Founding properties on measurement

Claudio Masolo
Laboratory for Applied Ontology, ISTC-CNR
masolo@loa-cnr.it

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1 Predication

- $\mathsf{1m}(x) \wedge \mathsf{2m}(y) \rightarrow x <_{\mathsf{L}} y$
- $\mathsf{Red}(x) \wedge \mathsf{Orng}(y) \rightarrow x \sim_{\mathsf{C}} y$
- $\mathsf{Red}(x) \wedge \mathsf{Orng}(y) \wedge \mathsf{Blue}(z) \rightarrow \mathsf{Closer}_{\mathsf{C}}(x, y, z)$

2 Reification

- $\mathsf{Length}(x, \mathsf{1m}) \wedge \mathsf{Length}(y, \mathsf{2m}) \wedge \mathsf{1m} < \mathsf{2m}$

  $$\mathsf{1m}(x) \triangleq \mathsf{Length}(x, \mathsf{1m})$$

  $$x <_{\mathsf{L}} y \triangleq \exists l_1 l_2(\mathsf{Length}(x, l_1) \wedge \mathsf{Length}(y, l_2) \wedge l_1 < l_2)$$

- $\mathsf{Color}(x, \mathsf{red}) \wedge \mathsf{Color}(y, \mathsf{orng}) \wedge \mathsf{red} \sim \mathsf{orng}$

  $$\mathsf{Red}(x) \triangleq \mathsf{Color}(x, \mathsf{red})$$

  $$x \sim_{\mathsf{C}} y \triangleq \exists c_1 c_2(\mathsf{Color}(x, c_1) \wedge \mathsf{Color}(y, c_2) \wedge c_1 \sim c_2)$$

- Not necessarily extensional: the set of objects that have lenght 1m could be identical to the set of objects that have color red.

3 A general framework

- $\mathbf{ob}(x)$: x is an object;
- $\mathbf{tm}(t)$: t is a time;
- $\mathbf{sp_i}(r)$: r is an region in the space $\mathbf{i}$;
- $\mathsf{L}(r, x, t)$: the region r is the **location** of the object x at time t.

- $\mathsf{Length}(x, \mathsf{1m}, t) \triangleq \mathbf{sp_L}(\mathsf{1m}) \wedge \mathsf{L}(\mathsf{1m}, x, t)$
- $\mathsf{Color}(x, \mathsf{red}, t) \triangleq \mathbf{sp_C}(\mathsf{red}) \wedge \mathsf{L}(\mathsf{red}, x, t)$
- $x \sim_{\mathsf{C},t} y \triangleq \exists c_1 c_2 (\mathbf{sp_C}(c_1) \wedge \mathbf{sp_C}(c_2) \wedge \mathsf{L}(c_1, x, t) \wedge \mathsf{L}(c_2, y, t) \wedge c_1 \sim c_2)$

4 Ontological neutrality

- This general framework is quite weakly characterized therefore it is compatible with different theories of properties:
  - regions as *universals*; location as *instance of* (**Universalim**);
  - regions as *classes of resembling tropes*; location as a composition of *inherence and membership* (**Trope Theory**);
  - regions as *classes of resembling objects*; location as *membership* (**Resemblance Nominalism**);

5 Towards a more empirical approach

- Is it possible to provide a more empirical or epistemic interpretation of this general framework?
- Is it possible to classify and compare objects in a **communicable** and **inter-subjective** way (allowing for useful **predictions**) without making powerful assumptions about their conformity with 'ontological properties'?

6 Giving a central role to measurement

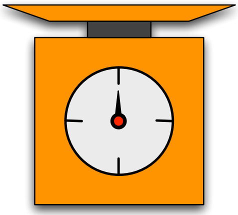
- **Hypothesis**: an object is classified as '1m long' if and only if the result of its length measurement is **1m**.
- Roughly:
  - spaces are related to measurement instruments;
  - regions in a space correspond to the values of a measurement instrument related to this space;
  - the location relation corresponds to the result of the measurement of an object by means of this instrument.

7 Towards an empirical theory of measurement

- *Representational Measurement Theory*, RMT (Suppes, Krantz, Luce, and Tversky), is one of the best known measurement theories.
- *Empirical Measurement Theory*, EMT (Frigerio, Giordani, and Mari) explicitly considers the epistemic/empirical aspect of measurement.
- I extend EMT by
  - providing a formal account of the *measurement standards* and of the *calibration* process and
  - considering time and *diachronic* comparisons.

(**!**) Later, I will motivate why I prefer EMT to RMT.

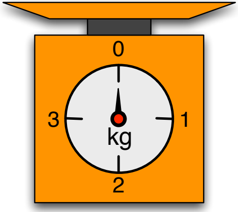
8 Measurement system: support



- m is the (physical) **support**
  - m is the scale in this case;
- $\mathcal{E} = \langle U, R_1, \ldots, R_n \rangle$ is the **empirical structure**: the set of empirically discernible internal states of m (after any possible interaction with an object) and the relations between them
  - U is the set of 4 states $\{s_0, s_1, s_2, s_3\}$ that correspond to any alignment between the indicator and one notch (discrete scale);
  - R is the order established (in U) by the clockwise order of notches:

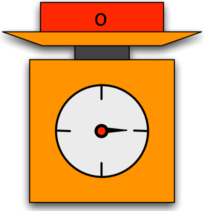
    $$s_0 \prec s_1 \prec s_2 \prec s_3$$

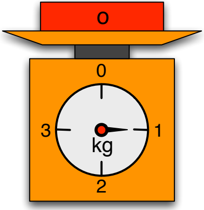
9 Measurement system: symbolization



- $\mathcal{S} = \langle V, S_1, \ldots, S_n \rangle$ is the **symbolic structure** necessary for abstracting from and refer to the internal states of the support m
  - $V = \{0\text{kg}, 1\text{kg}, 2\text{kg}, 3\text{kg}\}$
  - S: $0\text{kg} < 1\text{kg} < 2\text{kg} < 3\text{kg}$
- λ: $U \to V$ is the **symbolization function**
  - $\lambda(s_n) = n\text{kg}$
  - $n\text{kg} < m\text{kg}$ iff $s_n \prec s_m$

10 Measurement system: interaction





- κ: $O \to U$ is the **interaction function** that associates to an object $o \in O$ the internal state of the complex system $m \bullet o$
  - $\kappa(o) = s_1$, then
  - $\lambda(\kappa(o)) = 1\text{kg}$

it describes as the support interacts with the environment.

11 RMT vs. EMT

- RMT conceives measurement as the building of a homomorphism from an *empirical structure* $\mathcal{O} = \langle O, R_1^O, \ldots, R_n^O \rangle$ to a *numerical* structure $\mathcal{S} = \langle V, S_1, \ldots, S_n \rangle$.

- In EMT, it is the structure of the support that *induces* (via an interaction process) a structure on objects:
  - U gives the *resolution* of the MS

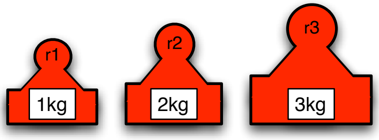
    $o \approx o'$ iff $\kappa(o) = \kappa(o')$
  - each R_i induces a relation on objects

    $R_i^O(o_1, \ldots, o_n)$ iff $R_i(\kappa(o_1), \ldots, \kappa(o_n))$

  i.e. an MS (and the measurement procedure) provides a specific 'point of view' on reality.

12 Measurement standard (mST)

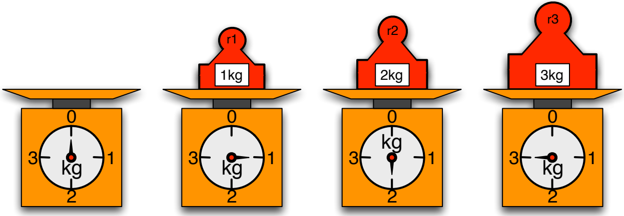
- a set R of *reference objects*: $\{r_0, r_1, r_2, r_3\}$;
  (in the example we have the problem of the 'null object' r_0)
- a *symbolic structure* $\mathcal{R} = \langle M, S_1^M, \ldots, S_n^M \rangle$;
  - $M = \{0\text{kg}, 1\text{kg}, 2\text{kg}, 3\text{kg}\}$;
  - $0\text{kg} < 1\text{kg} < 2\text{kg} < 3\text{kg}$;
- α: $R \to M$ is a one-to-one function that conventionally assigns to each object in R a symbol in M: $\alpha(r_n) = n\text{kg}$



13 Calibration

MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda \rangle$ is calibrated w.r.t mST $\langle R, \mathcal{R}, \alpha \rangle$ iff:

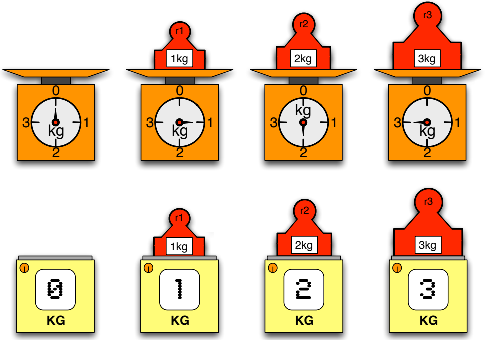
- $\mathcal{S} = \mathcal{R}$ (or more generally, there is a one-to-one relation between $\mathcal{S}$ and $\mathcal{R}$, i.e. the MS resolves the reference objects of the mST);
- for each $r, r_1, \ldots, r_n \in R$
  - $\lambda(\kappa(r)) = \alpha(r)$ and
  - $S_i(\lambda(\kappa(r_1)), \ldots, \lambda(\kappa(r_n)))$ iff $S_i^M(\alpha(r_1), \ldots, \alpha(r_n))$



14 Measurement framework

- A **measurement framework** is a couple $\langle \mathsf{s}, M^* \rangle$ where s is an mST, and M^* is a set of MSs calibrated with respect to s.

▶ Abstract from the physical implementation/relatization of the MSs

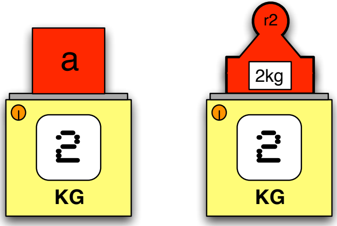


15 a has P

- Given an mST s with symbolic structure $\langle M, S_1^M, \ldots, S_n^M \rangle$, it is possible to **associate** to each $s_p \in M$ a property P:

  'a has P' if and only if there exists an MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda \rangle$ calibrated with respect to s such that $\lambda(\kappa(a)) = s_p$

- e.g. a has the property of 'being 2kg heavy' iff $\lambda(\kappa(a)) = 2$kg:

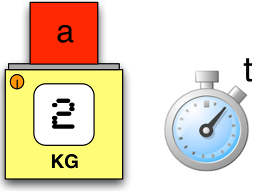


16 a has P at t

- Given an mST s with symbolic structure $\langle M, S_1^M, \ldots, S_n^M \rangle$, it is possible to **associate** to each $s_p \in M$ a property P:

  'a has P at t' if and only if there exists an MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda \rangle$ calibrated (at t) w.r.t. s such that $[t](\lambda(\kappa(a)) = s_p)$ (that represents the fact that m and a interacted at t with the result s_p).

- at t a has the property of 'being 2kg heavy' iff $[t](\lambda(\kappa(a)) = 2\text{kg})$:



17 Measurement structure

A measurement structure, is a structure $\langle O, T, S, F \rangle$ where:

- O is a set of ‘objects’
- T is a set of ‘times’
- S is a set of ‘symbols’
- F is a set of *measurement frameworks*

18 The general framework in terms of MSs

Given the measurement structure $\langle O, T, S, F \rangle$:

Objects	$\mathbf{ob}^{\mathcal{I}} \subseteq O$
Times	$\mathbf{tm}^{\mathcal{I}} \subseteq T$
Regions of space i	$\mathbf{sp_i}^{\mathcal{I}} \subseteq M_i$ (the set of symbols of the mST s_i in an MF of F)
Location	$\mathsf{L}^{\mathcal{I}} \subseteq S \times O \times T$ and $\langle r, o, t \rangle \in \mathsf{L}^{\mathcal{I}}$ iff there exists an MS $\langle m, \mathcal{E}, \kappa, \mathcal{S}, \lambda \rangle$ belonging to some M_i^* (in one measurement framework) such that $[t](\lambda(\kappa(o)) = r)$

19 Classification as measurement

- $\mathsf{EX}(o, t') \land \mathsf{L}(r, o, t) \land \mathbf{sp}_i(r) \rightarrow \exists r'(\mathsf{L}(r', o, t') \land \mathbf{sp}_i(r'))$
  if, at a given time t, an object o is located in a specific space $\mathbf{sp}_i$, then it is located in $\mathbf{sp}_i$ at every time at which o exists.

- Seems ontologically but not empirically plausible: the fact that o has been measured at t does not imply that o has been measured (w.r.t. the same dimension $\mathbf{sp}_i$) at every time at which it exists.

- Objects can be classified and compared only by measuring them.

- It is possible to introduce a potential aspect, i.e. if measured an object would produce a specific result, but this seems to require the difficult notion of *disposition*.

20 Measurement and realism

- Objects that interact with the support providing the same result ($\kappa(o) = \kappa(o')$) can, but do not necessarily need to, share an *ontological/physical* property.
  - In particular an MS with a coarse resolution is unable to distinguish some ontological properties.

- On the other hand, the states induced in the MS depend on the ontological properties of the objects.
  - MSs are **builded** because the classifications and the comparisons they provide allow us for (environmentally useful) **predictions**.

- Calibration and symbolization assure *inter-subjectivity* and *communicability*.

21 Change of mSTs and MSs

- **mSTs can change across time**
  A property is associated to a symbol of an mST that identifies a reference object. The diachronic alignment of the MSs relies on the calibration, at different times, w.r.t. **the same** mST. The change of reference objects of an mST invalidates the alignment.

- **MSs can change across time**
  Interaction and symbolization functions depend on the structure of the support m that can change across time. By (diachronically) calibrating an MS m w.r.t a stable mST s one assures the stability of m. Even assuming instantaneous measurement: (i) MSs are not re-calibrated every time they are used, and (ii) calibration and measurement cannot be synchronous.

22 Stable frameworks of objects

- If mSTs and MSs are **assumed** to be stable (at least from the calibration to the measurement), the state of $m \bullet a$ and the one of $m \bullet b$ depend exclusively on how a and b are.
- Only by assuming the stability of a framework of objects (mST and MSs) one can conclude that a and b *share* a property, that a *similarity* between them exists.
- Instead of re-identifying objects on the basis of a stable framework of properties, here we are 're-identifying properties' on the basis of a stable framework of objects.

23 Infinite regression

- **But** to empirically justify the stability of mSTs and MSs one needs to diachronically compare the supports and reference objects.
- To do that other mSTs and MSs, the stability of which, in turn, needs to be justified.

  Infinite regression! (or circularity)
- One can consider the *global framework* of all mSTs and MSs, the stability of which is determined on the basis of the **mutual relationships** between the components.
- This does not detect absolute change that maintain the mutual relationships.

24 Sensory systems

- Is it possible to establish a parallel between sensory systems and measurement systems?
- Is it possible to provide a more sensory oriented interpretation of the previous general framework re-using what we have done for measurement systems?
- How distal stimuli are compared or classed together? Is it still possible to avoid powerful assumptions about their conformity with external kinds/ontological properties?

25 Matthen's sensory classification thesis

- (I.a) Sensory systems classify and categorize; they sort and assign distal stimuli to classes [different from Sense Datum Theory].
- (I.b) Ideally they do so on some consistent basis.
- (I.c) The result of this activity have a lasting effect on the perceiver in the form of conscious memories (...).
- (II) A *sense-feature* is a property a stimulus appears to have by virtue of an act of sensory classification.
  - *Colours* are the properties **distal stimuli** appear to have when colour vision assigns them to classes.
  - Sensory systems create classificatory categories for the use of the organism of which they are a part.

26 Matthen's 3 stages sensory process (a)

- **Stimuli**: material objects and the packets of energy that they send to our sensory receptors.
- **Sensory classes**: the groups that the system makes of the stimuli, and sense-features, the properties that stimuli in a given sensory class share in virtue of belonging to that class.
  - Sensory classes can group physically different stimuli and their structure can mismatch the physical structure of stimuli: classification is useful for the acting and survival of species.
- **Sensations** (phenomenal or sensory experiences): the consciously available record of sensory classification, a label that identifies a distal stimulus as belonging to a particular class.
  - Classification is available not only to consciousness.

27 Matthen's 3 stages sensory process (b)

- The function of sensory experience is to provide us *access* to sensory classification for purposes of reasoning (and beliefs' formation).
- Through sensations, we come to know of distal objects that have been classified a certain way (awareness of external objects).
- Discriminability/indiscriminability of stimuli in sensory experience of them is a consequence of how sensory systems classify them.
- Sensation is how we come to know that our sensory systems have assigned a stimulus to a particular class.
- Sensation is the indicator, not the constitutive characteristic, of sensory classification.
- **First classification, then sensory appearance as label**.

28 Dretske: phenomenal vs. conceptual awareness

- Sensory experiences are different from knowledge, beliefs, judgments, etc.: *a child or an animal might be visually aware of the shirt's color without their knowing or thinking that the shirt is blue, without being conscious that anything is blue*.

- Two speedometers that have the same 'experience' (viz. of an axle rotation of N rpm) could give rise to different 'beliefs' (about speed, because the diameter of the connected wheels differs).

- Through learning, I can change what I believe when I see k, but I can't much change the way k looks (phenomenally) to me (...) We can, through learning, change our calibration.

▶ Experiences are states whose repr. properties are *systemic*, thoughts are states whose repr. properties are *acquired*.

29 The problem of objects

- When I have a red experience, do I have an experience of an object classified as red, or do I have the experience without any specific subject?
- Measurement theories assume a set of objects and the interaction function. What happens if I have access only to the internal states of the instrument (I don't see/know what I'm measuring)?
- Is this parallel correct?

- Link with *foci of attention*.
- Link with Pylyshyn's theory.

30 Auto-calibration

How the symbolization function can be stablished?

- Suppose to find an instrument without any symbol on it.
- Suppose to know how the instrument can interact with the environment and to discern its internal states.
- Suppose to write symbols in correspondence of internal states.
- Then, assuming the stability of the instrument, one can **compare** and **classify** objects.
- Without undertanding what she is measuring, she can observe that objects of kind A are 'good' while objects of kind B are 'bad'.
- Then, one starts to do some predictions on the environment.