Qualities in Possible Worlds

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Goal

Analyze how and under which assumptions it is possible to compare entities *living* in different worlds.

- ▷ The comparison is given with respect to a set of properties of objects like color, length, mass, shape, etc., called *quality kinds*.
- ▷ Quality kinds are builded from objects by means of a *local* (in a single world) *abstraction process* and they can have a local structure.
- ▷ **Problem**: how to tune (synchronize) quality kinds that has been locally builded in different worlds?
- ▷ **Motivation**: considering worlds as *contexts* or *information systems*, our framework provides a basic methodology to formally link quality systems that are localized in different information systems.

Scenario

- Let us suppose we have different satellites that make color measurement of some pieces of land.
  - Different kinds of satellites can have different color measurement systems.
  - The satellites can collect the data relative to one piece of land at different times.
- We want to motivate the assumption that what a satellite measures as, say, red at a certain time, corresponds to what another satellite (or the same satellite at a different time) measures as red.
- Our goal is to *tune* different measurements considering additional informations we have on the conditions these measurements have been made.

Substantiavalism vs. Relationism

- **Substantiavalism**: time is a container-like manifold and what happens occupies it *contingently*.
- **Relationism**: time is derived from relationships between events.
- Analogously for space: space as a container vs. space as a conceptual construction.

Parallelism with theories of properties

- **Substantiavalism and Universalism**.
  - Properties (called *universals*) are primitive and independent from their instances (*particulars*).
- **Relationism and Trope theory/Resemblance Nominalism**.
  - (*Trope theory*) Properties are classes of *exactly resembling* tropes.
  - (*Resemblance Nominalism*) Properties are classes of *resembling* objects.

Our setting (1/2)

Our goal is to compare objects in a relationist setting.

- ▷ In the case of time and space, tropes are not considered
  but
  the relations allowing to construct time from events are different from the relations used to construct space from physical objects.
- ▷ Resemblance nominalism admits just one resemblance relation
  but
  it has problems to differentiate co-extensional properties.

Our setting (2/2)

We begin with a system:

$\langle D, \equiv^1, \ldots, \equiv^n \rangle$, where $\equiv^i$ are resemblance relations on D,

which allows us to overcome the problems in resemblance nominalism, and to adopt a methodology similar to that of time/space construction.

- It is stronger than resemblance nominalism because of the presence of n different resemblance relations.
- It is weaker than trope theory because tropes cannot be reconstructed in it **but** tropes theorists can rephrase our formalization adopting:

$\langle D, T^1, \ldots, T^n, \mathsf{i}, \equiv \rangle$, where the T^j are disjoints sets of tropes, and i is the inherence relation

$$x \equiv^j y \;\; \textit{iff} \;\; \exists t, s \in T^j \, (\mathsf{i}(t, x) \wedge \mathsf{i}(s, y) \wedge t \equiv s)$$

Abstraction process

▷ $\mathcal{S} = \langle D, \equiv \rangle$ is a generic structure with one *equivalence* relation.

▷ $\mathcal{S}^e = \langle D^e, =^e \rangle$ is the *abstraction* of $\mathcal{S}$, where

- D^e is the set of (non-empty) equivalence classes of D;
- $=^e$ is the equality on D^e.

▷ Examples:

- different (punctual) events can be temporally co-localized from $\mathcal{E} = \langle E, \equiv_E \rangle$, E set of *events*, $\equiv_E$ *temporal coincidence* to $\mathcal{T} = \langle T, =^e \rangle$, T set of *times*.
- different objects can have the same color from $\mathcal{O} = \langle O, \equiv^c \rangle$, O set of *objects*, $\equiv^c$ *color resemblance* to $\mathcal{C} = \langle C, =_C \rangle$, C set of *color properties*.

Structuring (1/4)

- *Structural constraints* are normally introduced in modeling time (and space), e.g. a precedence relation can force time to be linear or branching, a congruence relation can constrain the metric, etc.
- These structural constraints are not uncommon for *quality kinds* like color, length, volume, shape, mass, etc. For example, a RGB structure can be assumed for colors, and weights are usually linearly arranged.
- Different quality kinds have different structures, therefore, in general, we will apply structural constraints separately for each quality kind.

Structuring (2/4)

- $\mathcal{S}' = \langle D, \equiv, R \rangle$ is the extension of $\mathcal{S}$ with the structuring relation R.
- $\mathcal{S}'^e = \langle D^e, =^e, R^e \rangle$ is the abstraction of $\mathcal{S}'$ where
  - $x^e \, R^e \, y^e$ *iff* $\exists a \in x^e, b \in y^e (a \, R \, b)$.
- In general, it is possible to have different structuring relations relative to the same abstraction process; for example:
  - $\mathcal{E}_{cg} = \langle E, \equiv_E, \lhd_E, \preceq_E \rangle$ is the event structure augmented with the *precedence relation* $\lhd_E$ and the (quaternary) *congruence relation* $\preceq_E$ among (punctual) events;
  - $\mathcal{T}_{cg} = \langle T, =^e, \lhd_T, \preceq_T \rangle$ is the associated abstraction, where: $\lhd_T$ is the abstraction of $\lhd_E$ while $\preceq_T$ is the abstraction of $\preceq_E$.

Structuring (3/4)

- ▷ The definition of R^e in terms of R:
  - $x^e \, R^e \, y^e$ *iff* $\exists a \in x^e, b \in y^e (a \, R \, b)$.

  is compatible with: $c \equiv a$, $d \equiv b$, $a \, R \, b$, $\neg(c \, R \, d)$, and $c^e \, R^e \, d^e$.
- ▷ Using an universal quantifier, the same problem arises considering 'negative' statements.
- ▷ It is possible to constrain 'homogeneity' by means of:
  - $x \equiv y \rightarrow \forall z (z \, R \, x \leftrightarrow z \, R \, y)$.

Structuring (4/4)

- ▹ Technically, structuring relations can be introduced in the *abstraction* structure. R can then be defined in terms of R^e:
  - $a\,R\,b$ *iff* there exist $x^e, y^e \in D^e(x^e\,R^e\,y^e$ and $a \in x^e, b \in y^e)$
- ▹ Philosophically, the introduction of structuring relations in the starting structure or in its abstraction, can reveal an objective/ontological vs. subjective/epistemological attitude towards these relations.
- ▹ To have a direct parallelism with the construction of time, we introduce all the structuring relations in the starting structure. This does not prevent us from considering them as “ontological” or as “epistemological” relations.

Extending our setting

- To each $\equiv^i$ we associate a set of structuring relations, obtaining:

  $$\langle D, \equiv^1, \ldots, \equiv^n, R^1_1, \ldots, R^1_{m_1}, \ldots, R^n_1, \ldots, R^n_{m_n} \rangle$$

  that allows for the comparison of entities in the same world.

- To compare entities 'living' in different worlds, we need to extend the formalism with:
  - a set of possible worlds W;
  - the relation $a{\downarrow}_w$ standing for "$a \in D$ is in the world w":

  obtaining:

  $$\langle D, W, \downarrow, \equiv^1, \ldots, \equiv^n, R^1_1, \ldots, R^1_{m_1}, \ldots, R^1_1, \ldots, R^n_{m_n} \rangle$$

A classical puzzle

Let us assume that:

1. $\equiv^i$ is independent from $\downarrow$, in particular it is a cross-world relationship;
2. entities can change, with respect to the quality kind i, through worlds.

For example, let us assume a persistent entity a (an entity that is in two different worlds, i.e. $\exists w, w'(a{\downarrow_w} \wedge a{\downarrow_{w'}} \wedge w \neq w')$), that is red in w and yellow in w'.

- ▹ We get a contradiction if we include a in the class of the red entities as well as if we put it in the class of the yellow ones.

Each view has its solution

- **Lewis & stage theory**. Entities are world bounded and modality is interpreted by means of the *counterpart* (C) relation: $a\downarrow_w \wedge a'\downarrow_{w'} \wedge \mathsf{C}(a', a)$ and a is red while a' is yellow.
- **Perdurantism**. An entity a has different *world stages* $a_{/w}$ in each world w to which it belongs: "a is red *at* w", because it has a world stage $a_{/w}$ that it is red. Analogously for "a is yellow *at* w'".
- **Endurantism**. Cross-world change requires the introduction of a world argument in the properties: a is not red in general, it can be red relatively to a world which must be specified.
  Criticism: *de facto* negation of *intrinsic properties*, all the properties become relations with the worlds.

Our approach (driven by the info systems' scenario)

▷ Equivalence classes of resembling entities are *localized* in single worlds, i.e. a world argument is added to resemblances:

- $a \equiv_w^i b$ stands for "*a* *i*-resembles *b* in the world *w*".

⇒ **Weak endurantism**: we only know the classes of objects that, in a given world, are indistinguishable with respect to one quality kind, but we don't have any cross-world relation between these equivalence classes (called *qualities*) .

- Given the class of red objects in one world, one has no way to infer which is the red class in a different world.

- We are interested in understanding whether and on which assumptions an equivalence at the level of qualities in the two worlds can be established without additional primitives.

Gathering ideas from the construction of time in branching-worlds

- **Forbes**: in each world, times and relations on times are abstracted from (*i*) the set of punctual events and, (*ii*) the *coincidence* ($\equiv_E$), *precedence* ($\lhd_E$), and *distance* (d_E) relations.
- *Branching-worlds* share an initial segment of their course of history, i.e. they share at least two (punctual) *times* that fix a common *origin* and *unit* of measure allowing for the definition of a unique d_T on times in branching worlds.
- A correspondence between localized times in *different* branching-worlds can be established in the following way:
  - $t_1 \equiv_T t_2$ *iff* $\mathsf{d}_T(t_1, t) = \mathsf{d}_T(t_2, t) \wedge t \lhd_T t_1 \wedge t \lhd_T t_2$.

Tuning systems

Tuning systems: aligning, finding correspondences between, **qualities** (i.e. equivalence classes of objects) in different worlds.

- Following Forbes, one should assume the existence of objects that, with respect to the quality kind considered, are invariant across (branching) worlds (the shared segment).
  By means of these invariant objects, correspondences between equivalence classes can be established.
- Our goal is to extend (and weaken) this notion of ‘shared segment’ to general worlds (that is, to worlds where a branching relationship is not defined) to make it applicable to objects and qualities.

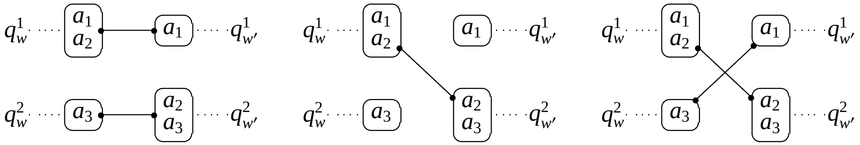
Basic structure

To illustrate the tuning mechanism, we just consider:

- two worlds w and w' and one (world indexed) resemblance relation $\equiv_w$, s.t. $x \equiv_w y \rightarrow (x{\downarrow_w} \wedge y{\downarrow_w})$;
- a *correspondence* relation ($\multimap\!\circ$) between qualities:
  $q_w \circ\!\!-\!\!\circ q_{w'}$ stands for "quality q_w (an equivalence class in w) corresponds to quality $q_{w'}$ (an equivalence class in w')";
- the set of *common facts* in w and w':
  $F_{w,w'} = \varepsilon(\equiv_w) \cap \varepsilon(\equiv_{w'})$, where $\varepsilon(\equiv_w) = \{\langle x, y\rangle \mid x \equiv_w y\}$ is the *extension* of $\equiv_w$.

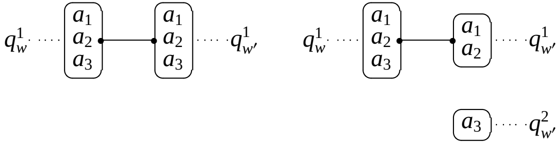
Then we will add a localized precedence relations $\lhd_w$ and the consequent extension of $F_{w,w'}$ that includes the common facts relative to $\lhd_w$.

Example with only trivial common facts



(a) (b) (c)

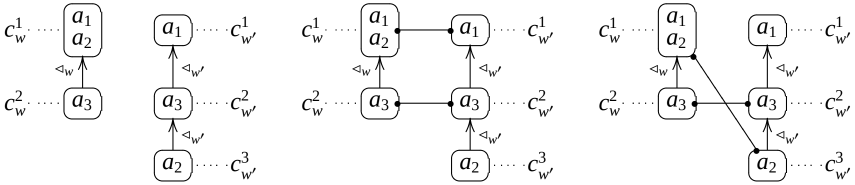
Example with non trivial common facts



(a) (b)

- **Minimal object change hypothesis** (mOCH): the systems is tuned forcing the minimal number of changes in the objects.
- **Example**. Two satellites that make color measurement of the same piece of land at the same time.

Adding local structuring precedences



(*a*) (*b*) (*c*)

- ▷ **Minimal structural change hypothesis** (mSCH): the system is tuned forcing the maximal structural similarity in different worlds.
- ▷ **Example**. Satellites of the same kind (or the same satellite) make color measurement at different times (or of different pieces of land).
- Both (*b*) and (*c*) satisfy (mOCH) but only (*b*) satisfies (mSCH).

Cross-worlds equivalence relations

- Once a correspondence between qualites in w and qualities in w' has been reached, a cross-world equivalence relation between objects can be defined:
  - $x \equiv_{w,w'} y \triangleq x \in q_w \wedge y \in q_{w'} \wedge q_w \multimapboth q_{w'}$
- Structural relations allow for finding additional correspondences between qualities:

  IF $q^1_w \multimapboth q^1_{w'} \wedge q^3_w \multimapboth q^3_{w'} \wedge q^1_w \lhd_w q^2_w \lhd_w q^3_w \wedge q^1_{w'} \lhd_{w'} q^2_{w'} \lhd_{w'} q^3_{w'}$

  AND assuming the same number of qualities in different worlds and that there is only one quality between classes q^1_w and q^3_w

  THEN the correspondence $q^2_w \multimapboth q^2_{w'}$ can be established.