c) Calculate the cumulative value of y in that interval, y_{cum} ;
 - d) Calculate the mean value, \bar{y} , of y , in the interval Δt ;

- e) Calculate the simple arithmetic mean of y in the interval Δt ;
- f) Verify that the arithmetic mean of y is equal to the mean value, \bar{y} , and equal to the central value, y_{central} , of y in that interval.
- g) Verify that in the linear model, the $\text{amr}(y) = \text{air}(y) = \text{constant}$. To do that, calculate, for the above mentioned interval, Δt , the $\text{amr}(y)$ and the $\text{air}(y)$ and compare the results.

Repeat exercise 3. considering the interval from $t = 0$ to $t = 10$.

8.4 EXPONENTIAL MODEL (2.4)

Consider a model that relates the characteristic y with time t , through the following basic assumption:

$$\text{rir}(y) = -0.4 \quad \text{for } 0 < t < \infty$$

Adopt the initial condition : for $t = 0$, $y = 100$

1. Write the general expression for the value of the characteristic y at the instant t ;
 - a) Calculate the value of y at the instants $t = 1, 2, 3, 4, 5, 6$.
 - b) Represent, graphically, the values of y calculated above, against the corresponding values of t .
 - c) Represent, graphically, the values of $\ln y$ against the given values of t .
2. Considering the interval of time $\Delta t = (3, 6)$
 - a) Calculate the variation of y , Δy , during the interval Δt .
 - b) Calculate y_{central} in the interval Δt .
 - c) Calculate the value of y_{cum} in the interval Δt .
 - d) Calculate \bar{y} in the interval Δt .
 - e) Show that the geometric mean of the values of y for $t = 3$ $\underline{\text{y}}$ $t = 6$ is equal to y_{central} and approximately equal to \bar{y} in that interval.
 - f) Show that, in that interval, $\text{rmr}(y)_{\text{relativeto}\bar{y}_i} = \text{rir}(y) = -0.4$
3. Consider the interval of time from $t = 0$ to $t = 10$. Repeat the calculations of questions 2 item a), c) and d) for this interval.

8.5 COHORT – EVOLUTION IN NUMBER

(3.2)

GROUP I

From the stock of megrim, *Lepidorhombus whiffiagonis*, from Divisions VIIIc and IXa of the ICES, the Assessment Working Group of ICES (ICES, 1997a) estimated that the fish recruit to the exploitable phase, at the beginning of age 1 year, and that in 1996 the instantaneous rate of total mortality during the exploitable phase was 0.7 year^{-1} .

Consider 1000 individuals of a cohort of megrim, recruited to the exploited phase, which starts at the beginning of age 1 and finishes at the end of age 7 years.

1.

- a) What is the value of the r of the variation of N_t in this interval ?
- b) What is the value of the r of the mortality of N_t in this interval ?
- c) Calculate the annual rate of survival during the interval.
- d) Calculate the annual rate of mortality during the interval.

2.

- a) Calculate the number of survivors at the beginning of each age of that interval.
- b) Calculate the number of survivors at the end of 7 years of age.
- c) Draw the graph of the number of survivors in each age of that interval.
- d) Calculate the number of deaths in each age of the interval.
- e) Calculate the number of deaths during all the exploitable phase.
- f) Determine the percentage of the initial number of 3 year-olds that survive until the beginning of their 6th year.
- g) Determine the percentage of the initial number of 3 year-olds that die before the beginning of their 6th year.
- h) Calculate the mean number of survivors during each age of the given interval.
- i) Calculate the cumulative number of survivors during ages 3 to 5.
- j) Calculate the mean number of survivors between the beginnings of ages 3 and 6.

GROUP II

The Working Group of ICES which evaluated the stock of iberic sardine, *Sardina pilchardus*, estimated the mortality rates in each age of 1995 (ICES, 1997b), presented in the following table :

Age Group	0	1	2	3	4	5	6
Annual Rate of Mortality	0.36	0.43	0.54	0.63	0.66	0.68	0.72

Suppose that these rates correspond to a cohort.

1. As previously seen, there may be several types of rates (ex: amr, rmr, air and rir and the relative rates were referred to several values of the characteristics). What type of rate is the annual rate of mortality?
2. Calculate the survival rate in each age class.
3. Calculate the total mortality coefficient for each age class.
4. Calculate the survival rate between the beginning of age 1 and the end of age 4.
5. Calculate the annual mean rate of survival in the same interval of ages.

GROUP III

Consider a cohort of a certain species for which the number of survivors at the beginning of age 2 years is 4325, while the number of survivors at the end of age 2 years is 2040.

1. Calculate the mean number of 2 year-old individuals and the number of individuals at the age of 2.5 years. Compare the results.
2. If the annual rate of mortality of this cohort during the ages 3 and 4 is 70 and 60 percent respectively, calculate the percentage of the initial number of individuals at age 3 that will survive until the end of age 4.

Give the relation between the survival rate during the period that covers the ages 3 and 4 years and the annual survival rates of ages 3 and 4 years.

8.6 COHORT – CATCH IN NUMBER (3.3)

GROUP I

According to the Assessment Working Group of ICES (ICES, 1996a) the relative instantaneous rates of total and natural mortality for the age of 3 years, of the stock of blue whiting, *Micromesistius poutassou*, in 1995, were estimated as being, respectively, $Z_3 = 0.4 \text{ year}^{-1}$ and $M_3 = 0.2 \text{ year}^{-1}$. In that year, the number of survivors at the beginning of age 3 year was 2600 million individuals.

1. Calculate, for the cohort of 1992 and for the age of 3 years :

- a) The total annual rates of survival and of mortality.
- b) The relative instantaneous rate of fishing mortality.
- c) The exploitation rate.
- d) The number of deaths during the age.
- e) The mean number of survivors during the age.
- f) The total catch in number of 3 years old individuals.
- g) The number of survivors at the end of the age.

GROUP II

The 4 year-old age group of the stock (Div. ICES VIIe-h) of whiting, *Merlangius merlangus*, is simultaneously exploited by the crustaceans trawl fleet and demersal fish trawl fleet.

The Working Group of ICES that evaluates this stock estimated (ICES, 1996a) that, in 1995, the total instantaneous mortality rate of that age (4 years) was $Z = 1.3 \text{ year}^{-1}$. Suppose, for this exercise, that the instantaneous rate of fishing mortality caused by the crustaceans trawl fleet was $F_c = 0.5 \text{ year}^{-1}$, while the corresponding value for the demersal fish trawl fleet was $F_f = 0.6 \text{ year}^{-1}$. The Group also considered the natural instantaneous mortality rate, $M = 0.2 \text{ year}^{-1}$.

1. In 1995 17.66 million individuals recruited at 4 years of age.

- a) Calculate the total number of deaths during that age.
- b) Calculate the mean number of survivors during the age.
- c) Calculate the exploitation rate of each fleet.
- d) Calculate the total exploitation rate.
- e) Calculate the catch in number by each fleet and the total catch in number.
- f) Calculate the number of survivors at the end of the age. (The solutions to the questions can be done in any order).

GROUP III

For this exercise, suppose that in 1990, the mean number of survivors of the cohort of a stock of anchovy, *Engraulis encrasicolus*, during the period of 2 years of age was calculated as about 50 million individuals. During 1992, 70 million individuals were caught, from which, 40 percent were caught by the national fleet, and it is estimated that 80 million died of natural causes.

1. For this age and this cohort:
 - a) Calculate the total exploitation rate and the exploitation rate of the national and foreign fleets.
 - b) Calculate the total, natural and fishing mortality coefficients.
 - c) Calculate the fishing instantaneous mortality rates caused by the national fleet and by the foreign fleet.
 - d) Calculate the number of survivors at the beginning and at the end of the age.
(The solutions to the questions can be done in any order).

GROUP IV

According to the Assessment Working Group of ICES (ICES, 1997a) the fishing mortality coefficients applied to the 1976 cohort of the stock of common sole, *Solea vulgaris*, of the Celtic Sea were estimated in each age from 2 to 8 years (following table). The natural mortality coefficient for this stock is considered constant and equal to 0.1 year^{-1} . It was estimated that, at the beginning of age 6 years there were 1112 million survivors of this cohort.

Age	2	3	4	5	6	7	8
F_i	0.07	0.22	0.33	0.41	0.45	0.41	0.74

1. Calculate the numbers of survivors of this cohort at the beginning of each of the above mentioned ages.
2. Calculate the number of deaths in each age referred in the table.
3. Calculate the exploitation rates of this cohort in each age.
4. Calculate the mean numbers of survivors during each of the above ages.
5. Calculate the catches, in numbers, extracted from this cohort during each of the ages mentioned above, using two different methods.

8.7 INDIVIDUAL GROWTH IN LENGTH AND WEIGHT (3.4)

GROUP I

The parameters of the von Bertalanffy length growth equation of the stock of European anglerfish (Div. VIIIc and IXa of ICES), *Lophius budegassa*, were estimated as (Duarte *et al.*, 1997):

Asymptotic length = 101.69 cm
Coefficient of growth in length = 0.08 year⁻¹
Theoretical age, when the length is zero = -0.2 year

1. Calculate the theoretical length corresponding to the age 3.84 years.
2. Calculate the length at the beginning of the ages 1 to 12 years.
3. Calculate, for each of the above mentioned ages, the central length.
4. Represent, graphically, the Bertalanffy curve of growth in length for this stock.

GROUP II

Using the growth parameters given in Group I:

1. Calculate the length that corresponds to each age interval between 1 and 12 years as being the simple arithmetic mean of the length at the beginning and at the end of each class.
2. Calculate the mean length in each age for the same interval from 1 to 12 years accordingly to the von Bertalanffy model.
3. Compare the lengths obtained in 1) with those obtained in 2) and with the central values in each age of the interval, calculated in Group I-3.

GROUP III

The data presented in the following table represents the mean length (cm) by age (years) obtained from direct age reading of individuals of the stock of European anglerfish, *Lophius budegassa*, (Div. VIIIc and IXa).

t	L _t (cm)	T	L _t (cm)
1	9.2	7	44.4
2	16.5	8	49.0
3	22.9	9	52.3
4	28.8	10	55.0
5	34.7	11	60.8
6	38.6	12	63.4

Based on this data the parameters of the growth equation were estimated according to the Gompertz model as being :

Gompertz: $L_{\infty} = 73.7$ cm; $k = 0.22$ year⁻¹ $t^* = -2.76$ year

(t* is the age corresponding to L=1 cm)

1. Represent, graphically, the observed values.
2. Calculate, for the interval 1-12 years, the values of the length at the beginning of each age, according to the Bertalanffy growth model and draw the corresponding growth curve.
3. Calculate, for the interval 1-12 years, the values of the length at the beginning of each age, according to the Gompertz growth model and draw the corresponding growth curve. Determine the inflection point of the curve.
4. Say which growth model you consider more appropriate for this case and justify your answer.

GROUP IV

The data presented on the following table concern the stock of European anglerfish *Lophius budegassa* (Div. VIIIc and IXa).

Table of individual weights by length class, taken from the samples of European anglerfish, *Lophius budegassa* collected by IEO and IPIMAR in 1994.

Li (cm)	W _{meani} (g)	n	Li (cm)	W _{meani} (g)	n
20-	129	3	50-	1685	28
22-	163	2	52-	1896	30
24-	219	4	54-	2107	24
26-	265	14	56-	2345	41
28-	320	8	58-	2569	41
30-	397	10	60-	2848	32
32-	486	9	62-	3126	35
34-	545	57	64-	3407	28
36-	664	60	66-	3700	19
38-	773	61	68-	4056	23
40-	890	58	70-	4411	17
42-	1027	64	72-	4764	13
44-	1122	56	74-	5203	8
46-	1334	50	76-	5587	4
48-	1503	37	78-	5982	3

The mean of the observed weights and the number (n) of individuals, are indicated for each length class.

Based on this data, the parameters of the weight-length relation were estimated for this stock as :

$$a = 0.021$$

$$b = 2.88$$

1. Calculate the theoretical weight for each length class.
2. On a graph, plot the observed and the theoretical weights against the length classes.

3. Suppose you want to use the weight-length relation, with $b=3$ (the constant estimated for this relation being $a=0.013$). Calculate, for this case, the theoretical weights for each length class. Compare these values with the theoretical weights estimated in 1).
4. Using the results obtained up to now, write the Bertalanffy growth equation, in weight, for this stock.

8.8 COHORT DURING ALL LIFE – BIOMASS AND CATCH IN WEIGHT (3.6)

GROUP I

The recruitment to the exploitable phase of horse-mackerel, *Trachurus trachurus*, distributed in the Ibero-Atlantic waters (Div. VIIIc and IXa) occurs at age 1. In order to make the calculations, let us consider the exploitable phase between the ages 1 and 10 years and the recruitment of a cohort equal to 1 000 individuals.

The parameters of the von Bertalanffy equation were estimated by the Working Group of ICES (ICES, 1998a), using the mean lengths, at age of the catch, as being:

$$\begin{aligned}L_{\infty} &= 34.46 \text{ cm} \\ K &= 0.225 \text{ year}^{-1} \\ t_0 &= -1.66 \text{ year}\end{aligned}$$

The weight-length relation was also estimated, using the mean weights by age adopted by the WG for the long-term projections (ICES, 1998a), as being:

$$W(g) = 0.011 L(cm)^{2.90}$$

The mortality of this stock is characterized as :

- A constant natural mortality coefficient during all the exploitable phase : $M = 0.15 \text{ year}^{-1}$.
- Fishing mortality coefficients in 1996 (ICES, 1998a) are different with age as :

Age (year)	1	2	3	4	5	6	7	8	9	10
$F_i(\text{year}^{-1})$	0.24	0.26	0.10	0.08	0.06	0.09	0.12	0.14	0.18	0.24

1. Organize the calculations in a spreadsheet, in order to calculate, for each age of the life of the cohort (from age 1 to 10):
 - a) The number at the beginning of the age
 - b) The individual weight at the beginning of the age
 - c) The biomass at the beginning of the age
 - d) The number of deaths during the age

- e) The mean number of survivors during the age
- f) The mean individual weight during the age
- g) The mean biomass during the age
- h) The Catch in number during the age
- i) The Catch in weight during the age

2. Determine:

- a) The cumulative number of survivors during all the exploitable phase
- b) The cumulative biomass of the cohort during all the exploitable life
- c) The total catch, in numbers, from the cohort during all the exploitable life;
- d) The total catch, in weight, from the cohort during all the exploitable life;
- e) The mean weight of the individuals caught, during all the exploitable life;
- f) The mean weight of the individuals of the cohort, during all the exploitable life.

GROUP II

Present the histograms of :

- 1. Mean numbers of survivors of the cohort in each age, during all the exploitable life.
- 2. Mean biomasses of the cohort at each age, during all the exploitable life.
- 3. Catches in number by age of the cohort, during all the exploitable life.
- 4. Catches in weight by age of the cohort, during all the exploitable life.

GROUP III

Suppose now, that one intends to analyse the case in which all the fishing mortality coefficients were 30 percent bigger than those indicated in the table of Group I.

- 1. Calculate the characteristics of the Group I-2 that could be obtained from the same cohort during all its life, in this alternative situation.
- 2. Compare the values of the characteristics obtained under these conditions with those obtained in Group I-2, calculating the percentage of variation of those characteristics in relation to the corresponding values of the previous situation.

8.9 COHORT DURING ITS LIFE – SIMPLIFICATION OF BEVERTON AND HOLT MODEL

(3.7)

GROUP I

The recruitment to the exploitable phase of horse-mackerel, *Trachurus trachurus*, in Ibero-Atlantic waters (Div. VIIIc and IXa) occurs at age 1.

The recruitment at the exploitable phase was simplified by adopting the age $t_c=2$ year.

The parameters of the von Bertalanffy equation estimated for this stock are the following :

$$\begin{aligned}L_{\infty} &= 34.46 \text{ cm} \\K &= 0.225 \text{ year}^{-1} \\t_0 &= -1.66 \text{ year}\end{aligned}$$

The weight-length relation: $W(g) = 0.011 L(cm)^{2.90}$

The mortality of this stock is characterized as :

- A constant natural mortality coefficient during all the exploitable phase $M = 0.15 \text{ year}^{-1}$;
- Fishing mortality coefficient, $F = 0.14 \text{ year}^{-1}$, constant during all the exploited phase.

1. Calculate, using the simplification of Beverton and Holt :

- a) The recruitment R_c to the exploited phase.
- b) The number of deaths during all the exploited life.
- c) The cumulative number of survivors during all the exploited life.
- d) The cumulative biomass during all the exploited life.
- e) The catch in number during all the exploited life.
- f) The catch in weight during all the exploited life.
- g) The mean weight of the individuals of the cohort during all the exploited life.
- h) The mean weight of the individuals caught during all the exploited life.

GROUP II

The data presented in Section 8.8 – Group I show a great variety in the values of F . However, in Section 8.9, the simplification of an F constant was adopted, $F = 0.14 \text{ year}^{-1}$. The purpose, now, is to compare the results from Section 8.8 with those obtained in this exercise, using the simplification of Beverton and Holt.

1.
 - a) Determinate the cumulative number, the cumulative biomass, the total catch in number, the total catch in weight and the mean weight in the catch.
 - b) Compare the results with those obtained in Section 8.8 (Group I-2).
2.
 - a) Using a 30 percent bigger value of F calculate the cumulative number, the cumulative biomass, the total catch in number, the total catch in weight and the mean weight in the catch and compare with the corresponding values obtained in the Group I-1.a).
 - b) Compare the percentages of variation of the characteristics obtained in 2.a) with those obtained in Section 8.8 (Group III).

8.10 STOCK – SHORT-TERM PROJECTION (4.3)

The mortality parameters and the exploitation pattern of the Iberic Stock of hake, *Merluccius merluccius*, from Divisions VIIIc and IXa of ICES were estimated by the Assessment Working Group of ICES (ICES, 1998b) as :

- Natural Mortality Coefficient = 0.20 year^{-1}
- Fishing Mortality Level in 1996 = 0.24 year^{-1}

Exploitation pattern in 1996

age	0	1	2	3	4	5	6	7	8
s_i	0.00	0.09	0.29	1.31	1.25	1.12	1.32	1.55	1.55

The growth parameters were estimated as being :

Growth parameters of von Bertalanffy (CE, 1994):	Weight length relation, $W_t(g) = a.L(cm)^b$ (Cardador, 1988):
$L_{\infty} = 100 \text{ cm}$ $K = 0.08 \text{ year}^{-1}$ $t_0 = -1.4 \text{ year}$	$a = 0.004$ $b = 3.2$

At the beginning of 1996 the considered stock had the following age structure, with i representing the age and N_i the number of survivors at the beginning of the age i , expressed in millions of individuals :

age	0	1	2	3	4	5	6	7	8
N_i	83	27	41	30	22	11	6	3	5

GROUP I

1. Estimate the individual weights at the beginning of each age, the total number of individuals and the total biomass of the stock at the beginning of the year.
2. Estimate the individual mean weight, the mean number of survivors, the mean biomass, the total catch in number, the total catch in weight and the mean weight in the catch and in the stock, during 1996.
3. Supposing there was a recruitment in 1997 equal to 100 million individuals, calculate the following, for the beginning of 1997:
 - a) The age structure of the stock, in number
 - b) The age structure of the stock, in biomass
 - c) The total number of individuals of the stock
 - d) The total biomass of the stock

(notice that the stock at the beginning of 1997 is equivalent to the stock at the end of 1996, except for the 1997 recruitment).

GROUP II

The scientists drew attention to the fact that the fishing level in 1996 was very high and should, therefore, be reduced. They suggested a reduction of about 40 percent so that one could get adequate catches in weight and biomasses in the future.

As an alternative to improve the exploitation of this stock, they also suggested increasing the mesh size of the fishing nets.

1. Fishing managers asked the scientists to evaluate the mean biomass of the stock, the catch in number, the catch in weight and the mean weight in the catch during 1997 in case of :
 - a) Maintaining the 1996 fishing pattern (status quo situation)
 - b) Reducing the 1996 fishing level by 40 percent.

Making the necessary calculations in order to estimate a) and b), present the results and comment on the changes in the catch in weight and mean biomass, relative to 1996.

2. Fishing managers also asked the scientists to evaluate the catch in weight and the mean biomass that would result from maintaining the fishing level of 1996 in 1997, but using a new mesh size of the fishing net. To solve this question, the scientists proposed the following new exploitation pattern:

age	0	1	2	3	4	5	6	7	8
s_i	0.00	0.00	0.03	1.50	1.65	1.75	1.80	1.80	1.80

Make the calculations, present the results and make your comments on the changes in the catch in weight and the mean biomass relative to 1996.

3. Evaluate the effects on the catch in weight and on the mean biomass, resulting from the simultaneous adoption (in 1997) of the reduction by 40 percent of the fishing level in 1996 and the introduction of the new exploitation pattern. Present the results and make your comments .

8.11 STOCK – LONG-TERM PROJECTION (4.4)

The mortality parameters of the Iberic Stock (Div. VIIIc e IXa) of Sardine, *Sardina pilchardus*, were estimated by the Assessment Working Group of ICES (ICES, 1997b) as :

- Natural Mortality Coefficient = 0.33 year^{-1}
- Fishing Mortality Level in 1996 = 0.56 year^{-1}

Assume that the exploitable phase occurs from the beginning of age zero until the end of age six.

Relative pattern of exploitation, s_i , during 1996

s_0	s_1	s_2	s_3	s_4	s_5	s_6
0.21	0.41	0.79	1.18	1.34	1.43	1.68

The parameters of the individual growth and the weight length relation of this stock were estimated (Pestana, 1989) as :

**Von Bertalanffy
growth parameters**

$$L_{\infty} = 22.3 \text{ cm}$$

$$K = 0.40 \text{ year}^{-1}$$

$$t_0 = -1.6 \text{ year}$$

Weight length relation

$$W = a.L^b$$

with L in cm and W in g

$$a = 0.0044$$

$$b = 3.185$$

GROUP I

1. Calculate the evolution in number and the biomass of a cohort during its life at the beginning of each age, supposing that growth, natural and fishing mortality parameters are the given values.
2. The recruitment at age 0 in 1996 was estimated as being 4300 million individuals. Calculate the cumulative number, the cumulative biomass, the catch in number and in weight, during all the life of the cohort.

GROUP II

It was estimated that, at the beginning of 1996, the considered stock had the following age structure, in number, representing i the age and N_i the number of survivors at the beginning of age i , expressed in millions of individuals :

I	0	1	2	3	4	5	6
N_i	4300	279	591	233	561	384	180

To make a long-term projection of the stock, all the mortality and growth parameters will be considered stable during the following years. Consider, also, that the recruitment in those future years will be equal to the recruitment of 1996.

1. Based on these assumptions, project the numbers of survivors at the beginning of each year and age, until 2006.
2. Compare the structures of the stock in the years 2003 and 2006.
3. Compare the evolution of the 1996 cohort with the structure of the stock at the beginning of 2003.

8.12 STOCK – RECRUITMENT RELATION

(4.5)

During the meeting of the assessment Working Group (ICES, 1998b) of the Iberic stock (Div. VIIIc and IXa) of hake, *Merluccius merluccius*, the following stock parameters were estimated for the period 1982-1996:

Year	N (age 0) (million)	Spawning Biomass (thousand tons)
1982	125	59.8
1983	107	61.4
1984	136	58.8
1985	97	44.1
1986	104	26.4
1987	97	24.2
1988	84	22.8
1989	56	18.9
1990	59	19.4
1991	69	20.5
1992	86	21.5
1993	70	21.0
1994	63	16.5
1995	32	15.2
1996	83	18.0

The annual recruitment at the exploitable phase is considered as being the number of individuals with age 0.

1. Draw the dispersion graph of the resulting recruitments, against the parental spawning biomass.
2. The parameters of Shepherd, Ricker, Beverton & Holt and Deriso S-R relations were estimated and are shown in the following table, as well as the respective determination coefficients, r^2 :

Parameters	Shepherd	Ricker	Beverton & Holt	Deriso
α (R/g)	3.50	4.43	4.91	4.40
k (1000 tons)	64.94	78.13	45.39	106.27
C	3.52			0.896
r^2	0.71	0.68	0.66	0.75

- a) Calculate the expected recruitments in each year of the period 1982-1996 using the four S-R models.
- b) The determination coefficient, r^2 , can be used as an indicator of a good or bad adjustment of the model to the observed data, depending on the number of observations (r^2 can be interpreted as a percentage of variation of the observed points that is explained by the model. Values close to 1 indicate a good adjustment and values close to zero indicate a bad adjustment). Using this indicator, make your comments about the adjustments of each model.

8.13 F_{\max}

(5.2.1)

Consider the stock of cod fish, *Gadus morhua*, in the Irish sea (Div. VIIa). The following mortality and biological parameters were estimated by the Working Group of ICES (ICES, 1998c):

Natural Mortality Coefficient: $M = 0.20 \text{ year}^{-1}$

Fishing Level in 1996: $F_{96} = 0.58 \text{ year}^{-1}$

Mean weight (kg) in the catch and in the stock :

Age	0	1	2	3	4	5	6	7
\bar{W}_i	0.001	0.883	1.778	3.597	5.695	7.904	8.502	9.200

GROUP I

Assume that the stock was constituted by the age groups 0 to 7 years and that it was exploited with the following exploitation pattern :

Age	0	1	2	3	4	5	6	7
s_i	0.80	0.90	0.96	1.00	1.00	1.00	1.00	1.00

1. Calculate, for 1000 recruits, the long-term annual catch in weight and the annual mean biomass, corresponding to the fishing level of 1996.
2. Adopting the factor F_{factor} between 0 and 2.5 year^{-1} with intervals of 0.1 year^{-1} draw the curve of the annual catch in weight against F_{factor} . Represent, in the same graph, the curve of the mean biomass against F_{factor} .
3. Calculate the biological reference point F_{\max} .

GROUP II

1. Answer the previous questions, but considering the following exploitation pattern.

Relative pattern of exploitation adopted by the Working Group:

Age	0	1	2	3	4	5	6	7
s_i	0.00	0.20	0.96	1.30	1.12	0.67	0.58	0.58

- Compare the value of F_{\max} of this exercise with the one obtained in the exercise in Group I.
- Calculate the biological reference point F_{\max} knowing that the Working Group considers the group of age 7 as a group of cumulative ages (7+). The mean weights in the catch and in the stock presented at the beginning of the text are maintained, except for the last group of ages which will now be the age group 7+ with the mean weight equal to 10.873 kg. The exploitation pattern is also maintained with the value 0.58 for the group 7+.
- Compare the two calculated values of F_{\max} , considering the last age group as 7 or as 7+.

8.14 $F_{0.1}$

(5.2.2)

Consider the Iberic stock (Div. VIIIc e IXa) of four-spot megrim, *Lepidorhombus boscii*.

The following parameters were estimated by the Working Group of ICES (ICES, 1998b):

Natural Mortality Coefficient: $M = 0.20 \text{ year}^{-1}$

Fishing mortality in 1996: $F_{96} = 0.36 \text{ year}^{-1}$

Exploitation pattern:

Age	1	2	3	4	5	6	7+
s_i	0.06	0.43	0.89	1.65	1.66	1.22	1.22

Mean weight (kg) in the catch and in the stock:

Age	1	2	3	4	5	6	7+
\bar{W}_i	0.037	0.067	0.086	0.109	0.144	0.188	0.244

1. Adopting the value 1000 for the recruitment to the fishing area, calculate the long-term annual catch in weight, the annual mean biomass, and the annual mean weight in the catch, corresponding to the fishing mortality level of 1996.
2. Adopting the factor F_{factor} between 0 and 2.5 year^{-1} with intervals of 0.1 year^{-1} draw the curve of the annual catch in weight against F_{factor} . Represent, in the same graph, the curve of the mean biomass against F_{factor} .
3. Calculate the biological reference point $F_{0.1}$
4. Calculate the biological reference point F_{max}
5. In the graph of the curves of the annual catch in weight and of the mean biomass against F_{factor} , mark the Biological reference points $F_{0.1}$ and F_{max} which were previously calculated. Make your comments.
6. Calculate the long-term mean biomass, the catch in weight and the mean weight in the catch for $F_{0.1}$. Compare those characteristics with the values obtained in question 1 and make your comments.

8.15 F_{med} and F_{MSY}

(5.2.3) and (5.2.4)

During the meeting of the Assessment Working Group (ICES, 1998b) on the Iberic stock (Div. VIIIc and IXa) of hake, *Merluccius merluccius*, the following population parameters were estimated :

Natural mortality coefficient: $M = 0.2 \text{ year}^{-1}$;

Fishing mortality in 1996: $F_{96} = 0.24 \text{ year}^{-1}$

Mean weight in the catch (g):

Age (year)	0	1	2	3	4	5	6	7	8+
$\bar{W}_i(\text{g})$	4	37	106	205	358	517	706	935	1508

Maturity ogive (percent):

Age (year)	0	1	2	3	4	5	6	7	8+
% mature _i	0	0	1	6	20	49	76	91	100

For the long-term projections, the Working Group adopted the mean exploitation pattern for the period 1994-1996, as shown in the next table :

Age (year)	0	1	2	3	4	5	6	7	8+
s_i	0.001	0.11	0.398	1.3	1.261	1.019	1.473	1.874	1.874

1. Calculate the long-term annual mean biomass, the spawning biomass, the annual catch in weight and the mean weight in the catch.
2. Draw the curve of the annual catch in weight and the mean biomass against F , for values of F_{factor} between 0 and 2.5 year^{-1} , with intervals of 0.1 year^{-1} . Calculate the TRPs F_{max} and $F_{0.1}$.
3. Calculate the biological reference point F_{med} , knowing the recruitments (million individuals at age 0) and the spawning biomasses (thousand tons) between 1982 and 1996, estimated by the Working Group, presented in the following table :

Year	N (age 0) (million)	Spawning biomass (1000 tons)
1982	125	59.8
1983	107	61.4
1984	136	58.8
1985	97	44.1
1986	104	26.4
1987	97	24.2
1988	84	22.8
1989	56	18.9
1990	59	19.4
1991	69	20.5
1992	86	21.5
1993	70	21.0
1994	63	16.5
1995	32	15.2
1996	83	18.0

4. Calculate the long-term mean biomass, the spawning biomass, the catch in weight and the mean weight in the catch for F_{med} . Compare with the values obtained in question 1 and make your comments.
5. Adopting the Ricker stock-recruitment relation, estimated by the Working Group ($\alpha = 4.43 \text{ R/kg}$ and $K = 78.13$ thousand tons), calculate F_{MSY} , B_{MSY} and Y_{MSY} . and compare the different F -target estimated.

8.16 MBAL AND B_{loss}

(5.3.4 & 5.3.5)

During the meeting of the Assessment Working Group (ICES, 1998b) on the Iberic stock (Div. VIIIc and IXa) of hake, *Merluccius merluccius*, the recruitment (million individuals at age 0) and the spawning biomass (thousand tons) was estimated for the period 1982-1996. The values obtained are presented in the following table :

Year	N (age 0) (million)	Spawning biomass (1000 tons)
1982	125	59.8
1983	107	61.4
1984	136	58.8
1985	97	44.1
1986	104	26.4
1987	97	24.2
1988	84	22.8
1989	56	18.9
1990	59	19.4
1991	69	20.5
1992	86	21.5
1993	70	21.0
1994	63	16.5
1995	32	15.2
1996	83	18.0

1. Using the spawning biomasses and the resulting recruitments, calculate the Biological Reference Limit Points, MBAL and B_{loss} .

8.17 F_{loss} and F_{crash}

(5.3.5 & 5.3.6)

GROUP I

During the meeting of the Assessment Working Group (ICES, 1998b) on the Iberic stock (Div. VIIIc and IXa) of hake, *Merluccius merluccius*, the recruitment (million individuals at age 0) and the spawning biomass (thousand tons) was estimated for the period 1982-1996. The values obtained are shown in the following table :

Year	N (age 0) (million)	Spawning biomass (thousand tons)
1982	125	59.8
1983	107	61.4
1984	136	58.8
1985	97	44.1
1986	104	26.4
1987	97	24.2
1988	84	22.8
1989	56	18.9
1990	59	19.4
1991	69	20.5
1992	86	21.5
1993	70	21.0
1994	63	16.5
1995	32	15.2
1996	83	18.0

The Shepherd S-R relation was adjusted to the pairs of values in the table ($r^2=0.71$), and the relation parameters are the following :

$$\begin{aligned}\alpha &= 3.5 \quad \text{Kg}^{-1} \\ k &= 64.94 \text{ thousand tons} \\ c &= 3.52\end{aligned}$$

1. Draw the dispersion graph of the resulting recruitments, against the parental spawning biomasses.
2. Calculate the expected recruitments in each year of the period 1982-1996 according to the Shepherd S-R model and in the previous dispersion graph, draw the respective curve.

GROUP II

1. Calculate the annual catch in weight and the spawning biomass per recruit for the stock of hake, using the mortality and biological parameters estimated by the Working Group for the long-term projections (given in Section 8.15), namely :

Natural mortality coefficient : $M = 0.2 \text{ year}^{-1}$
 Fishing mortality in 1996: $F_{96} = 0.24 \text{ year}^{-1}$

Mean weight in the catch (g):

Age (year)	0	1	2	3	4	5	6	7	8+
\bar{W}_i	4	37	106	205	358	517	706	935	1508

Maturity ogive (percent):

Age (year)	0	1	2	3	4	5	6	7	8+
% mature _i	0	0	1	6	20	49	76	91	100

Mean relative pattern of exploitation of the period 1994-1996:

Age (year)	0	1	2	3	4	5	6	7	8+
s_i	0.001	0.11	0.398	1.3	1.261	1.019	1.473	1.874	1.874

GROUP III

1. Using the results of Groups I and II, calculate biological reference Limit-Points, F_{loss} and F_{crash} .

8.18 PRODUCTION MODELS (EQUILIBRIUM) – SCHAEFER (6.7.1)

The following table presents the annual total catch Y , (t) and mean biomass \bar{B} , (t) for the fishery of the Iberic stock (Div. VIIIc and IXa) of sardine, *Sardina pilchardus*, between 1977 and 1996, used by the Working Group of ICES (ICES, 1998a).

Year	Y (t)	\bar{B} (t)
1977	125750	750289
1978	139990	759192
1979	153441	763313
1980	191682	804765
1981	214133	842091
1982	204504	802573
1983	181139	713376
1984	202686	794856
1985	204107	810539
1986	180606	679808
1987	168825	547179
1988	158540	481295
1989	137126	431719
1990	139157	368099
1991	127756	316365
1992	126054	453161
1993	138795	539096
1994	132800	416842
1995	121384	368158
1996	111431	246037

GROUP I

1. Calculate F_i corresponding to each year i .
2. Calculate the biomasses, B_i , at the beginning of each year. (Use the procedure proposed by Schaefer, that is, the biomass at the beginning of a year is approximated by the arithmetic mean of the mean biomasses of the previous and the following year).
3. Calculate the equilibrium catches, Y_E , which would correspond to the observed values of F .

4. Calculate the equilibrium mean biomasses, \bar{B}_E , which would correspond to the observed values of F .
5. Draw the graph of \bar{B}_E against F_i .

GROUP II

The Schaefer model was adjusted and the following parameters were estimated :

$$k = 1562851 \text{ t}$$

$$r = 0.426 \text{ year}^{-1}$$

1. Calculate the equilibrium biomasses and the equilibrium catches corresponding to the fishing levels observed in each year, using the Schaefer model.
2. Draw, in the graph of Group I point 5, the equilibrium biomasses calculated with the Schaefer model.
3. Calculate F_{MSY} , B_{MSY} and Y_{MSY}
4. Calculate $F_{0.1}$, $B_{0.1}$ and $Y_{0.1}$.

8.19 PRODUCTION MODELS (EQUILIBRIUM)

(6.7.1 & 6.7.2)

The following table presents the annual total catches and the corresponding total fishing efforts of a shrimp stock in the Arabian Sea during the period 1969 to 1978 (Sparre and Venema, 1992).

Year	Catch, Y (t)	Total effort (1000 days)
1969	546.7	1.224
1970	812.4	2.202
1971	2493.3	6.684
1972	4358.6	12.418
1973	6891.5	16.019
1974	6532.0	21.552
1975	4737.1	24.570
1976	5567.4	29.441
1977	5687.7	28.575
1978	5984.0	30.172

GROUP I

1. Draw a graph of the annual abundance index against the corresponding fishing mortality index.
2. The Fox model was adjusted to the data. The following parameters were obtained :

$$a = 6.150 \quad b = -0.028 \text{ with a determination coefficient, } r^2 = 0.78.$$

- a) Calculate and draw the curves of the equilibrium conditions of the abundance index and of the total catch against fishing effort.
- b) Determine the target-points MSY and 0.1.
- c) Determine the parameters (kq), (r/q) and (kr).

GROUP II

Knowing that the adjustment of the Schaefer model to the same set of data, resulted in the following values of the parameters :

$$a = 444.454$$

$$b = - 8.055$$

$$(r^2 = 0.77)$$

1. Repeat the calculations of the previous Group.

8.20 PRODUCTION MODELS – SHORT-TERM PROJECTION (6.8)

The following table presents the annual total catches (in tons) and the corresponding catches by fishing unit effort (kg/ fishing day of the fleet PESCRUL) of the stock of Deepwater rose shrimp, *Parapenaeus longirostris*, of the Algarve during the period 1983 to 1994 (Mattos Silva, 1995).

Year	Y (t)	cpue (kg/day)
1983	538	235
1984	638	131
1985	431	63
1986	99	22
1987	37	8
1988	62	21
1989	437	77
1990	146	28
1991	126	26
1992	53	25
1993	91	41
1994	232	66

Using the following parameters of the integrated Fox model and Yoshimoto & Clarke (1993):
 $k = 1580 \text{ t}$, $q = 0.39 \text{ thousand days}^{-1}$ and $r = 0.55 \text{ year}^{-1}$:

1. Project the cpue and the catch in weight for the year 1995, supposing that the fishing effort will be maintained equal to that of 1994 (situation status quo).
2. Determine the target reference points, Y_{MSY} , \bar{B}_{MSY} and F_{MSY} and the indices \bar{U}_{MSY} and f_{MSY} .
3. Determine the target reference points, $Y_{0.1}$, $\bar{B}_{0.1}$ and $F_{0.1}$ and the indices $\bar{U}_{0.1}$ and $f_{0.1}$.
4. Determine the percentages of the carrying capacity corresponding to the target points F_{MSY} and $F_{0.1}$.
5. Based on the results obtained in the previous questions, make your comments on the state of the stock and of its exploitation.
6. Suppose that one intends to reduce the fishing effort in 1995 by about 20 percent relative to the effort of 1994. Project the catch in weight for 1995 and present the variations resulting from that effort reduction on the catch in weight and on the biomass.

8.21 SIMPLE LINEAR REGRESSION – ESTIMATION OF THE PARAMETERS OF THE W-L RELATION AND GROWTH PARAMETERS (FORD-WALFORD, GULLAND AND HOLT AND STAMATOPOULOS AND CADDY) (7.2)

GROUP I

Consider the following 10 pairs of values of x and y:

x_i	2	6	7	8	11	15	16	18	19	21
y_i	13	40	52	56	78	105	111	130	132	149

1. Estimate the constants A and B of the straight line.
2. Estimate the values of Y corresponding to the given values of x.
3. Calculate the determination coefficient r^2 .
4. Draw a graph with the observed values and with the estimated line. Observe the adjustment and say if you consider the linear model adequate.

GROUP II

The data presented on the following table represent the individual weights by length class of European anglerfish, *Lophius budegassa* samples from the Iberic coast in 1994.

L_i (cm)	\overline{W}_t (g)	L_i (cm)	\overline{W}_t (g)
20-	129	50-	1685
22-	163	52-	1896
24-	219	54-	2107
26-	265	56-	2345
28-	320	58-	2569
30-	397	60-	2848
32-	486	62-	3126
34-	545	64-	3407
36-	664	66-	3700
38-	773	68-	4056
40-	890	70-	4411
42-	1027	72-	4764
44-	1122	74-	5203
46-	1334	76-	5587
48-	1503	78-	5982

- Using the simple linear regression model, estimate the parameters of the weight-length relation for this stock, considering that $W_{\text{central}} = W_{\text{mean}}$. (Notice that the $\ln W_{\text{central}}$ is linear to the $\ln L_{\text{central}}$).

GROUP III

The data presented in the following table represent the mean length (cm) of the individuals at the beginning of the age (years), obtained from direct age reading of the individuals of the stock of European anglerfish, *Lophius budegassa*, (ICES Div. VIIIc and IXa), Section 8.7

T	L_t	t	L_t
1	9.2	7	44.4
2	16.5	8	49.0
3	22.9	9	52.3
4	28.8	10	55.0
5	34.7	11	60.8
6	38.6	12	63.4

From this data and using the simple linear regression model, estimate the growth parameters K and L_{∞} with the expressions :

1. Ford-Walford (1933-1946)
2. Gulland and Holt (1959)
3. Stamatopoulos and Caddy (1989). (The expressions for 1), 2) and 3), were studied in the Chapter concerning Individual Growth).
4. Comment on the results obtained in the previous questions with the values of the parameters given in Section 8.7.

8.22 MULTIPLE LINEAR MODEL – REVISION OF MATRICES – (7.3) ESTIMATION OF THE PARAMETERS OF FOX INTEGRATED MODEL (IFOX)

REVISION OF MATRICES

GROUP I

Consider the matrices A and B :

$$A = \begin{pmatrix} 2 & 3 & 0 & 1 \\ 1 & 1 & 4 & 1 \\ 0 & 4 & 2 & 2 \\ 1 & 0 & 3 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 0 & 3 \\ 1 & 3 & 2 & 5 \\ 2 & 1 & 6 & 0 \\ 2 & 2 & 1 & 0 \end{pmatrix}$$

1. Using a spreadsheet, calculate: $A + B$, $A * B$, $\text{Det}(A)$, $\text{Det}(B)$, A^{-1} e B^{-1}
2. Show that $(A.B)^{-1} = B^{-1}.A^{-1}$
3. Show that $(A.B)^T = B^T.A^T$

GROUP II

Let the Matrices:

$$M_{(4,4)} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad O_{(4,4)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad I_{(4,4)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1. Verify that the matrix null 0 is idempotent.
2. Verify that the matrix identity I is idempotent.
3. Verify that the matrix M is idempotent.

4. What are the traces of M and I?
5. Calculate the ranks, r , of M and of I.
6. What is the value of the determinant of M and I?

GROUP III

1. Verify that the product Mx , where x is the vector given by $x^T = (3 \ 4 \ 8 \ 1)$, is a vector with all elements equal to the arithmetic mean, \bar{x} , of the 4 elements of vector x .
2. Verify that $(I-M)x$ is the vector of the deviation.
3. Verify that the sum of the squares of x_i , $-(x_i^2)$ can be written as : $x^T \cdot x$
4. Verify that the sum of the squares of the deviations, $-(x_i - \bar{x})^2$, can also be written in a matricial form, as : $x^T(I-M)x$

GROUP IV

1. Consider the vector $x = 2 + \theta$ where θ is an unknown parameter.

$$\begin{matrix} 3\theta \\ 5 - \theta \end{matrix}$$

a) Write the derivative $\frac{dx}{d\theta}$ of the vector x

b) Calculate $x^T x$

c) Calculate $\frac{d}{d\theta} (x^T x)$

d) Show that $\frac{d}{d\theta} (x^T x) = 2 \left(\frac{dx}{d\theta}\right)^T x$

2. Consider the vector $x = 2 + 4\theta_1 - 5\theta_2$ where θ_1 and θ_2 are two unknown constants.

$$1 + \theta_1 + \theta_2$$

$$\theta_1^2 + 4\theta_2$$

a) Write the derivative matrix $\frac{\partial x}{\partial \theta}$ (take θ_1 and θ_2 as variables)

b) Calculate $x^T x$

c) Transpose $\frac{\partial}{\partial \theta} (x^T x)$

- d) Show that the transposed matrix $\frac{\partial}{\partial \theta} (x^T x) = 2 \left(\frac{\partial x}{\partial \theta} \right)^T x$

GROUP V

Consider the following system of 2 equations with 2 unknowns;

$$\begin{aligned} 5 &= 2A + 3B \\ 4 &= A - 2B \end{aligned}$$

1. Show that the equation system can be written in matrix form as,

$$Y_{(2,1)} = X_{(2,2)} \theta_{(2,1)}$$

where Y is the vector of the independent terms (5 e 4) of the system,

θ is the vector of the unknowns A and B
and X is the matrix of the coefficients of the unknowns

2. Verify that the solution of the system can be given as $\theta = (X^T X)^{-1} X^T Y$
3. Show that X is a square, non singular matrix, and then that the solution of the system can be $\theta = X^{-1} Y$

ESTIMATION OF THE PARAMETERS OF THE YOSHIMOTO AND CLARKE MODEL (1993)

4. Estimate the parameters k , q and r , of the Fox integrated model (IFOX) and of Yoshimoto & Clarke (1993) using the following data :

Year	Y (t)	CPUE (kg/day)
1983	538	235
1984	638	131
1985	431	63
1986	99	22
1987	37	8
1988	62	21
1989	437	77
1990	146	28
1991	126	26
1992	53	25
1993	91	41
1994	232	66

which represent the total annual catches (in tons) and the respective catches by fishing effort unit (kg/fishing day of the fleet PESCRUL) of the stock of Deepwater rose shrimp, *Parapenaeus longirostris* of the Algarve during the period 1983 to 1994 (Mattos Silva, 1995).

Comment on the obtained results comparing them with those presented in Section 8.20.

8.23 NON LINEAR REGRESSION – ESTIMATION OF THE GROWTH PARAMETERS AND OF THE S– R RELATION (GAUSS–NEWTON METHOD) (7.4)

The data in the following table represent the mean length (cm) of the individuals at the beginning of the age (years), obtained from direct age reading of individuals of the stock of anglerfish, *Lophius budegassa*, (Div. VIIIc and IXa), Section 8.7.

t	L_t	t	L_t
1	9.2	7	44.4
2	16.5	8	49.0
3	22.9	9	52.3
4	28.8	10	55.0
5	34.7	11	60.8
6	38.6	12	63.4

GROUP I

1. Represent graphically the values of L_t against t.
2. Estimate the growth parameters K, L_∞ and t_0 for the Bertalanffy growth model using a non-linear regression model.
3. Estimate the values of L_t corresponding to given values of t and mark on a graph the observed values and the estimate curve. Comment on the adjustment.
4. Compare the results obtained in the previous questions with the values of the parameters given in Section 8.7.

GROUP II

Using the data on spawning biomass and on recruitments of hake, presented in Section 8.12, estimate the parameters of the Beverton and Holt, Ricker, Deriso and Shepherd S-R models.

1. Compare the obtained values with those presented in Section 8.12.

8.24 ESTIMATION OF M

(7.6)

GROUP I

1. Estimate the 5 percent Tanaka survival curve for natural mortality coefficients between 0.0 and 5.0 year⁻¹ and longevity between 0 and 30 years.
2. Calculate the values of M corresponding to the longevity 1, 2, 3, 10, 15 and 30 years.

GROUP II

The parameters of von Bertalanffy equation for the Iberic stock (Div. VIIIc and IXa of ICES) of horse-mackerel, *Trachurus trachurus*, are the following :

$$L_{\infty} = 34.46 \text{ cm, TL (carapace length)}$$
$$K = 0.225 \text{ year}^{-1}$$

1. Estimate the value of M for this stock, knowing that from 1985-95 in the distribution area of this stock in the Iberic Peninsula, the mean temperature of the sea water at the surface, was : $\bar{T} = 13^{\circ} \text{ C}$

GROUP III

Consider a certain fishing stock for which the first maturity mean age was estimated as 2.3 year.

1. Obtain an approximate estimate of M for this stock.

GROUP IV

The reproductive biology of the stock of Atlantic mackerel, *Scomber scombrus* was studied and it was estimated that the mean gonadosomatic index (gonad weight/total weight) of the mature females in the spawning period was 0.13.

1. Estimate an approximate value for the natural mortality coefficient M for this stock, assuming that it is constant for all the ages and years.

GROUP V

The scientists responsible for the evaluation of the stock of a certain fishing resource, implemented an acoustic cruise every year in January, to estimate the abundance of the stock by age classes. Fishery statistics are also needed to estimate the catch by age during the year.

The following table presents the estimations of the abundance of the stock, by age classes, obtained on the cruises in 1993 and 1994, as well as the structure of the catches during 1993.

Age	Number of survivals, N, in million		Total catch in 1993 (million)
	January 1993	January 1994	
2	243	353	11
3	99	189	15
4	86	67	20
5	37	52	9
6	13	22	3
7	6	8	1

Although this information is available, the scientists responsible for this stock have difficulties applying the assessment models to the resource because they do not have an estimation of M .

1. So, estimate the natural mortality coefficient for this resource in 1993, and help the group of scientists responsible for the resource.

GROUP VI

The following table presents the data on fishing effort, in million trawl hours, and the total mortality coefficient, Z , for a certain fishery for the period 1987 to 1995.

Years	Effort (10^6 hours)	Z (year ⁻¹)
1987	2.08	1.97
1988	2.80	2.05
1989	3.5	1.82
1990	3.6	2.32
1991	3.8	2.58
1992	-	-
1993	-	-
1994	9.94	3.74
1995	6.06	3.74

1. Determine M (natural mortality coefficient), assumed to be constant for the period 1987-1995.
2. Determine the catchability coefficient q .

8.25 ESTIMATION OF Z

(7.7)

GROUP I

A swept area cruise allowed the scientists of the Marine Research Institute in Bergen, Norway, to estimate the abundance of the different age classes of the stock of cod fish, *Gadus morhua*, in January of 1995 (following table).

Age (years)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
N ₉₅ (10 ⁹)	1984	440	160	103	82	65	54	43	33	27	26	21	17	13	10

1. Represent on a graph, the logarithms of the numbers of survivors against the age.
2. Select the age interval from which the total mortality coefficient, Z, can be taken as constant.
3. Estimate the total mortality coefficient, Z, of the stock in January 1995.

GROUP II

The following table presents the mean catches by age, in number, of plaice, *Pleuronectes platessa*, per 100 trawl hours in two periods, 1929-1938 and 1950-1958.

Age (years)		2	3	4	5	6	7	8	9	10
C/f	1929-38	125	1355	2352	1761	786	339	159	70	28
C/f	1950-58	98	959	1919	1670	951	548	316	180	105

1. Estimate the total mortality coefficient, Z, of the stock in each of the periods.
2. Consider that the mean fishing effort on the North Sea plaice during the two periods was 5 million hours of trawl in 1929-1938 and 3.1 million hours of trawl in 1950-1958. Estimate for each period:
 - a) the natural mortality coefficient, M;
 - b) the catchability coefficient, q;
 - c) and the fishing mortality coefficient, F.

GROUP III

The following table presents the annual composition of the catches by age from 1988 to 1994, in millions of individuals, for a certain resource :

CATCHES (million individuals)

Age	1988	1989	1990	1991	1992	1993	1994
0	599	239	424	664	685	478	330
1	678	860	431	1004	418	607	288
2	1097	390	1071	532	335	464	323
3	275	298	159	269	203	211	243
4	40	54	75	32	69	86	80
5	6	9	13	18	8	25	31
6	1	8	3	5	5	3	8
7	6	0	1	0	1	1	1

1. Calculate the mean annual composition during 1988-1994.
2. Estimate Z , based on that mean composition.
3. Estimate Z , based on the mean age of the mean composition of the catch.
4. Estimate Z for each year of the given period.
5. Compare the annual Z s with the values of Z obtained in questions 2 and 3.

GROUP IV

The following table shows the length composition, in equilibrium, of a certain resource, with $L_{\infty} = 100$ cm and $K = 0.2 \text{ year}^{-1}$.

Length class (cm)	35-	40-	45-	50-	55-	60-	65-	70-	75-	80-	85-	90-	95-
Catch (C_i) in million	7	10	20	51	46	44	41	36	33	28	23	17	8

1. Calculate the relative ages corresponding to the lower limit of each length class.
2. Determine the age interval corresponding to each length class.
3. From which class can one consider Z constant?
4. Determine Z using :
 - a) The catches in each class.
 - b) The cumulative catches.
 - c) The mean length in the catch.
5. Compare the values of Z obtained by the different methods of question 4.

GROUP V

The length compositions of the catches for three different periods of time are known for a certain fishing resource.

Period	Length classes (cm)	45-	50-	55-	60-	65-	70-	75-	80-	85-	90-	≥95
1960-69	Catch (C_i) in million	256	237	211	187	161	138	113	87	62	36	12
1970-79		268	226	180	141	105	76	50	30	15	6	1
1980-89		212	161	116	79	52	31	17	8	3	1	0

Consider the 45 cm length class as the first class completely recruited .

Adopt $K = 0.3 \text{ year}^{-1}$ and $L_\infty = 100 \text{ cm}$ as the von Bertalanffy growth parameters for this resource.

1. Estimate the values of the total mortality coefficient, Z , for each period and comment on the results.

8.26 AGE COHORT ANALYSIS (CA)

(7.9.1)

GROUP I

1. Consider a stock and an interval of time i , (t_i, t_{i+1}) . Knowing that for this interval of time:

$$M_i = 0.4 \text{ year}^{-1}$$

$$T_i = 2.3 \text{ year}$$

$$C_i = 230 \text{ million individuals}$$

- a) Adopt the value 0.5 year^{-1} for the fishing mortality coefficient for the interval and calculate the numbers of survivors at the beginning and end of the interval.

2. Consider the interval of time i , (t_i, t_{i+1}) . Knowing that in this interval of time:

$$M_i = 0.6 \text{ year}^{-1}$$

$$T_i = 0.9 \text{ year}$$

$$C_i = 98 \text{ million individuals}$$

Calculate the value of the fishing mortality coefficient, F_i , for the interval, taking the number of survivors, N_i , at the beginning of the interval, i , to be 172 million individuals.

3. Consider the interval of time (t_i, t_{i+1}) . Knowing that in that interval of time :

$$M_i = 0.5 \text{ year}^{-1}$$

$$T_i = 1 \text{ year}$$

$$C_i = 42 \text{ million individuals}$$

- a) Calculate the value of the fishing mortality coefficient for the interval, knowing that the number of survivors at the end of the year was $N_{i+1} = 85$ million individuals. Calculate the value of F_i using the Pope formula.

GROUP II

The data in the following table represent the catches in millions, of a cohort of hake, *Merluccius merluccius*, in the Iberic Peninsula waters.

Age (years)	0	1	2	3	4	5	6	7	8
C_i (million)	712	3941	8191	10311	5515	4149	3081	1185	549

Adopt a value of 0.2 year^{-1} for the natural mortality coefficient, constant for all the ages.

- Suppose that the value of the fishing mortality coefficient at the last age (8 years) was 1.0 year^{-1} . Calculate, by an iterative method and by the Pope method, for each age of the cohort :
 - The value of the fishing mortality coefficient.
 - The number of survivors at the beginning of the age.
 - Compare the results obtained by the two methods.
 - Represent, on a graph the values of F_i estimated against the age, and say what the recruitment of this cohort is at the exploited phase.

GROUP III

- Aiming to analyse the influence of the chosen F_{terminal} , repeat the calculations of question 1 of Group II, using one of the previous methods, with 0.3 and 1.5 year^{-1} for the value of F_{terminal} .
 - Draw a graph with the estimated values of F_i and N_i against the age.
 - Comment on the differences between the graphs for the different values of F_{terminal} .
- Aiming to analyse the influence of the choice of M , repeat the calculations of question 1 of Group II, using one of the previous methods, for values of M of 0.1 and 0.4 year^{-1} .

- a) Represent, on a graph, the estimated values of F_i and N_i against the age.
- b) Comment on the differences between the graphs for the different values of M .

GROUP IV

The annual catches by age class, of a certain resource, for the years of 1985 to 1994, are presented in the following table.

Catches by age class (Million individuals)										
Age (years)	Years									
	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
0	67	88	104	290	132	90	63	38	52	90
1	532	1908	1841	1671	4172	1915	1284	906	541	704
2	2070	1756	4424	3178	2534	6320	2826	1911	1322	741
3	728	4016	2256	4042	2499	1972	4742	2115	1382	890
4	353	945	3309	1273	1926	1170	883	2102	896	540
5	97	439	733	1730	558	827	479	356	807	316
6	16	107	300	333	656	207	291	166	117	243
7	25	8	73	136	126	243	73	101	54	35
8	5	7	5	33	52	47	85	25	33	16

The *modus operandi* of the fishing fleet was constant during the period, but the number of vessels increased significantly. It is considered that, at present, the resource is intensively exploited.

Besides the information on the fishery, the estimates of the growth parameters of this resource and of the natural mortality coefficient are also available:

$$\begin{array}{ll}
 L_{\infty} = 38.5 \text{ cm} & a = 0.021 \text{ of the relation } W(g)\text{-}L(\text{cm}) \\
 K = 0.25 \text{ year}^{-1} & b = 2.784 \text{ of the relation } W(g)\text{-}L(\text{cm}) \\
 t_0 = -0.51 \text{ year} & M = 0.3 \text{ year}^{-1}
 \end{array}$$

1. Estimate the fishing mortality coefficient and the number of survivors at the beginning of the year for each age class and each year. Use the Pope Cohort Analyses method.
 - a) Start by selecting $F_{\text{terminal}} = 0.5 \text{ year}^{-1}$ for the last age of every year and for all the ages of the last year.
 - b) After analysing matrix F obtained in a), select new values for F_{terminal} and repeat the application of Pope's method.

2. Besides the information given in the previous question, it is also known that the spawning takes place in a restricted period, around the beginning of the year. Research cruises using acoustic methods took place during the spawning period, in order to estimate the spawning biomass (kg/hour of trawl). The results obtained are shown in the following table :

Years	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
Spawning biomass Index	1270	1613	1629	1424	1300	1209	1000	718	476	326

The biological information collected during those cruises was also used to estimate the maturity ogive of the stock at the spawning period:

Age (years)	0	1	2	3	4	5	6	7	8
% Matures	0	1	20	50	80	100	100	100	100

- Calculate the spawning biomass in the spawning period of each year from 1985 to 1994 using the results of the Cohort Analyses obtained in question 1.b.
- Use the information of the acoustic cruises to tune the Cohort Analyses.
- Comment on the tuning results.

8.27 LENGTH COHORT ANALYSIS (LCA)

(7.9.2)

GROUP I

The following table presents the annual catch length composition, of a cohort of a resource with $L_{\infty} = 130$ cm and $K = 0.1 \text{ year}^{-1}$.

Length classes(cm)	Catch, C_t (million)
6-	1823
12-	14463
18-	25227
24-	8134
30-	3889
36-	2959
42-	1871
48-	653
54-	322
60-	228
66-	181
72-	96
78-	16
84-	0

The natural mortality coefficient was estimated as being $M = 0.3 \text{ year}^{-1}$.

1. Using the Pope method, and adopting $E=0.5$ as being the exploitation rate in the (78-) length class of the catch, estimate the number of survivors at the beginning of each length class, the fishing mortality coefficient F and the exploitation rate E in each class.
2. Calculate the mean number of survivors of the cohort.

GROUP II

The following tables 1 and 2 present the basic information on a hypothetical stock during the years 1985 to 1994.

1. Apply the slicing technique to the Catch matrix and comment on the validity of applying cohort analyses by ages.
2. Estimate the matrices $[F]$ and $[N]$ by length classes and years.
3. Calculate the matrix $[F_{sep}]$ and comment on the hypothesis that the exploitation pattern can be considered to be constant during those years.

Table 1. Growth parameters of the von-Bertalanffy curve, L_∞ and K Natural Mortality Coefficient, M and constants a and b of the weight/length relation

Growth		Natural Mortality		Weight/length relation $w_i = a \cdot (L_i)^b$	
L_∞ (cm)	42	M (year ⁻¹)	0.8	a	0.0023
K (year ⁻¹)	0.5			b	3

Table 2. Catch matrix in thousands of individuals, by length classes and years in the period 1985-94

Age (sliced)	Length classes (cm)	Years									
		1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
0	20-	35	41	30	17	49	69	34	61	46	29
	21-	338	400	292	167	472	662	327	593	442	276
	22-	805	952	699	400	1127	1575	777	1404	1053	657
	23-	1500	1766	1317	757	2108	2923	1436	2574	1962	1220
	24-	1901	2222	1702	985	2688	3678	1795	3175	2485	1535
	25-	2034	2357	1872	1093	2902	3900	1886	3276	2659	1627
	26-	1898	2175	1806	1067	2739	3600	1722	2925	2482	1502
1	27-	1951	1817	1228	1416	1445	2932	3376	1695	1785	2376
	28-	1664	1529	1091	1276	1250	2467	2801	1369	1523	1999
	29-	1382	1251	948	1125	1053	2018	2258	1071	1265	1636
	30-	1127	1003	812	980	873	1619	1782	818	1031	1312
	31-	900	787	684	841	710	1269	1372	607	823	1029
	32-	694	595	560	702	558	959	1017	432	635	778
2	33-	809	565	290	389	834	511	759	832	221	518
	34-	584	399	226	310	618	361	522	544	160	365
	35-	403	267	170	240	439	242	340	335	110	245
	36-	262	168	122	178	294	152	207	191	72	154
3	37-	165	168	66	71	175	214	93	128	75	46
	38-	86	84	40	45	96	107	44	55	39	23

Consider $L_a = 20$ cm and $t_a = 0$

(Extracted from : Cadima, E. & Palma, C., 1997. Cohort Analysis from annual length catch compositions. Working document presented to the Working Group of the Demersal Stocks Assessment of the South Shelf, held in Copenhagen from 1-10 September, 1997.)

8.28 EXAMINATION - WRITTEN TEST

TRAINING COURSE ON FISH STOCK ASSESSMENT INSTITUTO DE INVESTIGAÇÃO DAS PESCAS E DO MAR (LISBON, 3 NOV. -10 DEC. 1997)

QUESTION 1

Consider a certain stock with the following parameters:

Natural Mortality Coefficient: $M = 0.20 \text{ year}^{-1}$

Fishing mortality in 1996: $F_{96} = 1.08 \text{ year}^{-1}$

Exploitation pattern :

Age	1	2	3	4	5	6	7+
s_I	0.07	0.23	0.33	0.49	0.97	1.00	1.00

Mean weight (kg) in the catch and in the stock:

Age	1	2	3	4	5	6	7+
\bar{W}_I	0.053	0.076	0.111	0.125	0.158	0.204	0.337

Maturity ogive (percent):

Age (year)	1	2	3	4	5	6	7+
% matures _i	34	90	100	100	100	100	100

The estimated recruitments and the spawning biomasses between 1986 and 1996 are presented in the following table :

Table 1

Year	N (age 1) (thousands)	Spawning biomass (t)
1986	8751	1957
1987	8305	1591
1988	7123	1956
1989	7596	2073
1990	6013	2287
1991	5054	1506
1992	9713	1400
1993	5520	1275
1994	6000	980
1995	7329	675
1996	6840	917

1. Calculate the biological reference point $F_{0.1}$. Indicate the value of the virgin biomass that you estimated and calculate the percentage $B_{0.1}/B_{\text{virgin}}$.
2. Calculate the biological reference point F_{max} .
3. Calculate the biological reference point F_{med} .
4. Estimate the parameters of the S-R Ricker model and indicate what the value of the spawning biomass/recruit is, corresponding to F_{crash} .
5. Comment on the present state of the stock and its exploitation.

QUESTION 2

Consider a stock where the length compositions of the catches during 1986-1995 are known (Table 2).

The following parameters were estimated from this stock :

Natural mortality coefficient = 1.2 year^{-1}

Asymptotic length = 39.8 cm

Growth coefficient = 0.8 year^{-1}

Table 2 – Catch Matrix (thousand individuals)

Length classes (cm)	Year									
	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
14-	10	9	8	9	7	18	13	12	15	8
15-	20	17	15	18	13	35	26	23	29	16
16-	29	25	23	26	19	52	39	34	42	24
17-	47	41	37	43	31	84	63	56	69	39
18-	92	80	72	83	60	164	122	108	134	76
19-	224	194	175	202	146	398	297	263	325	185
20-	261	226	203	234	169	461	343	305	376	214
21-	420	363	326	376	271	736	547	488	601	342
22-	335	525	603	506	511	365	983	571	657	741
23-	345	540	618	516	520	370	995	581	666	752
24-	422	661	751	622	625	444	1189	699	799	902
25-	442	693	781	642	643	454	1212	718	818	923
26-	415	650	726	592	590	415	1102	659	747	843
27-	388	607	672	542	537	376	995	601	677	765
28-	360	564	617	493	486	339	890	543	609	688
29-	332	520	563	444	435	301	788	487	543	613
30-	304	475	508	396	386	265	689	431	478	539
31-	275	430	453	348	337	230	594	377	414	468
32-	272	246	509	494	330	314	212	428	351	353
33-	239	216	439	419	277	261	174	359	292	294
34-	206	186	369	345	226	211	139	293	235	237
35-	171	155	300	273	176	162	106	228	181	182
36-	136	123	230	202	129	117	75	167	130	131
37-	82	99	116	196	147	91	80	40	117	83
38-	49	60	66	103	75	45	38	21	58	41
39-	0	0	0	0	0	0	0	0	0	0

1. Estimate the values of the fishing mortality coefficient in each length class of the cohort of 1987 (to simplify, adopt $F_{\text{terminal}} = 0.5 \text{ year}^{-1}$).
2. Estimate the corresponding values of the number of survivors at the beginning of each class.
3. Say what the recruitment of this cohort is.

QUESTION 3

The Fox Production model was adjusted to a certain stock and the following parameters were obtained :

$$\begin{aligned} K &= 300 \text{ thousand tons} \\ r &= 0.50 \text{ year}^{-1} \end{aligned}$$

1. Determine the biological reference points F_{MSY} and $F_{0.1}$.
2. Knowing that in recent years the biomass of this stock is about 30 percent of the virgin biomass, comment on the present state of the stock and its exploitation, based on the adopted model.